International Electronic Journal of Mathematics Education
2025, 20(2), em0824
e-ISSN: 1306-3030
https://www.iejme.com

MODESTUM

Research Article **OPEN ACCESS**

The advantage of using GeoGebra in the understanding of vectors and comparison with the classical method

Erëmirë R. Aliu [1], Teuta Jusufi Zenku [1], Egzona Iseni [1], Shpetim Rexhepi [1*]

[1] Mother Teresa University, Skopje, NORTH MACEDONIA
***Corresponding Author:** shpetim.rexhepi@unt.edu.mk

Citation: Aliu, F., Jusufi Zenku, T., Iseni, E., & Rexhepi, S. (2025). The advantage of using GeoGebra in the understanding of vectors and comparison with the classical method. *International Electronic Journal of Mathematics Education, 20*(2), em0824. https://doi.org/10.29333/iejme/16007

ARTICLE INFO

Received: 21 Aug 2024

Accepted: 30 Jan 2025

ABSTRACT

This study explores the effectiveness of GeoGebra in enhancing students' understanding of vectors in mathematics education. The research compares traditional teaching methods with the application of GeoGebra to determine its impact on conceptual and procedural knowledge acquisition. Vectors are often challenging for students to grasp due to their abstract nature and frequent use in physics rather than mathematics itself. This study investigates how visualization and interactivity through GeoGebra can facilitate comprehension. The research was conducted in two primary schools, where one group of ninth-grade students was taught vectors using the classical approach, while another group utilized GeoGebra. Findings indicate that students who learned with GeoGebra demonstrated a better conceptual and procedural understanding of vectors, improved problem-solving abilities, and higher engagement levels compared to those taught with conventional methods. The study also highlights challenges such as the availability of technology, teacher training, and implementation scalability, which can impact the effectiveness of digital tools in education. The results suggest that integrating GeoGebra into mathematics instruction can significantly enhance students' learning experiences. However, broader accessibility and teacher training are essential for its successful implementation. Future research should explore the long-term impact of digital learning tools in mathematics education and their application across different educational contexts.

Keywords: mathematics, difficulties, vectors, Geogebra

INTRODUCTION

Mathematics is often seen as a particular subject, whether by students, parents, or even by mathematicians themselves. However, mathematics as a teaching subject is basic and considered the main subject because it is related to many other sciences. The connection of mathematics as the queen of natural sciences with other subjects was made much more apparent by the explanation of vectors in mathematics. Vectors are one of the mathematical concepts taught in mathematics in detail and their application is in many fields, especially in the subject of physics. In physics, through vectors, students model forces, velocity, acceleration, etc. The concept of vector was one of the concepts that was difficult for students to understand (Widada et al., 2020).

Many studies have concluded that students who knew simple vector mathematics also had exceptional skills in basic physics. Vectors play a prominent role in many branches of physics (Bollen et al., 2017). Some previous studies have shown that students' difficulties in learning physics are mainly caused by a lack of skills and understanding of mathematics, especially vectors. A lack of understanding of vectors can cause many serious problems when vector concepts are embedded in almost all physics concepts. Therefore, to facilitate this difficulty in understanding vectors through this study, we compared the classical method of explanation and the application of GeoGebra during classes when vectors are explained, and we conducted studies through tests and interviews to see how GeoGebra affects the understanding of vectors and does it have a positive impact in this direction. So, this study aims to investigate how much effect the use of GeoGebra has on the understanding of vectors in general.

LITERATURE REVIEW

Understanding vectors has been a persistent challenge for many students, as evidenced by several studies found that most students use geometric definitions, which are often inadequate and do not align with the common mathematical definition of vectors. This suggests that fully grasping the concept of vectors is difficult for students (Tobias et al., 2018).

Similarly, the study conducted by Pavlakos et al. (2001) indicates that misconceptions about vector addition often arise because students, accustomed to manipulating real numbers, fail to recognize that vectors are different mathematical objects with unique operations for addition and multiplication.

On the other hand, the study by Wutchana et al. (2015) shows that students often understand that vectors have both magnitude and direction and that when adding two vectors, one vector's tail must be attached to the other's tip. However, while students demonstrate a good understanding of simpler concepts such as scalar multiplication and calculating magnitudes, interpreting dot and cross products remains challenging (Sabah, 2023).

This study highlights the benefits of integrating technology into mathematics education. Digital tools, specifically GeoGebra, enhance student engagement and comprehension (Tobias et al., 2018). The interactive nature of such tools provides students with a more effective approach to learning mathematical concepts.

Students using GeoGebra demonstrated better conceptual and procedural understanding of vectors compared to those who followed traditional instructional methods. The ability to visualize vector operations helped students grasp concepts more effectively, leading to higher test performance (Pavlakos et al., 2001). Access to technology played a significant role, with students in urban schools benefiting more due to better resources. (Tobias et al., 2018).

Previous research has emphasized the importance of visualization and interactive learning in mathematics education. Studies have shown that students struggle with abstract concepts like vectors due to a lack of direct physical representation (Pavlakos et al., 2001). Research by Pavlakos et al. (2001) and Tobias et al. (2018) highlights the challenges in understanding vector operations. This study reinforces these findings, showing that GeoGebra provides an effective solution through its visual and interactive features (Tobias et al., 2018).

Equipping teachers with professional training is crucial for the successful integration of technology in classrooms. Schools should provide educators with opportunities to develop their skills in using tools like GeoGebra (Tamam & Dasari, 2021). Additionally, improving access to technology, especially in rural schools, can help bridge the educational gap and ensure equal learning opportunities for all students (Tamam & Dasari, 2021).

Further research should explore the long-term effects of digital tools on student learning. Investigating how GeoGebra and similar platforms impact different mathematical concepts and student retention over time can provide deeper insight into their educational value (Wassie & Zergaw, 2019). Through its user-friendly interface, GeoGebra facilitates a hands-on approach to learning, offering students a practical and engaging environment to rectify misconceptions and deepen their understanding of mathematical principles (Tuda & Rexhepi, 2024).

The Use of Technology and Visualization Software

Technology has increasingly been employed to enhance students' understanding of vectors, with positive results. One example is VectorPad, a visualization tool that uses pen-based technology to visualize vectors in 3D. Bott and LaViola (2010) showed that this tool was positively received by participants for its ability to dynamically demonstrate vector operations, though there was room for usability improvements, such as automatic camera rotation.

Another powerful tool for improving mathematical skills is GeoGebra. Numerous studies have shown that using GeoGebra has a positive impact on students' acquisition of mathematical concepts (Tamam & Dasari, 2021; Wassie & Zergaw, 2019). This software provides robust tools for exploring, visualizing, and constructing mathematical concepts and has proven to be a valuable pedagogical resource for both students and teachers (Majerek, 2014).

The mathematization process of students understand the concept of vectors through learning realistic mathematics and ethnomathematics (Widada et al., 2020), while Mollakuqe et al. (2021) suggest further improvements for GeoGebra, such as the use of smartphones in the absence of computers, its positive impact remains undeniable.

Synthesis and Discussion

Through these studies, it becomes clear that the challenges in understanding vectors are numerous and varied. Some studies focus on fundamental misconceptions and the lack of basic knowledge that hinder students from fully understanding vectors. On the other hand, the development and use of technological tools like GeoGebra and VectorPad offer great potential for overcoming these challenges and significantly improving students' comprehension. However, there is still room for further research to determine how these tools can be used most effectively and to address the ongoing challenges in teaching vectors (Tuda & Rexhepi, 2024).

Meaning of Vector

All students can grow and develop critical thinking skills when learning math (Aliu et al., 2021). Even vectors as part of mathematics develop students' critical thinking since vectors are widely used in many other sciences including computer science to build different games.

The concept of vector is multidimensional and has to do with many levels of knowledge construction. This concept is combined with many contexts that appear in algebra, geometry, and physics, it creates many problems in teaching as well as in its understanding (Pavlakos et al., 2001).

The segment $[AB]$ whose edges we consider as ordered pairs of points is called a vector and is denoted by $\overrightarrow{AB}$.

1. The straight line $[AB]$ is called the carrier line and determines the direction of the vector $\overrightarrow{AB}$.
2. We call the distance between points A and B the intensity (length) of the vector $\overrightarrow{AB}$ and denote it by $|\overrightarrow{AB}|$
3. The orientation of the vector $\overrightarrow{AB}$ is from the starting point to the ending point.

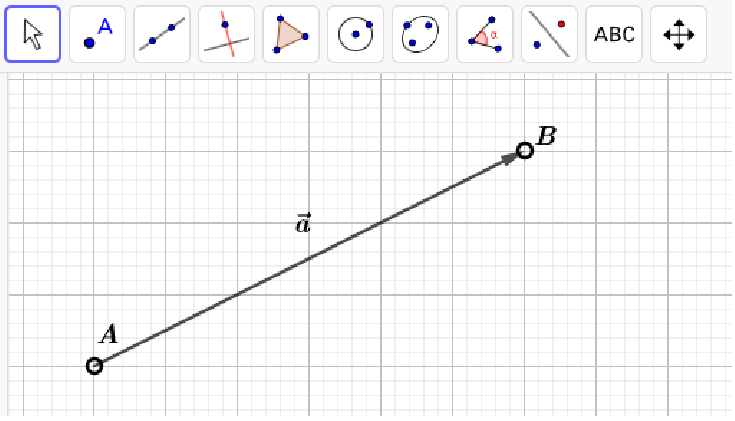

Figure 1. Meaning of vector (Source: Authors' own elaboration)

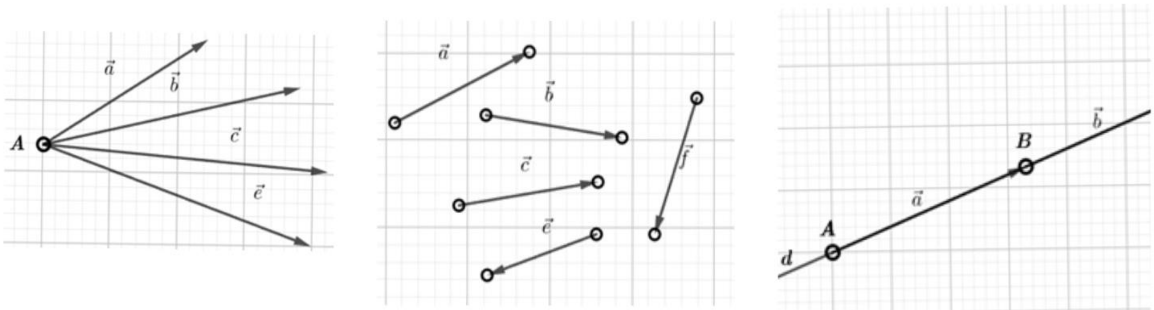

Figure 2. Bound vectors, free vectors, and slip vectors (Source: Authors' own elaboration)

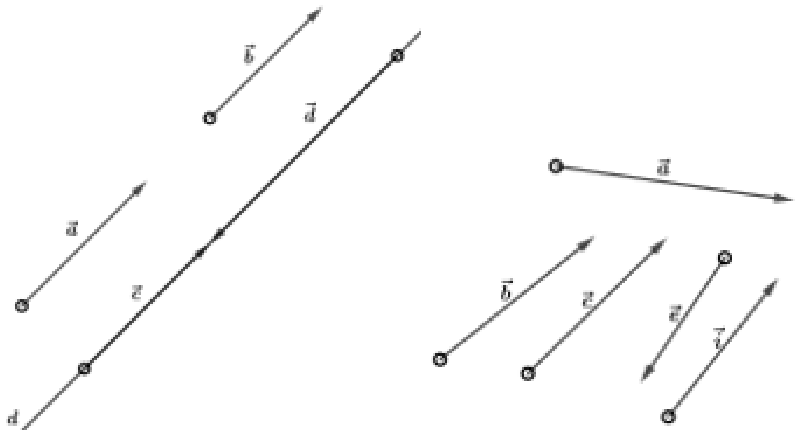

Figure 3. Collinear vectors, coplanar vectors (Zaka, 2018)

The vector concept was presented geometrically (**Figure 1**) as an infinite set of arrows of equal length and direction (Tobias et al., 2018). Vectors can be in three forms, as bound vectors, free vectors and sliding vectors. These three types of vectors are shown in **Figure 2**. The vector whose intensity is zero is called the zero vector and is marked with $\vec{0}$, while the vector whose intensity is one is called the unit vector (Zejnullahu et al., 2006). According to the position of the vectors, we distinguish collinear vectors and coplanar vectors (**Figure 3**). Collinear vectors are vectors that have the same direction or meet a line.

Addition of vectors

The usual addition of vectors is performed using an axiom that states that for every two vectors $\vec{a}, \vec{b}$ there is a third vector $\vec{a}+\vec{b}$ which satisfies other properties, such as $\vec{a}+\vec{b} = \vec{b}+\vec{a}$ (Watson et al., 2003). To add two vectors, we have two rules. Triangle rule and parallelogram rule (**Figure 4**).

Properties for vector addition:

- $\vec{a}+\vec{b}=\vec{b}+\vec{a}$ Commutative property
- $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$ Associative property

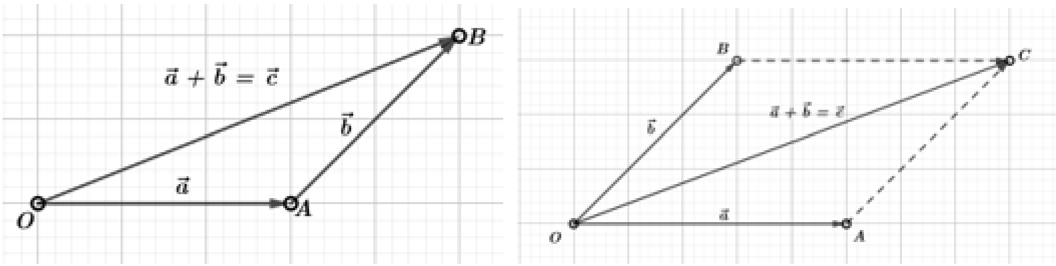

Figure 4. The triangle and parallelogram rule for adding vectors (Minir et al., 2010)

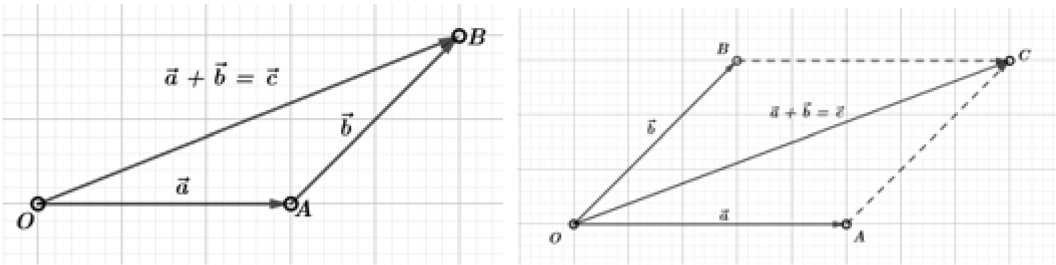

Figure 5. The triangle and parallelogram rule for vectors subtraction (Minir et al., 2010)

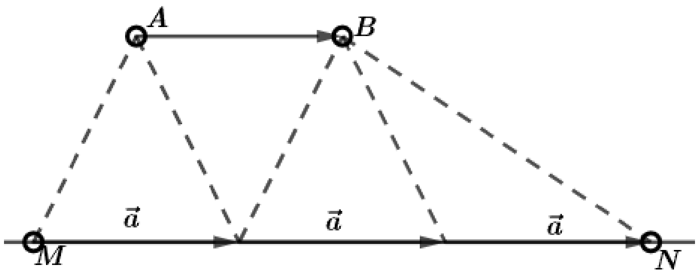

Figure 6. Vector multiplication by scalar (Source: Authors' own elaboration)

- $\vec{a} + \vec{0} = \vec{a}$ Element zero
- $\vec{a} + (-\vec{a}) = \vec{0}$ The opposite element

Vector subtraction

Vector subtraction is understood when students can understand the opposite vector. Vectors are called opposite when they are the same length but only in different directions. We also have two rules for subtracting vectors. Triangle rule and parallelogram rule (**Figure 5**).

Vector multiplication by scalar

Not only the term vector is a term unknown to students. Even the term scalar (**Figure 6**) continues to be a term not very clear to students when they hear it for the first time. Scalar i.e. number. And this scalar or number indicates how many times a vector will be repeated.

Using Geogebra Software for Understanding Vectors

Historically, it is known that the subject of mathematics has rarely been preferred by students, due to the difficulty of understanding it. Mathematics teachers using the classic means of explaining mathematics such as paper and pencil seem to be insufficient to clarify mathematics concepts and make mathematics attractive. In today's times, everyone has access to technological devices and the classic teaching methods seem to no longer have the value they once had, as students are no longer interested in this method. Technological devices are part of students' daily life and learning mathematical concepts through these devices, according to mathematics teachers, can increase the motivation and participation of students in learning mathematics and offer different ways in teaching mathematics.

The use of computers and the Internet in education, together with computational tools (educational software), based on educational paradigms, can contribute to the teaching and learning process (Bulegon, 2015). Therefore, it was important for teachers to understand better how students learn, using new working techniques so that students have easier access to the understanding of vectors (Widada et al., 2020).

Figure 7. Connecting vectors to everyday life by students (https://www.testsconducir.es/b/se%C3%B1ales-de-circulacion/marcas-viales - left; https://www.testsconducir.es/b/se%C3%B1ales-de-circulacion/marcas-viales - right)

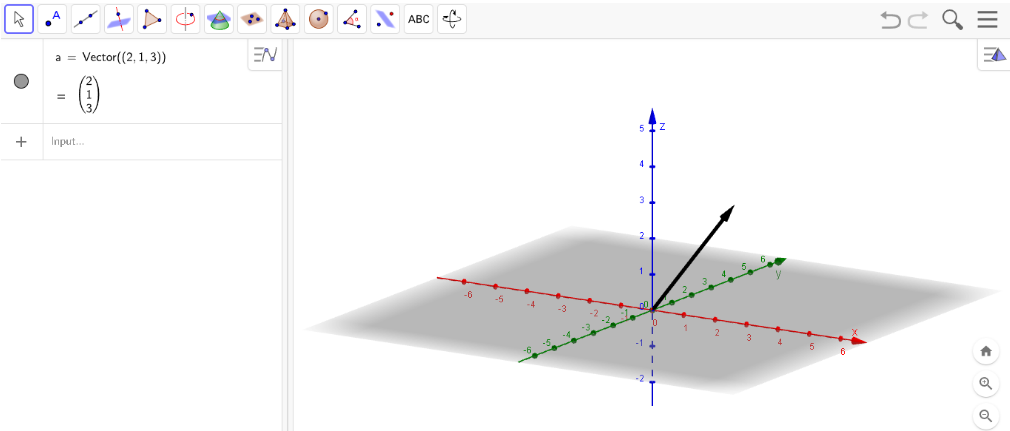

Figure 8. Illustration of vector concept using GeoGebra software (Source: Authors' own elaboration, using GeoGebra)

In this paper, we have focused only on one mathematical software, considering it as the most suitable to clarify the concept of vectors. GeoGebra is used as very practical and very easy-to-access software. GeoGebra is a free, open-source, highly practical, and easy-to-use software that provides a versatile tool for visualizing mathematical ideas (Hohenwarter & Jones, 2007).

This software provides multiple representations of mathematical objects, it can help students discover the connections between mathematical objects and their graphical representation. Students take more responsibility for their learning and the dynamic of classroom activity shifts to more discussion, interest, motivation to study, and collaborative learning (Dikovic, 2009).

Through GeoGebra, can be visualized, analyzed, and understood exponential growth and decay like never before. This software not only allows us to observe the behavior of exponential functions but also experiment with various parameters, transforming, and translating these functions to gain a more profound insight into their properties (Tuda & Rexhepi, 2023). The illustrations have a greater impact compared with using the program GeoGebra and Mathematica for solving exercises by students than solving exercises on the whiteboard, since students of lower classes hardly find themselves when working with GeoGebra, compared to higher classes (Kamberi et al., 2020).

Comparison of the Classical Method of Explanation with the Modern Method

To carry out this comparison in the best possible way, the authors took as a study two ninth-grade classes where the theme of the vector was developed, including the meaning of the vector as a concept, the parts of the vector, and operations with vectors. In one class the authors used the classical method as a form of explanation, that is, in group form or with the teacher at the center of the explanation to teach the vector concept, while in the other class, we used GeoGebra software as a form of explanation to teach this new concept.

Vector explanation using GeoGebra software

In the first group, the vector concept using the GeoGebra software was explained. To explain the vector concept, a classroom was used, where it is equipped with technological equipment and where everyone has access to GeoGebra software. With the help of GeoGebra software, we started to present the vector, and then the types of vectors. For the students, the term vector was a new term. But by presenting the vector in GeoGeber, especially in the three-dimensional view, they could understand very clearly what a vector looks like. By showing the parts of the vector such as the direction, start, and end point through the software the students started to think about vectors and by relating them to everyday life examples. They said that vectors are like arrows on the road, showing the direction of the road, or even like the EXIT sign showing the direction of the exit (**Figure 7**). Naturally, these thoughts were not expressed to the students, but seeing what the vector looks like through the 3D image in GeoGebra (**Figure 8**), the students' imagination was stimulated, and they began to connect the vector concept with everyday life.

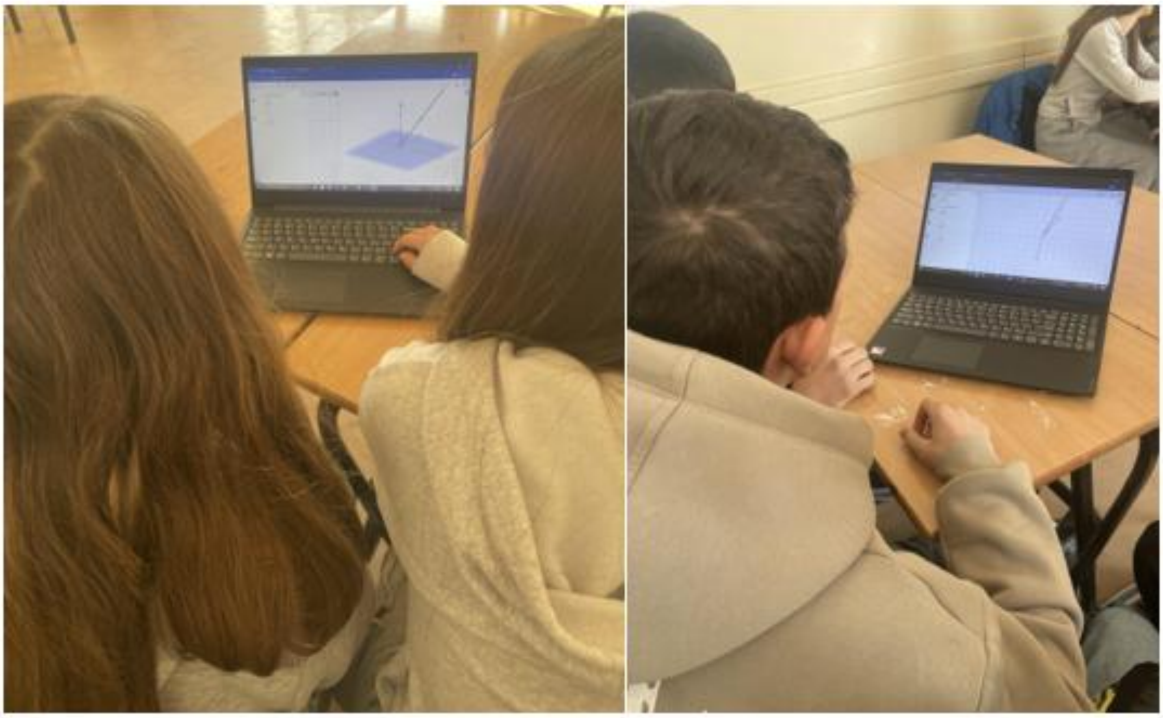

Figure 9. The use of GeoGebra during the lesson (Source: Field study)

Although the vector concept is completely new to the students and unheard of before, through Geogebra a curiosity was sparked to study this concept in detail. The students, understanding the vector and operations with vectors using GeoGebra, were much more accurate in the tasks they chose individually. Because they see every kind of task graphically and by means of pictures and colors, they understand much more easily the examples we took in class (**Figure 9**).

An interview with the best student in the group where the Geogebra software was used is provided below:

Professor: What do you think about vectors, do you think you have understood them enough?

Student: At first, of course, I had a lot of confusion, since the vector is an unknown concept for us before, but now after the lessons on the meaning of the vector and operations with vectors are over, I have quite well understood the vector as a concept, the operations with it and the role his.

Professor: What was the biggest difficulty while learning the concept of vectors?

Student: The biggest difficulty was understanding the addition and subtraction of vectors using the triangle and parallelogram method.

Professor: How difficult was learning to use the GeoGebra software?

Student: Even though it was the first time I had access to GeoGeber, it was very easy to use GeoGeber and learn its tools.

Professor: Do you think vectors have applications in other fields?

Student: In fact, we had vectors everywhere in other subjects, in Physics, Mechanics, even when walking, we are walking in the direction of the vector, so it is a term we use every day.

Vector explanation in classical form

In the second group, we used the classical method of explaining the vector concept (**Figure 10**). So, with the teacher in the center or group form, with didactic materials such as charts, papers, and pencils. For ninth-grade students, the term "vector" is new, and they initially felt confused using this term. During the explanation of the chapter on vectors, the curiosity of the students was not stimulated to make a reaction about vectors or to connect the learning unit on vectors to everyday life. Many students called the vector an arrow since they could not imagine how a vector might look in space. Clarification of the vector as a concept and vector actions according to the students were completely unnecessary. Vectors were too abstract for them, and they couldn't imagine it, despite the teacher giving his best with the tools he had.

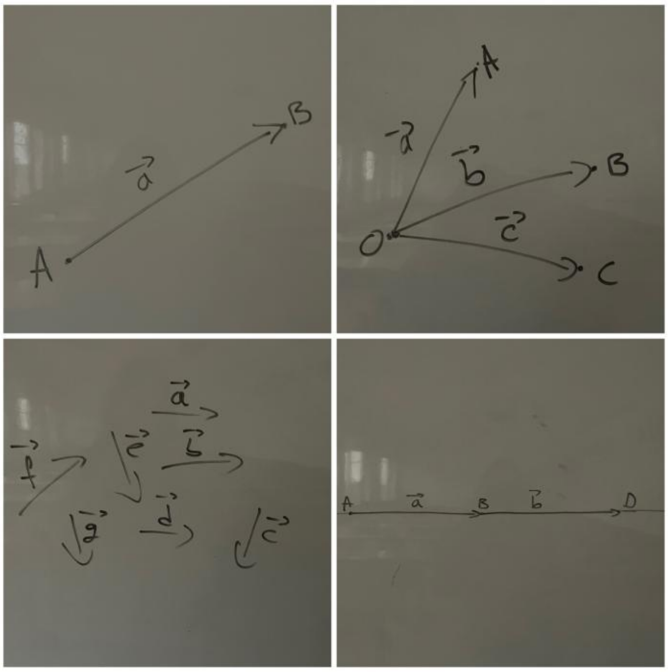

Figure 10. Concept of vector and types of vectors being explained by classical method (Source: Field study)

When all the lessons were finished about vectors and operations, the best student was interviewed in the group:

Professor: What do you think about vectors, do you think you have understood them enough?

Student: I think I have a confusion in my head about vectors, I can manage to understand all operations with vectors but I have not managed to sufficiently clarify the vector as a concept and its properties for addition, subtraction or scalar multiplication.

Professor: What was the biggest difficulty while learning the concept of vectors?

Student: The biggest difficulty is imagining the vector in space, so what it looks like in reality and not how we see it, like an arrow.

Professor: How understandable was the explanation of the vector concept during the lessons?

Student: During the explanation, there were deficiencies in the visualization of vectors and actions with vectors and this made it difficult to understand

Professor: Do you think vectors will be taught in other subjects?

Student: It seems to me that vectors have nothing to do with mathematics either, as I see them as unusable, I don't know about other subjects.

Difficulties of Understanding Vectors

Little interest of students to learn about vectors, as well as poor teaching techniques, are problems for vector learning. Vector is an abstract concept, and it is also a new concept.

The types of vectors and their classifications are recognized by the students in the example, but they cannot be defined easily. In collecting the vector, the students had difficulties in applying the rules and presenting the graph. Also, the scalar product is not able to follow its properties to choose an example.

In the following, we will present some of the difficulties of understanding vectors and operations with vectors by students where the Geogebra software was not used when explaining vectors, but the classical method of explanation was used, and we will show the help that this software offers you students for easier understanding of vectors

In an assignment during the lesson, students were asked to present a vector with starting point A and ending point B (**Figure 11**).

Students had difficulty representing the vector by drawing only a straight line and placing two points A and B as the start and end points, and not specifying the direction of the vector, but only representing it as a segment.

This may be because the students do not have very deficient knowledge in the field of geometry and find it very difficult to understand the basic concepts of geometry as in the previous case there was confusion between the segment as a basic concept of geometry and vector as a completely new concept in mathematics.

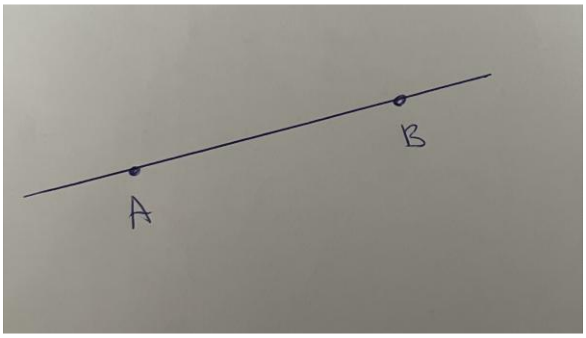

Figure 11. Students' difficulties in vector understanding (Source: Field study)

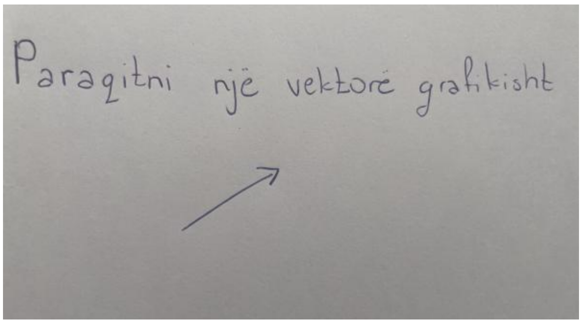

Figure 12. Presentation of the vector graphically by the student in classical fashion (Source: Field study)

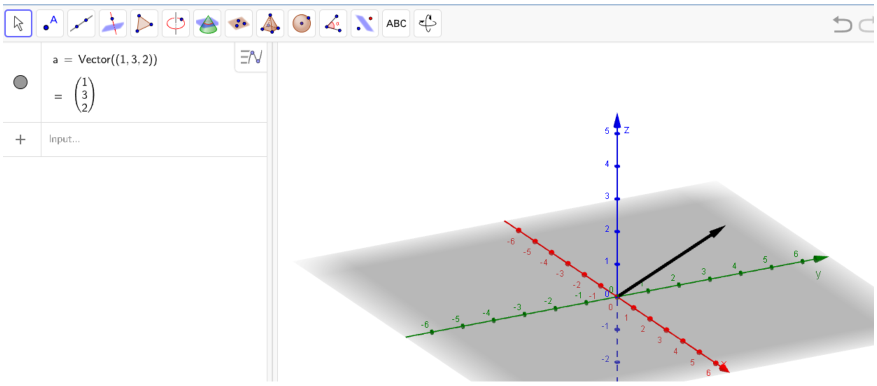

Figure 13. Using GeoGebra for vector representation (Source: Authors' own elaboration, using GeoGebra)

Figure 14. Representation of sliding vectors (Source: Authors' own elaboration)

Another question related to this example addressed to the students was what a vector looks like in space, most of the answers were that an arrow was drawn on a plane (paper, blackboard) (**Figure 12**).

Through the 3D view of the GeoGebra software (**Figure 13**), students change their thinking and see that the position of the vectors changes in space depending on the values they have in the x, y, z coordinates.

In another case, the students were asked to find the starting and ending points of the vectors on a carrier line where two vectors are placed (**Figure 14**).

A significant part of the students could not understand that two vectors can be placed on a carrier line, and they considered as a single vector the beginning of the vector $\overrightarrow{AB}$ point A, and the end of the vector $\overrightarrow{CD}$ point D. In the following, the error of the students during the solution of this given task is presented in **Figure 15**.

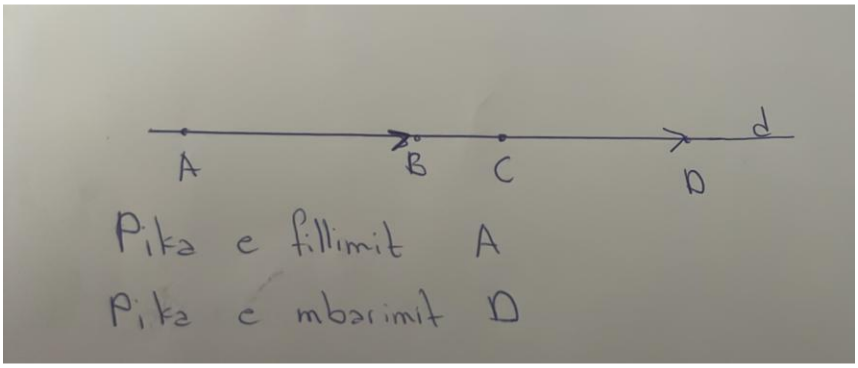

Figure 15. Students' difficulties with sliding vectors (Source: Field study)

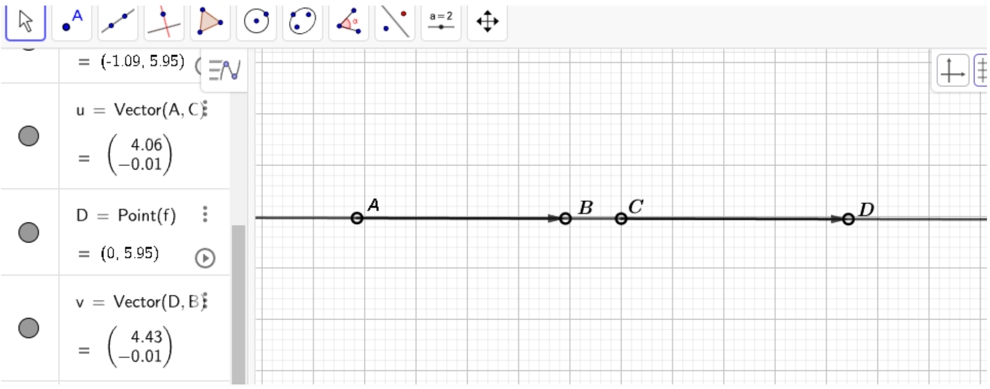

Figure 16. Using GeoGebra to understand vector problems (Source: Authors' own elaboration, using GeoGebra)

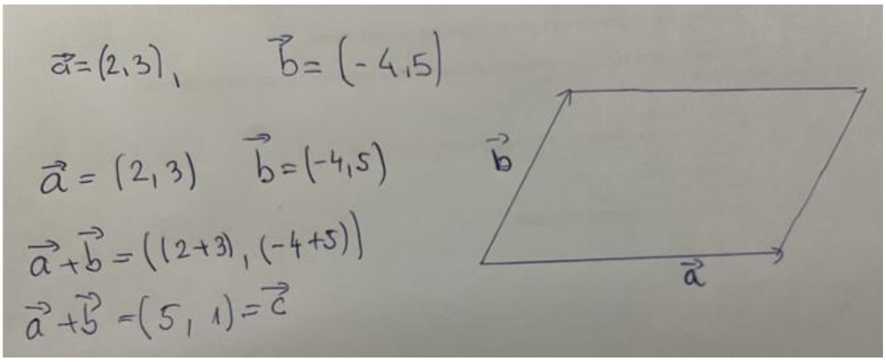

Figure 17. Student's difficulties in understanding the addition of vectors (Source: Field study)

With the GeoGebra software, students have an easier approach to this task (**Figure 16**). In another task, students were asked to add the vectors $\vec{a}$ and $\vec{b}$ using the parallelogram method. The students collected the coordinates of the same vector among themselves and only presented a parallelogram naming the vectors $\vec{a}$ and $\vec{b}$ on its sides (**Figure 17**). With the help of the GeoGebra software, the students understood the addition of vectors more easily since this software, by entering data, graphically and in 3D expresses the way of adding vectors, in our example with the parallelogram method (**Figure 18**).

In another task during the lessons, it was requested to multiply the vector $\vec{a}$ (2,3) by the scalar -2. Most students solve the problem like shown in **Figure 19**. The student did not present the vector in coordinates, and to operate on the vector $\vec{a}$ (2,3). With the scalar -2, he went back to two units and changed the direction of the vector. Using GeoGebra, this action is much easier to understand. First, you see the vector $\vec{a}$ (2,3) placed on the coordinate axis and then the action with the scalar -2. Using GeoGebra in the following **Figure 20**, the actions are seen and touched and do not leave room for the student's imagination to think wrongly about the logic of the task.

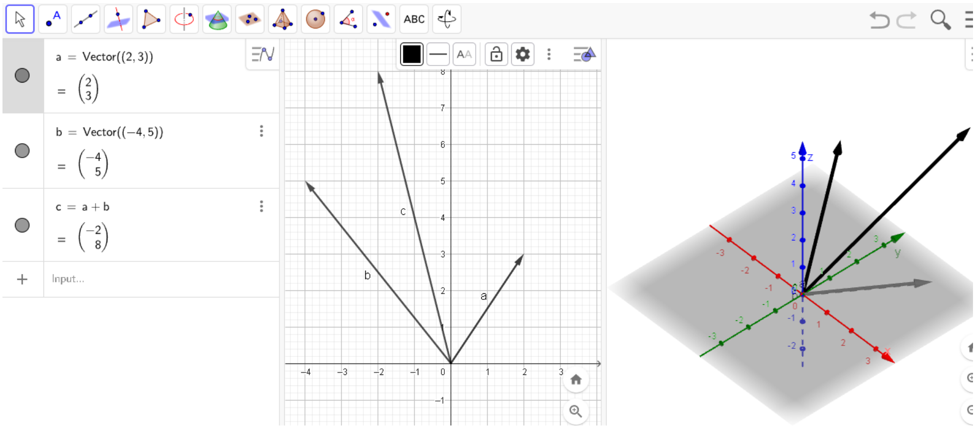

Figure 18. Using GeoGebra in tasks with the addition of vectors (Source: Authors' own elaboration, using GeoGebra)

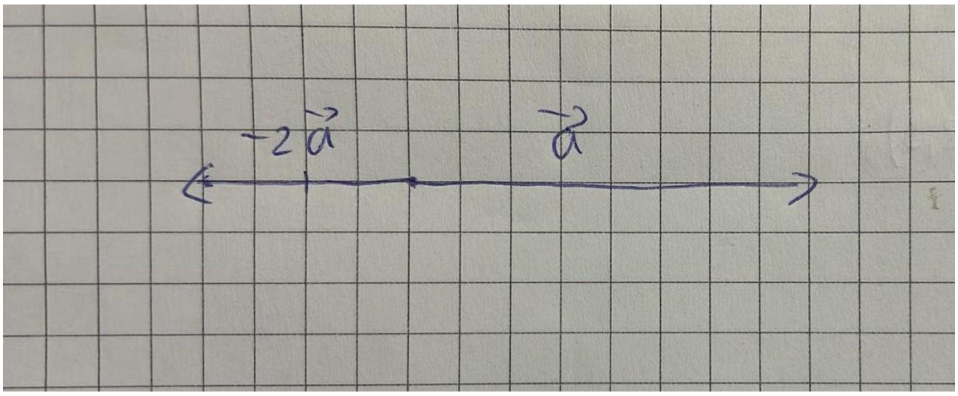

Figure 19. Difficulty during the example for vector multiplication by a scalar by the student (Source: Field study)

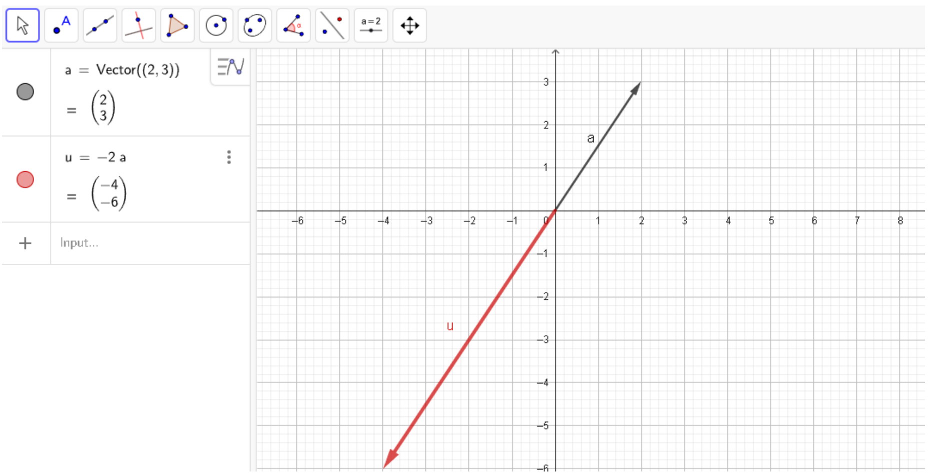

Figure 20. Solving the same example using GeoGebra (Source: Authors' own elaboration, using GeoGebra)

Another difficulty for students (**Figure 21**) is the difference between equal vectors.

We see that the student has incorrectly assigned the vectors, writing that $\overrightarrow{AD} = \overrightarrow{CB}$ and $\overrightarrow{AB} = \overrightarrow{DC}$.

$\overrightarrow{AB} = \overrightarrow{DC}$. has been compared well, but the vectors $\overrightarrow{AD} = \overrightarrow{CB}$ have not been well compared since the student has only compared the lengths of the vectors and verified that they are parallel to each other. However, it has not been evaluated whether the vectors have the same direction

To avoid this mistake, it is enough to remind students of the definition of equal vectors.

Definition: Two vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are equal, if they have equal lengths, are parallel, and have the same direction. They are denoted by $\overrightarrow{AD} = \overrightarrow{DC}$

To see the impact of GeoGebra on the understanding of vectors and actions with vectors, we conducted a test with the students of class IX-1, in the class where we explained the vector concept through Geogebra. After the students were introduced to GeoGebra for the first time, we first conducted a pre-test to show the students how the test would be conducted and then conducted the main test.

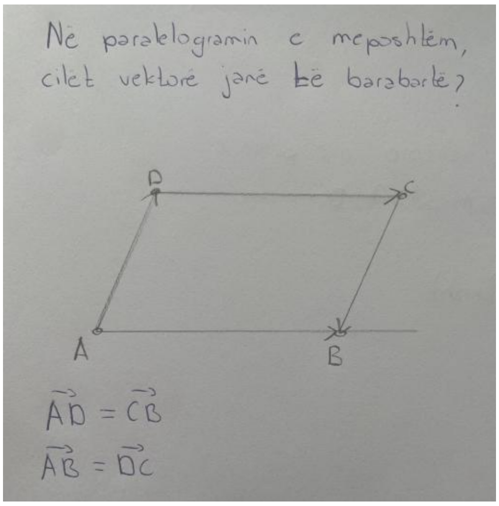

Figure 21. Difficulties of the student during the solution of the task with the comparison of vectors (Source: Field study)

Research Questions and Hypotheses

This study seeks to address the following research questions:

RQ1 How does the use of GeoGebra enhance students' conceptual and procedural understanding of vectors compared to traditional teaching methods?

RQ2 What specific aspects of GeoGebra (visualization, interactivity, ease of manipulation) contribute most to students' improved comprehension of vectors?

RQ3 How does the integration of GeoGebra impact students' engagement and motivation in learning vector concepts?

RQ4 Are there significant differences in problem-solving abilities between students who use GeoGebra and those who rely on classical instructional methods?

Hypotheses

To explore these research questions, the following hypotheses were formulated:

H1 Students who use GeoGebra will perform significantly better in conceptual and procedural understanding of vectors than students taught using traditional methods.

H2 The interactive and visual features of GeoGebra will enhance students' ability to grasp complex vector operations more effectively than classical methods.

H3 Students who learn vectors through GeoGebra will demonstrate higher levels of engagement and motivation compared to those using traditional instruction.

H4 The use of GeoGebra in vector learning will lead to a statistically significant improvement in students' problem-solving skills compared to traditional teaching methods.

By testing these hypotheses, this study aims to determine whether GeoGebra serves as an effective pedagogical tool for improving vector comprehension and problem-solving skills in students.

To ensure comparability between the groups, students were randomly assigned to either the GeoGebra-assisted or traditional instruction group. A pre-test was administered to assess prior knowledge of vectors before the intervention. The pre-test included conceptual and procedural questions to measure students' baseline competency levels. The results were analyzed to confirm that both groups had similar initial knowledge, ensuring that any differences observed in post-test results could be attributed to the teaching method rather than pre-existing disparities. This approach strengthens the validity of the study by minimizing biases in student grouping.

METHOD

The research that was done in this paper was carried out in two elementary schools, in the ninth grade, since vector understanding is taught in the ninth grade. This research was carried out in Kosovo, and unfortunately in Kosovo, not all schools use technology during lessons, especially in villages, most schools do not have technological equipment, and in schools located in the city, a large part of them use technology during lessons, therefore during the research process we chose a school located in the village called "Hasan Ali-Remniku", to see the classical method of explanation and to analyze the application of technology, especially GeoGebra during mathematics lessons, we chose a school in the city by the name of "Musa Zajm". This research was developed simultaneously in both schools, during the two weeks in February 2024.

The educational units that were developed during this research on vectors were the Meaning of Vectors, Operations with Vectors, Types of vectors, the opposite vector, etc. In the ninth grades, according to the curriculum of Kosovo, 4 hours a week are held in the subject of Mathematics, therefore, in these 2 weeks, the teaching units were explained fluently by the mathematics teacher in the respective schools, and we only observed and made the differences between the two schools, where one used the classical method, at the "Hasan Ali-Remniku" school and at the "Musa Zajm" school, where the method with the application of GeoGebra was used during the lesson.

Fortunately, the ninth grade that was chosen randomly in the school where the lesson was developed with the application of GeoGebra, the teacher of the subject was very familiar with the GeoGebra software because he applied it throughout the year in the classroom, for all units in the subject of mathematics. He had learned GeoGebra in a self-taught manner without any special training.

Methodological and Analytical Considerations

To ensure methodological rigor, several steps were taken to address potential biases and limitations:

Student Grouping & Baseline Knowledge:

To maintain comparability, students were randomly assigned to the groups.

A pre-test was conducted to assess prior knowledge and ensure that both groups had similar initial competency levels in vector mathematics.

Data Collection & Assessment:

A combination of qualitative (interviews, classroom observations) and quantitative (test scores, statistical analysis) data collection methods were employed.

Standardized tests were used to measure conceptual understanding, procedural fluency, and problem-solving skills.

Analytical Framework:

The study employed descriptive and inferential statistical analyses (ANOVA) to determine the significance of differences between the two groups.

The reliability of the test instruments was assessed using Cronbach’s Alpha.

Limitations and Challenges:

The study was conducted over a short two-week period, which may not fully capture long-term retention effects.

Technological accessibility varied between urban and rural schools, potentially affecting engagement levels.

The reliance on a single software (GeoGebra) may not account for alternative technological tools that could enhance vector learning.

By addressing these methodological and analytical considerations, this study aims to provide a balanced and evidence-based evaluation of the impact of GeoGebra on students’ understanding of vectors.

Population Study

The students' research was done in two schools, the elementary school "Hasan Ali-Remniku" in Remnik and the school "Musa Zajm" in Gjilan. This research only includes ninth graders. In the primary school "Musa Zajm" in Gjilan, we researched the first group which included 50 students (n=50) who used the GeoGebra software during the lessons when the Understanding of Vector was explored, while the second group we carried out the research in the primary school "Hasan Ali-Remniku" in Remnik, which includes 52 students (n= 52).

In the second group, no application was used to solve the tasks, and the teaching and evaluation of the students was done traditionally. A pre-test was also conducted for the students so that the students are informed and prepared for the form of the test and also so that they do not have problems with the post-test.

Algebraic Concepts of the Subject

The paper was formulated using the mathematical software GeoGebra. This paper focuses on three basic points: understanding the concepts, procedural knowledge, and solving the tasks of the chapter Understanding vector. In each of these points, a study was done, and they were divided into groups the GeoGebra group and the group that did not use any application. Also, the general test was done, and it was studied in these three points mentioned above.

Study Application Mechanism

The statistics that we have extracted in this scientific paper were created using the IBM SPSS software and these statistics are part of the paper where we are based to see the results of the research.

1. Initially, the students are taught to install GeoGebra on their technological devices such as phones, iPads, or laptops. Then the students have learned how to use this software to then apply it when choosing tasks in the lessons, and for solving homework. Besides illustrations important role in understanding the unit is homework, classroom commitment, extra hours, and extra exercises (Aliu et al., 2021). We have provided additional material and a YouTube tutorial for students to learn more about this software. The Internet resources as well as other illustrative and demonstration methods make them as attractive as possible (Kamberi et al., 2020). In the second group, the lesson was developed traditionally, and the assessment was also done in this way.
2. After the students have learned how to use GeoGebra, we have given them many tasks during the lesson to apply the software while solving the tasks, during this phase some of the students had problems with choosing the tasks as they had not learned how to use it very well of GeoGebra.
3. And finally, we conducted a test for the students, whereas in the following results the students who used GeoGebra have better results than those who chose the test in the traditional way.

Table 1. Means and Standard Deviations of the Two Groups

Variable	Group	N	Mean	SD
Conceptual Knowledge	GeoGebra	50	8.1800	1.15511
	NoApp	52	7.8846	1.21516
Procedural Knowledge	GeoGebraproc	50	8.1600	1.07590
	NoAppproc	52	7.4615	1.19577
Problem Solving	GeoGebraprob	50	8.0200	1.97917
	NoAppprob	52	7.2308	1.45002
Overall Test	GeoGebraTest	50	8.2200	1.03589
	NoAppTest	52	7.2692	1.34478

RESULTS

Table 1 summarizes the means and standard deviations for the different groups.

Conceptual Knowledge: The GeoGebra group had a higher mean score (8.18) compared to the NoApp group (7.88), indicating that students using GeoGebra had better conceptual understanding.

Procedural Knowledge: The GeoGebra group scored higher (8.16) than the NoApp group (7.46), suggesting better problem-solving skills.

Problem-Solving: Similar trend; students using GeoGebra outperformed the NoApp group.

Overall Test: The GeoGebra group (8.22) had a better overall performance than the NoApp group (7.26).

Interpretation and APA Reporting

Narrative of the results

Conceptual knowledge: The ANOVA analysis for conceptual knowledge showed a statistically significant difference between the group using GeoGebra and the group using traditional methods ($F (4, 45) = 6.985$, $p < 0.01$). Students using GeoGebra had a higher average score ($M = 8.18$, $SD = 1.16$) compared to those who did not use it ($M = 7.88$, $SD = 1.22$).

Procedural knowledge: The ANOVA results indicate that the differences in procedural knowledge are also statistically significant ($F (4, 45) = 12.073$, $p < 0.01$). The GeoGebra group had a higher average score ($M = 8.16$, $SD = 1.08$) compared to the traditional group ($M = 7.46$, $SD = 1.20$).

Problem solving: Differences in problem-solving abilities are also significant ($F (4, 45) = 4.690$, $p < 0.01$). The group using GeoGebra showed a higher score ($M = 8.02$, $SD = 1.98$) compared to those who did not use it ($M = 7.23$, $SD = 1.45$).

Overall test: The ANOVA results (**Table 2**) for the overall test also showed statistically significant differences ($F (4, 45) = 4.245$, $p < 0.01$). The GeoGebra group had a higher average score ($M = 8.22$, $SD = 1.04$) compared to the group that did not use the application ($M = 7.27$, $SD = 1.34$).

Table 2. ANOVA result

			Sum of Squares	df	Mean Square	F	Sig.
Conceptual knowledge	GeoGebra	Between Groups	25.045	4	6.261	6.985	0.00
		Within Groups	40.335	45	.896		
		Total	65.380	49			
	NoApp	Between Groups	24.545	4	6.136	6.366	0.00
		Within Groups	43.375	45	.964		
		Total	67.920	49			
Procedural knowledge	GeoGebra	Between Groups	29.361	4	7.340	12.073	0.00
		Within Groups	27.359	45	.608		
		Total	56.720	49			
	NoApp	Between Groups	27.205	4	6.801	7.415	0.00
		Eithin Groups	41.275	45	.917		
		Total	68.480	49			
Problem sloving	GeoGebra	Between Groups	13.822	4	3.456	4.690	0.00
		Within Groups	33.158	45	.737		
		Total	46.980	49			
	NoApp	Between Groups	34.484	4	8.621	7.601	0.00
		Within Groups	51.036	45	1.134		
		Total	85.520	49			
Overall test	GeoGebra	Between Groups	14.405	4	3.601	4.245	0.00
		Within Groups	38.175	45	.848		
		Total	52.580	49			
	NoApp	Between Groups	24.464	4	6.116	4.824	0.00
		Within Groups	57.056	45	1.268		
		Total	81.520	49			

To identify the significance of the differences, we use advanced software with the results from the students' tests. From the results, we have found that the group that used the GeoGebra software was the one that had better results in the test, while the group that did not use the application, had moderately good results.

DISCUSSION

This study confirms that GeoGebra enhances students' understanding of vectors compared to traditional methods. Students using GeoGebra demonstrated stronger conceptual and procedural knowledge, better problem-solving skills, and higher engagement levels. GeoGebra's interactive features facilitated real-time vector manipulation, making abstract concepts more tangible. Unlike static illustrations, its 3D visualization allowed students to explore vectors dynamically, improving comprehension.

Students using GeoGebra found lessons more engaging, promoting active participation and self-correction. In contrast, traditional methods led to disengagement due to reliance on memorization and limited interaction. Without visual aid, students struggled with spatial representations and vector operations. The absence of dynamic feedback hindered problem-solving accuracy and conceptual clarity. ANOVA results showed significant differences ($p < 0.01$) in conceptual knowledge, procedural understanding, and problem-solving abilities, with GeoGebra users consistently outperforming the traditional group.

GeoGebra should be incorporated into curricula to enhance learning, but its effectiveness depends on proper teacher training and equitable access to technology. Future studies should explore the long-term impact of GeoGebra, its applicability in other mathematical areas, and comparisons with other digital tools. Conceptual and procedural knowledge were measured using multiple-choice questions and problem-solving tasks. Conceptual understanding was assessed through open-ended questions requiring students to explain reasoning, while procedural knowledge was evaluated based on accuracy in mathematical operations and vector-based exercises. Pre-tests and post-tests demonstrated that GeoGebra users showed significant improvement in both areas.

CONCLUSION

This study highlights the effectiveness of GeoGebra in improving students' understanding of vectors. The findings indicate that students using GeoGebra demonstrated better conceptual and procedural knowledge, higher engagement, and enhanced problem-solving skills compared to those taught with traditional methods. The software's ability to provide interactive, visual learning experiences makes it a valuable tool for mathematics education.

The practical implications of these findings suggest that integrating GeoGebra into the curriculum can significantly enhance learning outcomes. By enabling students to visualize and manipulate vector concepts, GeoGebra fosters deeper comprehension and greater motivation. However, for its successful implementation, adequate teacher training and equitable access to technological resources must be ensured.

Future research should focus on addressing challenges related to teacher preparedness, resource availability, and the long-term impact of GeoGebra on student learning. Investigating how its use influences retention of mathematical concepts and how it can be effectively scaled across different educational settings will be essential in maximizing its benefits.

Author contributions: ERA: data curation, visualization, testing and investigation, writing – original draft, review & editing; **TJZ, EI & SR:** conceptualization, ideas, methodology, project administration, supervision, validation, writing – original draft, review & editing. All authors have sufficiently contributed to the study and agreed with the results and conclusions. All authors have agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Ethical statement: The authors stated that, according to applicable institutional regulations, the study does not require approval from an ethics committee since it is educational research based on standardized tests. Informed consent was obtained from the participants before the research was conducted; and they were informed about the research, anonymity and their rights about withdrawing from the study without adverse repercussions. All students participate in the study with desire, will and full awareness.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

REFERENCES

Aliu, A., Rexhepi, Sh., & Iseni, E. (2021). Analysis and comparison of commitment, homework, extra hours, preliminary grades and testing of students in Mathematics using linear regression model. *Mathematics Teaching Research Journal*, *13*(3), 21-52.

Bollen, L., Kampen, P., Baily, Ch., Kelly, M., & Cock, M. (2017). Student difficulties regarding symbolic and graphical representations of vector fields. *Physical Review Physics Education Research, 13*, Article 020109. https://doi.org/10.1103/PhysRevPhysEducRes.13.020109

Bott, J. N., & LaViola, J. J. (2010). A pen-based tool for visualizing vector mathematics. In *International Symposium on Sketch-Based Interfaces and Modeling*. https://doi.org/10.2312/SBM/SBM10/103-110

Bulegon, A. M. (2015). GeoGebra and eX e learning: Applicability in the teaching of physics and mathematics. *Journal of Systemics, cybernetics and Informatics, 9*(2), 61-66.

Dikovic, Lj. (2009). Implementing dynamic mathematics resources with GeoGebra at the college level. *International Journal of Emerging Technologies in Learning, 4*(3), 51-54. https://doi.org/10.3991/ijet.v4i3.784

Hohenwarter, M., & Jones, K. (2007). Ways of linking geometry and algebra: The case of GeoGebra. *Proceedings of the British Society for Research into Learning Mathematics*, 126-131.

Kamberi, S., Latifi, I., Rexhepi, Sh., & Iseni, E. (2020). The influence of practical illustrations on the meaning and operation of fractions in sixth grade students, Kosovo-curricula. *International Electronic Journal of Mathematics Education, 17*(4), Article em0717. https://doi.org/10.29333/iejme/12517

Majerek, D. (2014). Application of GeoGebra for teaching mathematics. *Advances in Science and Technology Research Journal, 8*(24), 51-54. https://doi.org/10.12913/22998624/567

Minir, E., Qamil, H., & Ramadan, L. (2010). *Matematika*. Pejë.

Mollakuqe, V., Rexhepi, Sh., & Iseni, E. (2021). Incorporating Geogebra into teaching circle properties at high school level and it's comparison with the classical method of teaching. *International Electronic Journal of Mathematics Education, 16*(1), Article em0616. https://doi.org/10.29333/iejme/9283

Pavlakos, G., Spyrou, P., & Gagatsis, A. (2001). The role of angle in understating the vectors. In *4th Mediterranean Conference on Mathematics Education* (pp. 201-213). Mediterranean Society for Mathematics Education.

Sabah, S. (2023). Science and engineering students' difficulties in understanding vector concepts. *Eurasia Journal of Mathematics, Science and Technology Education, 19*(8), Article em2310. https://doi.org/10.29333/ejmste/13431

Tamam, B., & Dasari, D. (2021). The use of GeoGebra software in teaching mathematics. *Journal of Physics*, *1882*(1), Article 012042. https://doi.org/10.1088/1742-6596/1882/1/012042

Tobias, M., Frank, F., & Rolf, B. (2018). A vector is a line segment between two points? -Students' concept definitions of a vector during the transition from school to university. *HAL*-open science, 2185-2192.

Tuda, Sh., & Rexhepi, Sh. (2023). Exploring exponential functions using GeoGebra. *Brillo Journal, 3*(1), 43-58. https://doi.org/10.56773/bj.v3i1.45

Tuda, Sh., & Rexhepi, Sh. (2024). GeoGebra impact in avoiding common mistakes students make in handling exponential functions. *Mathematics & Informatics*. https://doi.org/10.53656/math2024-4-7-geo

Wassie, Y., & Zergaw, G. A. (2019). Some ofthe potential affordances, challenges and limitations of using GeoGebra in mathematics education. *EURASIA Journal of Mathematics, Science and Technology Education*, 15(8), Article em1734. https://doi.org/10.29333/ejmste/108436

Watson, A., Spyrou, P., & Tall, D. (2003). The relationship between physical embodiment and mathematical symbolism: The concept of vector. *The Mediterranean Journal of Mathematics Education, 1*(2), 73-97.

Widada, W., Herawaty, D., Beka, Y., Sari, R. M., Riyani, R., & Nugroho, K. U. Z. (2020). The mathematization process of students to understand the concept of vectors through learning realistic mathematics and ethnomathematics. *Journal of Physics, 1470*, Article 012071. https://doi.org/10.1088/1742-6596/1470/1/012071

Wutchana, U., Bunrangsri, K., & Emarat, N. (2015). Teaching basic vector concepts: A worksheet for the recovery of students' vector understanding. *Eurasian Journal of Physics and Chemistry Education, 7*(1), 18-28. https://doi.org/10.51724/ijpce.v7i1.43

Zaka, O. (2018). *Gjeometria analitike* [Analytical geometry]. Vilamasi.

Zejnullahu, R., Gjergji, R., Berisha, F., Zejnullahu, A., & Limani, R. (2006). *Matematika 9*. Peje.