International Electronic Journal of Mathematics Education
2024, 19(4), em0792
e-ISSN: 1306-3030
https://www.iejme.com

MODESTUM

Research Article

OPEN ACCESS

Specialized knowledge of in-service primary education teachers to teach probability: Implications for continuous education

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Citation: Franco Seguí, J., & Alsina, Á. (2024). Specialized knowledge of in-service primary education teachers to teach probability: Implications for continuous education. *International Electronic Journal of Mathematics Education, 19*(4), em0792. https://doi.org/10.29333/iejme/15153

ARTICLE INFO

Received: 11 Jan. 2024

Accepted: 11 Aug. 2024

ABSTRACT

The knowledge of in-service primary education teachers to teach probability is evaluated based on the mathematics teacher's specialized knowledge (MTSK) model. Starting with a mixed methodology, the MTSK-probability questionnaire, validated beforehand, is used. Findings show insufficient specialized knowledge to teach probability: generally, the mean scores are 12.95 out of 38; more specific, the data indicate that scores in the sub-domains knowledge of topics, knowledge of practices in mathematics, and knowledge of mathematics teaching are significantly better than knowledge of the structure of mathematics, knowledge of features of learning mathematics, and the knowledge of mathematics learning standards, although the average norm scores in all knowledge subtypes are below 45 out of 100. We conclude that it is necessary to design continuous education programs that use different instruments and strategies to provide a more global approach to teacher's knowledge, for example, with reflection on practice or learning communities, with the aim of improving the specialized knowledge of in-service primary education teachers to teach probability.

Keywords: specialized knowledge of mathematics teachers, teaching probability, teacher professional development, continuous education, primary education

INTRODUCTION

In recent years, due to scientific and technological advances, the study of probability has been included in school curricula from an early age to promote numeracy in this area (Batanero, 2013; Batanero et al., 2021; National Council of Teachers of Mathematics [NCTM], 2000, Nunes et al., 2015; Vásquez & Alsina, 2014). These days, people are surrounded by all kinds of information where random phenomena, chance and uncertainty are present. Given this reality, it is especially important for students to acquire knowledge of essential elements of probability so as to have a critical vision and greater rigor when making decisions involving uncertainty (Gal, 2012). In some curricula in different countries, the contents of probability occupy the same level of importance as the rest of mathematical contents (Inzunza & Rocha, 2021).

It is clear that teaching staff must have adequate knowledge of probability if they are to teach this subject. In other words, teachers need a thorough understanding of the characteristics of mathematical knowledge (MK) so they can recognize uncertainty in real contexts through experimentation (Gal, 2012; Molina et al., 2022). Moreover, various researchers have confirmed the need to consider the practical and pedagogical problems involved in teaching probability (Batanero et al., 2016). In this sense, considering the notion of conditional probability, considering only the mathematical facet to understand how students reason under conditions of uncertainty is not adequate to provide a holistic view of students' understanding of probability (Kapadia & Borovcnik, 2010). Thus, by studying conditional probabilistic reasoning, it is possible to interpret whether teachers or future teachers accept or ignore what they learn during their training, and how this reasoning may influence their teaching in the future (Elbehary, 2020).

In generic terms, recent decades have seen an increased interest in the knowledge of teachers to teach the various subjects of the curriculum. Since the strategies of Shulman (1987), this interest has grown to become a significant topic for the mathematical education research community (Aguilar-González et al., 2018, p. 43).

In this study, we opted for the mathematics teacher's specialized knowledge (MTSK) model proposed by Carrillo et al. (2018) to investigate the specialization of teacher knowledge when teaching mathematics, where the specialization affects all the

The authors declare that this study is developed within the framework of a doctoral thesis.

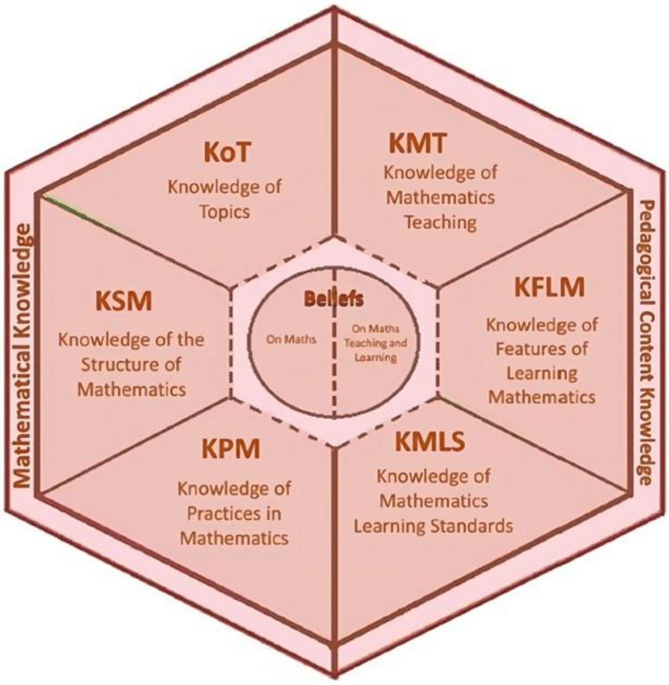

Figure 1. Theoretical model of mathematics teacher's specialized knowledge (Carrillo et al., 2018)

subdomains. For this reason, based on this model, we have characterized the knowledge that primary education teachers should mobilize for the teaching of probability.

Based on the theoretical tools provided by the MTSK, a preliminary study was designed, and a questionnaire was validated called MTSK-stochastic probability (Franco & Alsina, 2023) to evaluate the specialized knowledge of primary school teachers to teach probability. In this new study, this questionnaire was administered to 25 in-service primary education teachers in order to analyze the specialized knowledge that teachers at this stage mobilize to teach probability.

THEORETICAL FOUNDATION

Mathematics Teacher's Specialized Knowledge Model

MTSK is a theoretical model of the specialized knowledge of mathematics teachers that considers the advances of its predecessors (Ball et al., 2008; Godino et al., 2017; Shulman, 1986) in an effort to overcome their limitations. In this regard, specialization affects every subdomain (Carrillo et al., 2018). The MTSK model of Carrillo et al. (2018) focuses on the teaching of mathematics to understand the specialized knowledge of teachers and, in turn, serves as a methodological tool to analyze different practices across the different domains of knowledge. Additionally, with respect to its predecessors, the model considers conceptions and beliefs as the third domain (Carrillo et al., 2018). In short, it is crucial for teachers to have specialized knowledge in order to teach mathematics effectively, since this lets them adapt their teaching to the needs and skills of their students, thus encouraging more meaningful learning.

The MTSK model (**Figure 1**) thus proposes three domains of knowledge:

(1) MK, which considers knowledge of topics (KoT), knowledge of the structure of mathematics (KSM), and the knowledge of practices in mathematics (KPM),

(2) *Pedagogical content knowledge* (PCK), which comprises the knowledge of mathematics teaching (KMT), knowledge of features of learning mathematics (KFLM), and the knowledge of mathematics learning standards (KMLS), and

(3) *Beliefs* about mathematics and its teaching and learning.

Regarding the content standard analyzed, Escudero-Domínguez et al. (2016) underscore that, regardless of educational level, the standard of data analysis and probability is the least analyzed so far.

Considering this shortage of studies, the systematic review carried out by Franco and Alsina (2022a) yields an overview of the scientific articles published on the teaching of probability in primary school between 1997 and 2021. The research found during that period uses different knowledge models, such as CCDM (Godino et al., 2017) and MKT (Ball et al., 2008). By contrast, no research was found with the MTSK model. A recent, more detailed review of the literature that considered other papers besides scientific papers published in major databases found some research on the specialized knowledge of teachers for teaching probability. For example, León et al. (2020) analyzed seven elementary school teachers' knowledge of students' learning characteristics in conditional probability, revealing a low knowledge of KFLM and identifying teacher limitations to recognize bias, fallacies and confusion in student responses. In Cardeñoso and Azcárate (2004) point out a probabilistic knowledge of an intuitive and non-formalized nature, evidencing that the knowledge of the subjects (KoT) is scarce and the valuation of the world of

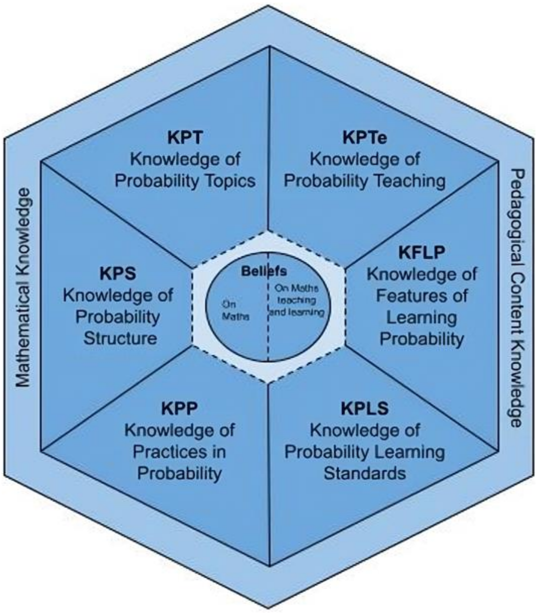

Figure 2. Adaptation of the MTSK model to the teaching of probability (Source: Authors' own elaboration)

uncertainty and its treatment can influence its teaching. In turn, in the research by Vásquez and Alsina (2017), 7 hypothetical situations are analyzed, dealing with the independence of events and the notion of randomness, the calculation and comparison of probabilities of events (elementary, not equiprobable, and random). Linking the results of this study with the KoT, a low mastery is observed, since the average number of correct answers on the understanding of the independence of events is 22.4%.

Specialized Knowledge of Primary Education Teachers to Teach Probability

Considering these preliminary data, considering the particularities of the MKT, CCDM, and MTSK models, in Franco and Alsina (2022b) link the facets and subdomains of knowledge. In addition, the authors characterize the knowledge that teachers should mobilize when teaching probability, from the perspective of the domains and subdomains of MTSK (**Figure 2**).

The different sub-domains adapted to the teaching of probability are summarized below.

Knowledge of probability topics (KoT/KPT)

Knowledge of probability topics focuses on the phenomenological aspects, meanings and definitions of concepts, examples, and so on, that characterize specific aspects of probability. Thus, for example, the historical-epistemological development of probability has given rise to the coexistence of different meanings: frequential, intuitive, Laplacian or classical, subjective and axiomatic meaning (Batanero, 2005). Elsewhere, Vásquez and Alsina (2019a, 2019b) state that all the meanings can be presented in primary education except the axiomatic.

Knowledge of probabilistic structure (KSM/KPS)

Knowledge of probabilistic structure encompasses different anticonceptual connections, in which teachers have to consider the different meanings of probability: in the *intuitive approach*: chance, variability, the event (guaranteed, possible and impossible), possibility; in the *classical approach*: game of chance, favorable cases (Laplace rule), equitable game; in the *frequency approach: test, absolute frequency*, relative frequency, estimated value of probability; in the *subjective approach*: verbal reasoning and casual language with no numerical support when processing and integrating experimental information (Gómez et al., 2014; Vásquez & Alsina, 2017)

Knowledge of practices in probability (KPM/KPP)

Knowledge of practices in probability considers the different ways of knowing, creating or producing probability. From this perspective, some authors state that teachers must use real, meaningful and motivational contexts for students (Alsina et al., 2020). In this sense, in real life, probability is related to the ability to effectively participate in and manage real-world demands, roles, and tasks that involve uncertainty and risk (Gal, 2012). The author proposes three characteristics to participate in and manage probability in real life: involve, handle, and uncertainty and risk.

- **Involve:** Prepare students to engage effectively in different real-life situations that pose mathematical and probabilistic demands. This requires a variety of cognitive skills, knowledge bases and beliefs, attitudes, and a critical stance. In short, for people to feel comfortable tackling tasks.
- **Handle:** Most arithmetic situations do not have solutions that can be classified as correct or incorrect. That is, adults handle situations and can decide what action to take based on their personal goals or the context in which the action takes place.

- **Uncertainty and risk:** Consideration should be given to the nature of the actual tasks facing adults, for example, issues relating to health-related decisions (Schapira et al., 2008) or the managing of financial affairs (Lusardi & Mitchell, 2007). The importance of probability lies in separately related ideas, such as the level of certainty or uncertainty one may have regarding the occurrence of future events, or their degree of predictability. In this regard, the degree of uncertainty experienced may be the basis of one's perception and ability to assess the risk associated with events or outcomes of relevance to their life (Gal, 2012).

Knowledge of probability teaching (KMT/KPTe)

Knowledge of probability teaching covers the different strategies that allow the teacher to promote the development of probabilistic skills. In this sense, to help students develop probabilistic reasoning, Batanero and Godino (2002) state that teachers must

(1) provide a wide variety of experiences that can be used to observe random phenomena and differentiate them from deterministic ones,

(2) stimulate the expression of predictions about the behavior of these phenomena and the results, as well as their probability,

(3) organize the collection of experimental data to give students the ability to contrast their predictions with the results produced and review their beliefs based on the results,

(4) highlight the unpredictability of each isolated result, as well as the variability of the small samples, by comparing the results of each child or in pairs, and

(5) help to identify the convergence phenomenon by accumulating results from the whole class and comparing the reliability of small and large samples.

Knowledge of features of learning probability (KFLM/KFLP)

Knowledge of features of learning probability refers to the process of students' understanding of the different contents, the language associated with each probabilistic concept, as well as potential errors, difficulties or obstacles in the student learning process. Students must build their knowledge through a gradual process, based on their mistakes and effort (Batanero, 2005). Against this backdrop, teachers must consider aspects related to students' beliefs and intuitions since the teaching of probability can be affected by biased beliefs that distort information and cause systemic errors in problem solving. In other words, by observing and interpreting the approaches used by students to solve probabilistic problems, teachers should develop strategies to prevent learning difficulties (González et al., 2017).

Knowledge of probability learning standards (KMLS/KPLS)

Knowledge of probability learning standards focuses on the probabilistic content required to be taught in a given course, making it possible to improve the practice of teachers who teach probability. In this regard, Vásquez and Alsina (2014) note certain essential aspects that should be considered when teaching probability based on a review of curricular approaches in the American, Spanish and Chilean primary education curricula:

(1) collects and classifies qualitative and quantitative data,

(2) makes and interprets very simple graphs,

(3) identifies random situations,

(4) makes conjectures and estimates about certain games,

(5) solves problems that imply mastery of the contents of probability, and

(6) reflects on the problem-solving process.

From this perspective, the current Spanish curriculum develops a proposal for the area of mathematics aimed at achieving the general objectives of the stage and the development and acquisition of the key competencies that students should achieve at the end of primary education (MEFP, 2022). **Table 1** shows the contents of the stochastic sense that the Spanish curriculum establishes as learning objectives at the end of the stage.

In synthesis, considering the mathematical and pedagogical knowledge described above, it is necessary for teachers to have different knowledge in order to carry out effective probability teaching (Franco & Alsina, 2022b). However, research on the knowledge of future teachers and in-service teachers in the teaching of probability is not very encouraging as it shows difficulties in different aspects of the content to be taught (Franco & Alsina, 2022a, 2022b). Consequently, the literature highlights the importance of reinforcing teacher education (e.g., Chick & Pierce, 2008; Estrada & Batanero, 2020; Franklin & Mewborn, 2006; Ruz et al., 2020). From this perspective, the aim of the study is to analyze the specialized knowledge of teachers in the teaching of probability, obtaining results that in the future will allow us to design education programs that are better adapted to the needs of teachers (Pascual et al., 2023).

METHODOLOGY

This research relied on a mixed approach methodology of an exploratory-descriptive nature that combines both quantitative and qualitative analyses (Creswell, 2014).

Table 1. Basic knowledge of the stochastic sense of the Spanish curriculum

Variable	Definition
Data organization and analysis	▪ Statistical data sets and graphs of everyday life: description, interpretation, and critical analysis. ▪ Strategies for conducting a simple statistical study: formulation of questions, and collection, recording and organization of qualitative and quantitative data from different experiments (surveys, measurements, observations …). ▪ Absolute and relative frequency tables: interpretation. ▪ Simple statistical graphs (bar chart, pie chart, histogram, etc.): data representation and selection of the most convenient one. ▪ Measures of centralization (mean and mode): interpretation, calculation, and application. ▪ Measures of dispersion (range): calculation and interpretation. ▪ Calculator and other digital resources to organize statistical information and perform different data visualizations. ▪ Relationship and comparison of two sets of data from their graphical representation.
Uncertainty	▪ Uncertainty in everyday situations: quantification, subjective estimation and verification. ▪ Calculation of probabilities in experiments, comparisons or investigations (Laplace's rule).
Inference	▪ Identification of a data set as a sample of a larger set and reflection to apply simple research findings.

Table 2. Relationship between items in the MTSK-stochastic questionnaire of probability and the subdomains of the MTSK model

Subdomains	Items				
	1	2	3	4	5
Knowledge of probability topics (KPT)	1a)	2a) and 2b)	3a)	4a) and 4c)	5a)
Knowledge of probability structure (KPS)	1d)	2b) and 2e)	3c)	4d)	5b)
Knowledge of practices in probability (KPP)	1a)	2a)	3a) and 3c)	4a)	5c)
Knowledge of probability teaching (KPTe)	1a) and 1c)	2d)	3c)	4c)	5c)
Knowledge of features of learning probability (KFLP)	1b)	2c) and 2d)	3b) and 3c)	4b)	5a) and 5c)
Knowledge of probability learning standards (KPLS)	1d)	2e)	3a)	4d)	5b)

Table 3. Content of the set of items that make up the MTSK-stochastic questionnaire of probability

Content	Items				
	1	2	3	4	5
Random	x	x	x	x	x
Possible, probable events	x	x	x	x	x
Probability calculation				x	x

Participants

The sample selected for this study consisted of 25 in-service primary education teachers who teach in different schools in Catalonia and the Balearic Islands, in Spain. The sample was selected through non-random convenience sampling to voluntarily respond to the previously validated MTSK-stochastic probability questionnaire (Franco & Alsina, 2023).

The sample of participating teachers consisted of 22 women (88%) and 3 men (12%). In terms of years of experience, the vast majority of the participants (47.6%) had under two years of experience, 28.6% had between 2 and 5 years, 14.3% had between 5 and 10 years and only 9.5% had more than 10 years of experience. When asked if they felt qualified to teach probability, 14.3% said they felt very qualified, 61.9% moderately qualified and 23.8% did not feel qualified. When asked if they teach probability in their courses, 47.6% answered affirmatively while 52.4% did not teach it. Likewise, 66.7% said they received training in probability during their university education, and 71.4% said they received training in the teaching of probability.

Design and Procedure

Data were obtained by administering the MTSK-stochastic probability questionnaire (Franco & Alsina, 2023), which consists of two sections: first, a section to obtain descriptive data on the participants, for example, questions about their age, gender, education, etc.; and second, a section dedicated to solving different problems that place teachers in hypothetical mathematics teaching scenarios. In total, it includes five items–some written by the authors and others reformulated from the research of Begg and Edwards (1999), Estrella (2010), Estrella et al. (2015), and Vásquez and Alsina (2015) to determine the different probability knowledge that teachers have. In addition, it should be noted that each question provides an answer to a subdomain of the MTSK as seen in the **Table 2**. The questionnaire containing 19 questions, meaning the maximum possible score is 38 points when assigning 2 points per correct answer. The items in the questionnaire are shown in **Appendix A**.

Table 3 shows the contents of the different items that comprise the questionnaire.

Data Analysis

Once the answers were collected, the data were analyzed so as to integrate the findings from both methodologies. The answers were coded based on how correct they are, assigning a 0 for a blank or incorrect answer, a 1 if it is partially correct and a 2 if it is correct for the quantitative data. The criteria for defining the category to which the answers belong were specified by means of a rubric. It should be noted that an answer is deemed to be correct when the teachers justify their answer, identifying various difficulties, content presented, curricular level of the questions and/or different teaching strategies. In the case of qualitative data, the teachers' responses were recorded in order to look for patterns and key issues in the data, then categorize them and analyze the relationship between the different responses.

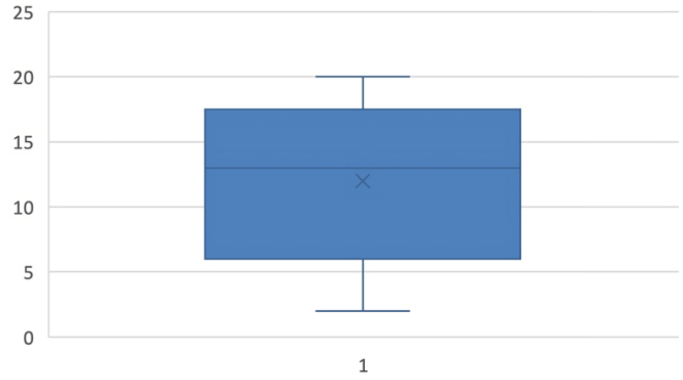

Figure 3. Distribution of total scores and average scores of the MTSK-probability questionnaire (Source: Authors' own elaboration)

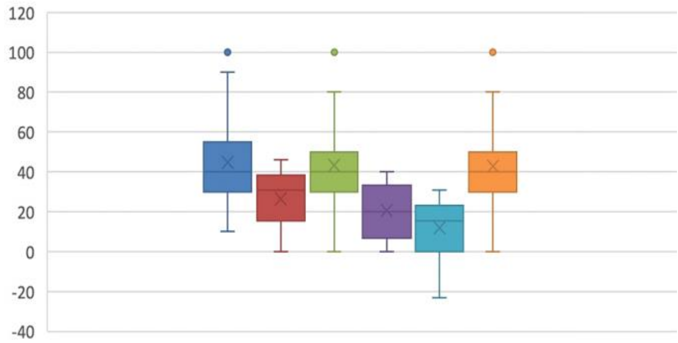

Figure 4. Distribution of normalized scores by subdomains of MTSK to teach probability (from left to right: KoT/KPT, KSM/KPS, KPM/KPP, KMLS/KPLS, KFLM/KFLP, & KMT/KPTe) (Source: Authors' own elaboration)

RESULTS

In keeping with the objectives of the study, we first conducted a general analysis of the total scores of the questionnaire and described the results obtained for the different domains and subdomains of MTSK. We then analyzed the possible influence of the teaching context on said knowledge.

Results of the Specialized Knowledge of Primary School Teachers to Teach Probability

General results

In order to analyze the total score of the questionnaire, the answers of the 25 teachers were categorized by how correct they were. The total scores obtained range between 2 and 20 points, which indicates that no teacher answered the questionnaire correctly (given that the maximum score was 38). The average score of the teachers was 12.95 points, with only two teachers obtaining a score of at least half (19 being half of the highest possible score), and the standard deviation being 0.14 points. In this regard, the average score is very low, given that it considers both correct and partially correct answers.

Figure 3 shows a low value for the median (13 points) and the values of the total scores are more concentrated in the lower area of the box. The amplitude of the lower whisker is higher than that of the top one, indicating an inequality between the extremes of the distribution of the total scores. In summary, these results indicate that primary school teachers have great difficulty answering the questions posed in relation to their specialized knowledge to teach probability.

Comparison of results of the domains and subdomains of MTSK to teach probability

In order to compare between the different subdomains of MTSK to teach probability, we recoded the total scores for each type of knowledge using a normalized scale of 0 to 100, since the number of items was different in each category. **Figure 4** compares these scores, which show that the teachers have poor specialized knowledge, since more than 50% of them failed to exceed the normalized scores. In this regard, we note that the teachers have a similar score on KoT (KoT-KPT), practices in probability (KPM-KPP) and probability teaching (KMT-KPTe), while the low level of knowledge of structure in probability (KSM-KPS), knowledge of probability learning standards (KMLS-KPLS) and features of learning probability (KFLM-KFLP) is surprising.

It should be noted that the teachers' specialized knowledge of probability is not directly observable, which is why we decided to administer the MTSK-stochastic probability questionnaire to, through their answers, deduce the teachers' level in the different subdomains, from the MK and the pedagogical knowledge of the content, that comprise the MTSK model (Carrillo et al., 2018).

We also conducted a qualitative analysis to identify in more detail the difficulties exhibited by the participants in the study. Below is the quantitative analysis of the various subdomains of knowledge from the items that give rise to partially correct or incorrect answers.

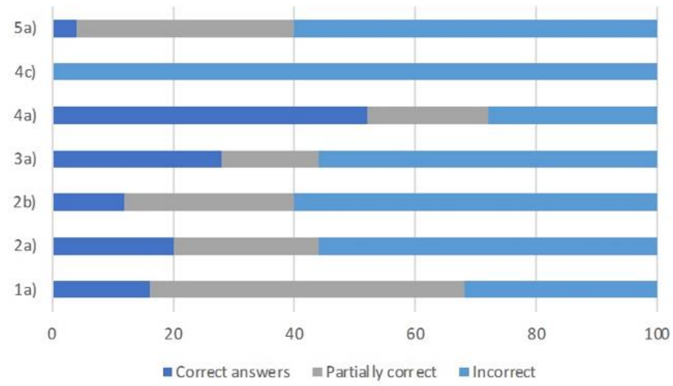

Figure 5. Composition of the different types of answers for the KoT/KPT by degree of correctness (Source: Authors' own elaboration)

Table 4. Concepts and/or properties identified by the teachers to solve the problem presented in item 4c)

Types of concepts and/or properties identified	Frequency (n)	Percentage (%)
Equally likely	19	76
Other	4	16
No answer	2	8

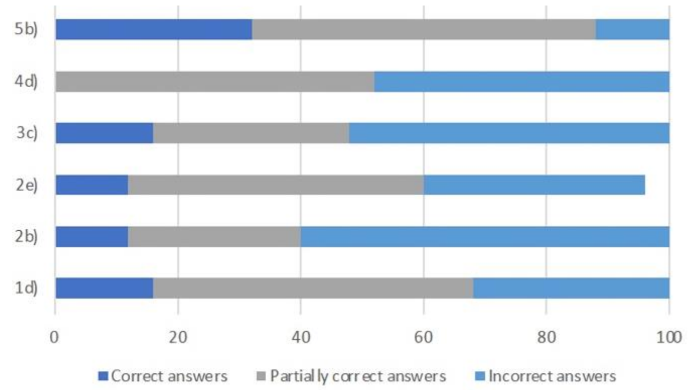

Figure 6. Composition of the different types of answers for KSM/KPS by degree of correctness (Source: Authors' own elaboration)

Knowledge of Probability Topics (KoT/KPT)

The teachers exhibit poor, and in some cases very poor, knowledge in almost all of the items that evaluate this type of knowledge, as shown in **Figure 5**. The set of items focused on evaluating the KoT delves into the answers given by the teachers when solving problems related to estimating results (1a and 3a), events (2a and 2b), the randomness of events (4a and 4c), and calculating the probability of an event (5a).

The percentage of correct answers involving KoT/KPT does not exceed 40% in most items, with only item 4a obtaining a higher score (52%). An analysis of the answers to item 1a shows a low percentage of correct answers (16%), while the partially correct answers (52%) are due to a lack of rigor in their answers, meaning only four teachers were able to accurately identify the different concepts and properties in question and correctly solve the problem involving an understanding of the range, arithmetic mean and mode.

By contrast, in the case of item 4a, more than half of the teachers (52%) managed to identify four or more correct answers from the students involving the solution to the problem. As regards items 2b and 5a, the results were very similar. In both cases, 60% of the participants failed to answer the situation presented. More specifically, the teachers gave very general answers to the different items, for example, by responding to different situations without proper justification.

When analyzing the incorrect answers to item 4c, we classified them as shown in **Table 4**. Note that the concept identified the most is "equally likely", at 76%, followed by "other", at 16% and "no answer" at 8%. One answer given was: "*All four are likely, equally likely*" (teacher 10).

Among the answers categorized as "other" were those that identified concepts that were not directly related to the notions that we sought to study through the situation presented.

Knowledge of Probability Structure (KSM/KPS)

To determine the knowledge of probabilistic structure, we analyzed the answers received for questions 1d), 2b), 2e), 3c), 4d), and 5b) of the MTSK-stochastic probability questionnaire. **Figure 6** shows that the teachers had little knowledge of the different conceptual connections that are involved in the items mentioned above. The items that exhibited the highest percentages of incorrect answers were 2b, 3c and 4d, whose objective was to evaluate the knowledge of the mathematical structure in relation

Table 5. Concepts and/or properties identified by the teachers to solve the problem presented in item 2e)

Types of concepts and/or properties identified	Frequency (n)	Percentage (%)
Numbering and calculation	4	16
Probability	17	68
Statistics	7	28
Other	2	8
No answer	1	4

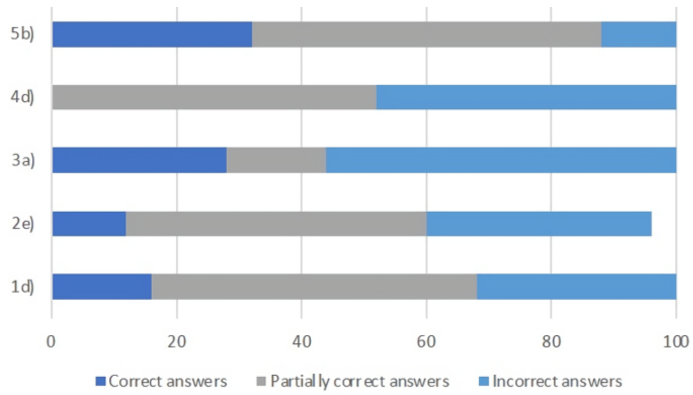

Figure 7. Composition of the different types of answers for the KMLS/KPLS by degree of correctness (Source: Authors' own elaboration)

to understanding the randomness of events. **Figure 6** shows that the teachers had difficulty solving problem situation 2b, since more than half of the participants (60%) were unable to identify cross-sectional and temporal concepts and/or properties. 28% managed to identify two or three concepts, while 12% identified four or more concepts and/or properties involved in solving item 2. Next, in the case of item 4d, the teachers had problems, since none of them managed to identify three or four concepts and properties related to the situation presented, while 52% identified two or three properties, and 48% recognized one or no concepts and/or properties related to the given situation.

Table 5 focuses on the content relevant to item 2e. Note that the concept that is involved the most is probability at 68%, relating concepts such as chance, random phenomena, guaranteed/probable/impossible result, followed by statistics at 28% and numbering and calculation at 16%. An example of a right answer is the following: "*Probability, conditional probability, number of cases, chance*" (teacher 19).

Knowledge of Probability Learning Standards (KMLS/KPLS)

To analyze the knowledge of probability learning standards, we focused on the following items: 1d), 2e), 3a), 4d), and 5b).

Figure 7 highlights the difficulties faced by teachers in understanding the school curriculum. More specifically, greater difficulty is observed in item 4d) and item 3a), since half of the participants (48% and 56%, respectively) answered the situation presented incorrectly. Even so, they differ in the right answers, since no participant answered item 4d) correctly, while 28% answered item 3a) correctly.

In this regard, we analyzed more specifically the answers to item 2e), classified as shown in **Table 6**. Note that the contents most identified by teachers were probability and statistics, at 68% and 28%, respectively. One example of an answer is, as follows: *"Probability, guaranteed/probable or impossible result"* (teacher 7).

Knowledge of Features of Learning Probability (KFLM/KFLP)

To analyze the knowledge of the KFLM/KFLP, we focused on items 1b), 2c), 2d), 3b), 3c), 4b), 5a), and 5c).

Figure 8 reveals greater difficulty with item 2c) and item 1b), since no right answers were received to the questions posed. By contrast, the results of item 2d) show that 28% of the teachers answered correctly, while more than half (56%) gave a partial answer to the problem.

Since it is the item with the highest number of incorrect answers, **Table 6** analyses more specifically the teachers' answers to item 3b), involving the possible difficulties that students may face when making estimates about likely and unlikely events when rolling dice. **Table 6** shows that the participants regard the concept of chance as the most common difficulty among students. One example of an answer is the following: "*Thinking that the first horse to move will be the winner*" (teacher 4).

Knowledge of Probability Teaching (KMT/KPTe)

In relation to the study of knowledge of probability teaching, items 1d), 2b), 2e), 3c), 4d), and 5b) were analyzed, since they refer to the different strategies used by teachers to promote the development of probability skills.

Accordingly, **Figure 9** shows a low KMT/KPTe, as evidenced by the fact that the number of correct answers did not exceed 32% for any of the aforementioned items. Moreover, the incorrect answers are in the range of 12% to 60%, with the teachers exhibiting

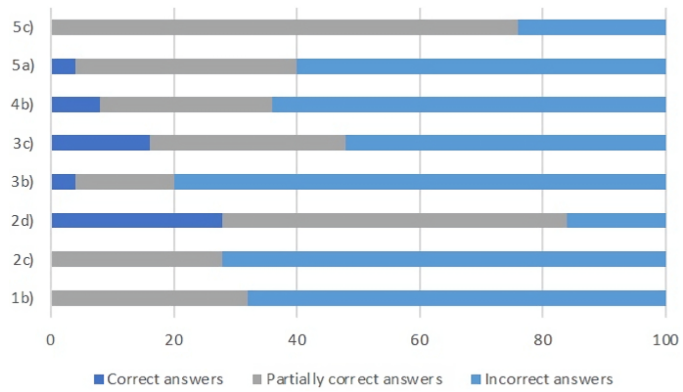

Figure 8. Composition of the different types of answers for the KFLM/KFLP by degree of correctness (Source: Authors' own elaboration)

Table 6. Difficulties identified by the teachers with solving the problem presented in item 3b)

Types of difficulties identified	Frequency (n)	Percentage (%)
Chance	11	44
Concepts in probability	4	16
Other (competitiveness, operation, etc.)	8	32
No answer	2	4

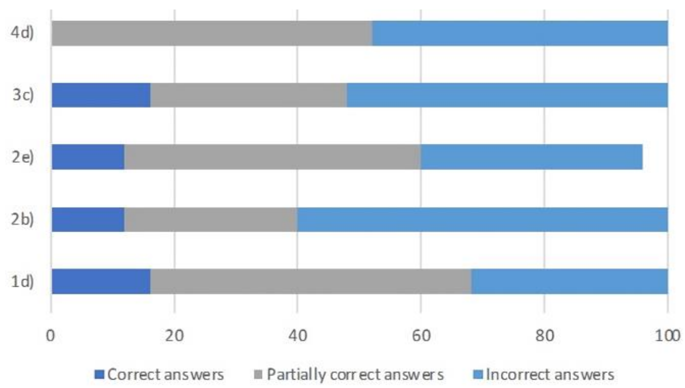

Figure 9. Composition of the different types of answers for the KMT/KPTe by degree of correctness (Source: Authors' own elaboration)

Table 7. Possible techniques used by teachers to assist in solving the problems presented in item 2b)

Types of techniques	Frequency (n)	Percentage (%)
Probability	15	60
Statistics	7	28
Other	2	8
No answer	2	8

problems proposing specific techniques to address the students' difficulties. More specifically, the item with the most difficulties was 2b), with 12% correct answers and 60% incorrect answers.

When analyzing the answers to item 2b), we classified them as shown in **Table 7**. Doing so revealed that the relevant concept that was identified the most was probability at 60%, followed by statistics at 28%. One example of an answer is as follows: "*Chance and probability: calculating the probability of an event. Combined probability: calculating the probabilities of multiple events*" (teacher 19).

Knowledge of Practices in Probability (KPM/KPP)

To analyze the KPM/KPP, we analyzed the answers obtained to questions 1a), 2a), 3a), 3c), 4a), and 5c) of the MTSK-stochastic probability questionnaire. **Figure 10** shows that the teachers have little knowledge of the processes (deductive or inductive thinking) that are involved in solving the items mentioned above. The items that exhibited the highest percentage of incorrect answers were 2a, 3a and 3c, whose goal was to evaluate the KPM in relation to the different techniques to solve situations involving estimates of events (possible/probable/impossible).

When analyzing the answers to item 2a), we classified them as shown in **Table 8**. Doing so revealed that the prevailing technique was the bar graph (28%), followed by the tree diagram. One example of an answer is the following: "*Tree diagram, so as to see in order the possible solutions and discards*" (teacher 20).

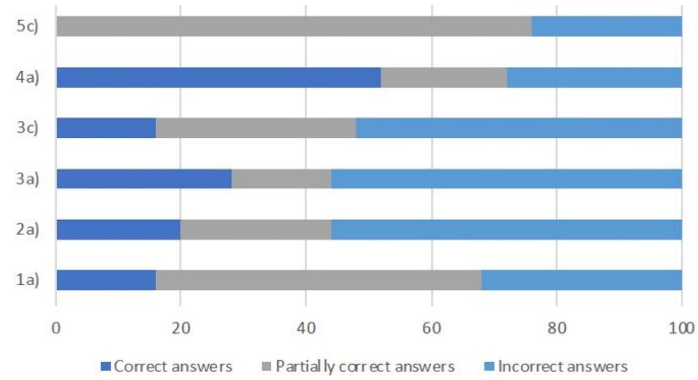

Figure 10. Composition of the different types of answers for the KPM/KPP by degree of correctness (Source: Authors' own elaboration)

Table 8. Possible techniques used by teachers to assist in solving the problems presented in item 2a)

Types of techniques	Frequency (n)	Percentage (%)
Pie chart	3	12
Bar chart	7	28
Tree	5	20
Other	7	28
No answer	1	4

DISCUSSION

This study analyzed the specialized knowledge for teaching probability mobilized by 25 in-service primary education teachers from the perspective of the MTSK model (Carrillo et al., 2018).

In general, the results revealed that their knowledge is insufficient, since the average of the total scores was 12.95 points out of a possible 38 points, with only 2 teachers scoring more than 50%. Previous research conducted with other knowledge models, such as Vásquez and Alsina (2019a), yielded similar results on the specialized knowledge of active teachers to teach probability. According to these authors, teachers cannot teach what they do not know, which ultimately has a negative impact on student learning. In relation to the above problem, it should be noted that probability was not included in the primary education curriculum until the late 1980s (Alonso-Castaño et al., 2021) when the NCTM (1989) decided to include "data and chance". In other words, one of the determining factors to understand the lack of preparation of in-service teachers is the fact that many of them were not taught probability from an early age, as has been proposed by different organizations and authors (Alsina, 2012, 2017; Alsina et al., 2020; Batanero, 2013; Bryant & Nunes, 2012; Fryre et al., 2013; Jones, 2005; NCTM, 2000; Nunes et al., 2015). More specifically, if we focus on the different subdomains of specialized knowledge, our study has shown that the results involving knowledge of probability topics (KoT/KPT), knowledge of practices in probability (KPM/KPP), and knowledge of probability teaching (KMT/KPTe) in the area of probability are significantly better than in the knowledge of probabilistic structure (KSM/KPS), knowledge of probability learning standards (KMLS/KPLS), and knowledge of features of learning probability (KFLM/KFLP). Even so, the specialized knowledge of teachers is deficient in all types of knowledge, since the average of the normalized scores in each was below 45 out of 100.

In light of these data, the study provides information of interest on the specialized knowledge of teachers for teaching probability. It does this by providing original results with in-service primary education teachers, since there is practically no research on teaching probability that relies on the MTSK model. As already noted, these results reveal the teachers' low knowledge, and consequently the need to be trained in the pedagogy of probability to improve the level of mathematical and didactic knowledge of primary education teachers. Likewise, the NCTM (2000) advanced the teaching of probability to the age of 3, which implies that teachers in the early childhood and primary education stage must have both didactic and content knowledge in order to effectively teach probability (Alsina, 2012, 2020).

The main limitations of the study involve the sample size and intrinsic characteristics of the questionnaire. On the one hand, due to various factors, including the lack of time to answer the questionnaire, only 25 in-service primary teachers participated in the study. Also, although it was not explicitly stated, another factor may have been a fear of feeling evaluated in an area that is outside their realm of expertise. In any case, the sample analyzed could limit the generalization of the quantitative and qualitative results. On the other hand, it is possible that the teachers' answers do not fully reflect their knowledge, given that for some items, the participants provided answers that were too broad or, on the contrary, too brief, meaning they could have been more specific and analyzed the finer points of their answers. In addition, it is possible that giving a verbal explanation for some of the questions asked in the items increases the level of difficulty compared to a schematic or numerical resolution. In this regard, future research could offer different alternatives, such as solving the question manually and uploading it with a photograph. From this perspective, the duration of the questionnaire (one hour or more, as the questions were open-ended) may have limited the sample, and thus their answers to the questionnaire. To overcome these limitations, in future research we will look for alternatives such as analyzing the data through video recording of the teachers when they present probabilistic content in the classroom to their students.

CONCLUSIONS AND IMPLICATIONS FOR CONTINUOUS EDUCATION

By way of conclusion, these results could help instructors of in-service teachers to design interventions to improve the specialized knowledge of teachers for teaching probability. From this perspective, given the importance of data analysis and interpretation in an increasingly computerized society (Batanero, 2001; Batanero et al., 2011), it is essential that teachers mobilize solid mathematical and didactic knowledge to promote the probabilistic numeracy of students from an early age (e.g., Alsina, 2017, 2021; Bryant & Nunes, 2012; Jones, 2005; NCTM, 2000; Nunes et al., 2015; and others).

To conclude, by identifying difficulties in the different subdomains for teaching probability, different authors highlights the importance of reinforcing teacher education (e.g., Estrada & Batanero, 2020; Ruz et al., 2020). In order to better understand the knowledge of teachers and the relationship of factors such as the teaching context, future research should continue to more broadly study the specialized knowledge of in-service teachers when teaching probability so as to evaluate the effects of their knowledge on the teaching and learning of students. In other words, a more comprehensive approach to teacher knowledge can be achieved by using a variety of teaching tools such as interviews, video recording, classroom observation, field notes, written productions, etc. used in most of the research analyzing teacher education strategies. For example: classroom observation allows for the identification of learning opportunities to improve the instruction of future teachers (Youngs et al., 2022); video recordings provide examples and opportunities for improving teacher noticing through the use and connection of mathematical representations, in a lesson study context (Suh et al., 2021); interviews allow for revealing teachers' beliefs about learning and teaching (Doruk, 2014). In conclusion, then, these instruments provide better information about teachers' knowledge and a route to expanding their probability literacy and thus moving towards more effective teaching of content.

Future studies will need to further investigate the effect of these tools to improve the specialized knowledge for teaching probability in order to design more specific education programs.

Author contributions: JFS & ÁA: study conception and design; **JFS:** material preparation, data collection, data analysis, and writing the first draft; **ÁA:** commented on previous versions of the manuscript. Both authors have agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Ethical statement: The authors stated that this study has the approval of the ethics committee of the university, the Academic Commission of the PhD program in education, evaluated favorably on 03/03/2021, its Research Plan. Written informed consents were obtained from the participants.

Declaration of interest: No conflict of interest is declared by authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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APPENDIX A

Table A1. The MTSK-stochastic probability questionnaire (Franco & Alsina, 2023)

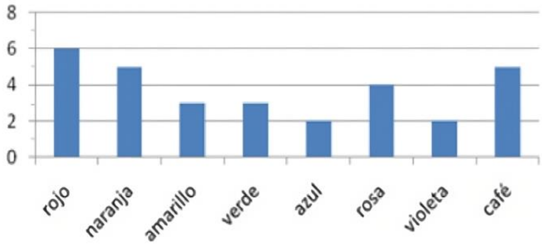
Items	Definition
Item 1	In the Balearic Islands, various opinion polls were conducted to determine the level of support for a candidate for president in the next election. Four newspapers conducted separate polls across the region. The results of the surveys of the four newspapers for the level of support for the candidate are shown below: - Newspaper 1: 36.5% (survey conducted on 6 January, with a sample of 500 randomly chosen citizens entitled to vote). - Newspaper 2: 41% (survey conducted on 20 January, with a sample of 500 randomly chosen citizens entitled to vote). - Newspaper 3: 39% (survey conducted on 20 January, with a sample of 1000 randomly chosen citizens entitled to vote). - Newspaper 4: 58.5% (telephone survey conducted on 20 January with 1000 readers of the newspaper). If the election were held on 25 January, which of the newspaper results would be the best predictor of the level of support for the presidential candidate? A. Newspaper 1, because it was a random sample at the beginning of the month B. Newspaper 4, because there were 1000 voters, and it was a telephone survey C. Newspaper 3, because it was close to the election, and it was a random sample of 1000 voters. D. Newspapers 3 and 4, because their surveys were on the same date and the sample size was the same in both surveys. *Questions:* a. Choose the option you think is right. Explain your reasoning and try to make a case to convince others of your answer. b. Describe the possible problems or obstacles that may lead students to doubt the best possible prediction. c. What teaching strategies would you use to help students who had difficulties or made errors to solve the problem presented? d. To what mathematical content can we relate the content involved in this problem?
	MTIA context: Informal context/real situation
Item 2	A box contains 4 red, 3 green and 2 white balls. How many balls have to be taken out to make sure that one ball of each color is picked? The answers received from some of the students are as follows: **Carla**: Three, because there are three different colors. **Antonio**: You would have to take them all out to be sure. **Raúl**: If you took out the red and green balls first, that would make seven, but since you want one of each color, then, eight **Karina**: To be sure, you would have to take out six balls, because if there are nine in total and there are three colors, you have to leave three balls in the box, one of each color. *Questions:* a. What graphic model would you use to solve the problem? Why? b. What mathematical concepts and/or properties should students use to give a correct solution to this problem? c. What difficulties can students have in finding the best possible prediction? d. How would you as a teacher intervene to help the students overcome their difficulties? e. To what mathematical content can we relate the content involved in this problem?
	MTIA context: Informal context/manipulatives
Item 3	The next activity will consist of having a horse race. Each student has to roll the two dice and move the corresponding horse figure. The first one to reach the end wins the race. Before presenting the problem to her students, the teacher will give them time to guess, play, decide the potential winner, etc. They can also spend a few minutes playing with the following applet: https://www.proyectodescartes.org/descartescms/matematicas/item/1236-la-nocion-de-probabilidad-y-la-carrera-de-animales The students must then calculate each horse's winning chances. Which horse has the best chances? The worst? Why? Once calculated, the students will play several rounds and see which horse wins in each case. *Questions:* a. Which answer(s) should be accepted as correct? Why? b. What difficulties might the students encounter in arriving at the solution? c. If a student argues that the winning horse will always be 6 because it is the number in the middle. Or, if a student says that horse 12 has zero chances of winning a race. The teacher could provide useful feedback to these answers with the aim of letting them develop their mathematical knowledge based on what they know and their reasoning.
	MTIA context: Games
Item 4	Consider at the following graphic:  There is a bag with colored candies. Trini's teacher lets her take a candy out of the bag, but without looking inside. The graph above shows the number of candies of each color. - What is the probability that Trini will pull out a red candy? *Questions* a. What difficulties might the students have solving the problem? b. For what educational level do you think the above question is best suited? What mathematical knowledge is involved? Why? *What techniques would you use to overcome the difficulties or errors of the students?*
	MTIA context: Formal contexts/graphic resources

Table A1 (Continued). The MTSK-stochastic probability questionnaire (Franco & Alsina, 2023)

Items	Definition
Item 5	With the app at https://pinetools.com/coin-flipper, you flip a coin five times. a. Which of the following sequences are the most and least likely to occur? - Head, heads, heads, tails, tails - Heads, heads, heads, tails, heads - Tails, heads, tails, tails, tails - Heads, tails, heads, tails, heads - All four sequences are equally likely or unlikely b. What would you expect on the sixth flip assuming the Tails, Heads, Tails, Tails, Tails sequence for the first five flips? Why? - Heads - Tails - Equally likely - Other *Questions:* c. Which answer(s) should be accepted as correct? Why? d. Describe the potential problems and errors that underlie the incorrect answers. *Do you know of any kind of definition of probability that you could use to explain this problem? How would you implement it in this case?*
	MTIA context: Technological resources