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Prior knowledge of a calculus course: The impact of prior knowledge on students' errors

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ABSTRACT

We conducted a descriptive study among first-year engineering students at the Anton de Kom University of Suriname. We analyzed students' errors regarding necessary prior knowledge in a calculus A exam. We found that the stage of the solution in which prior knowledge is required impacts the importance of prior knowledge. We also found that many errors concerned basic algebra and trigonometry concepts and skills. We concluded that even though the required prior knowledge concerns basic algebra and trigonometry, the stage of the solution in which prior knowledge is needed is of great importance.

Keywords: prior knowledge, error analysis, calculus, prerequisite

INTRODUCTION

Globally, undergraduates commonly struggle with first-year calculus courses. For example, the passing rate of calculus I in the United States was 73 (Bressoud et al., 2015 as cited in Hurdle & Mogilski, 2022). At the Faculty of Technological Sciences (FTeW) of the Anton de Kom University of Suriname, 55% of students passed the calculus A exam in 2019.

Calculus A is a first semester course for engineering students at the FTeW. To complete this course successfully, students require thorough knowledge of high school mathematics. Students are expected to have basic knowledge of algebra, equations, functions and trigonometry, as well as the ability to calculate simple limits and derivatives.

The course calculus A consists of the following subjects: proof of mathematical induction, complex numbers, the concept of limits and calculating limits, rules of differentiation, continuity and differentiability of functions and word problems concerning related rates.

Knowledge of all of these subjects was evaluated via a three-hour exam with open questions. The exam is taken after an eight-week module.

Many factors, such as insufficient knowledge of the subject matter, an incorrect approach to studying, language barriers and the teacher's didactics, may contribute to poor course results (Ancheta & Subia, 2020; Domondon et al., 2023).

Makamure (2021) and Segura and Ferrando (2023) focused on teachers' didactics in their research. In his study, Makamure (2021) emphasized the importance of a thorough error analysis, particularly the ability of teachers to identify, interpret and remediate students' errors. He concluded that most teachers lack this ability and stated that conducting a proper error analysis is part of good mathematics teaching. He also emphasized that a lack of prior knowledge is one of the causes of students' errors. In addition, Segura and Ferrando (2023) conducted a study on teachers' flexibility and performance in solving real open-ended problems. According to them, flexibility in solving a problem is the ability to choose different solution strategies for the same problem. This flexibility is very important because it allows the learner to build deep and connected knowledge; a highly flexible teacher will be a better educator and will be able to perform a decent error analysis.

The aim of this study was to investigate the role of prior knowledge in calculus A to understand students' poor results. We placed particular focus on topics of prior knowledge, where students made errors, rather than analyzing the errors themselves, which is research that can be pursued in a future study.

We address the following main research question:

Do students have sufficient prior knowledge to successfully solve calculus A exam problems?

We also address the following subquestions:

1. What prior knowledge is required for each topic of the exam?
2. Which prior knowledge errors do students make in the calculus A exam?
3. What is the impact of prior knowledge on solving each exam question?

THEORETICAL FRAMEWORK

Much research involving prerequisite knowledge and errors in first-year calculus has been conducted over the years (e.g., Hurdle & Mogilski, 2022; Khairani et al., 2019). In some studies, a pre-calculus test was developed to investigate the knowledge of students even before they started the calculus course (e.g., Ancheta & Subia, 2020; Rach & Ufer, 2020).

Hailikari et al. (2008), as cited in Rach and Ufer (2020), stated that the most significant factors influencing students' success in their studies are their cognitive prerequisites, such as their prior knowledge (e.g., Hailikari et al., 2008; Kosiol et al., 2019).

In this paper, we investigated the impact of prior knowledge on a calculus exam and defined prior knowledge as all the techniques, procedures and concepts that have been discussed in the students' previous education.

In his study, Radatz (1979) investigated the information processing of mathematics. According to Radatz (1979), a lack of prerequisite skills, particularly prior knowledge, is a possible cause of student error. He distinguished five categories of errors by investigating the way students learn mathematics, that is, how they obtain, process, retain, and reproduce mathematical content. Radatz (1979) identified the following categories of errors: Errors due to language difficulties, which occur in the translation from a problem in natural language to mathematical language. Errors due to difficulties in obtaining spatial information thus a lack of spatial awareness. Errors due to deficient mastery of prerequisite skills, facts, and concepts. These students lack the necessary skills and prior knowledge to solve a problem. Errors due to incorrect associations or rigidity of thinking. Students make incorrect links in problem-solving activities. They apply general rules and procedures without first checking the necessary conditions for a specific case. For example, students use the rule $a.b=0 \Leftrightarrow a=0 \vee b=0$ even though the constant was not zero. Errors due to the application of irrelevant rules or strategies. Students use incorrect methods or procedures while solving problems. For example, they use the quadratic formula to solve a third-order equation.

Rach and Ufer (2020) stated that the relation between the level of prior knowledge, and a successful start of the mathematics program was not yet specified in research. They also concluded that previous research only emphasized that the more prior knowledge students have, the better they will perform. In their study they developed a model of four different levels of mathematical knowledge.

The first level concerns knowledge of facts, and problems at this level can be solved using standard procedures. The next two levels require conceptual knowledge, and there are no routine procedures for the solution of problems at these two levels. Problems in level 2 require students to use a known representation of a concept, while students need to link multiple representations of a concept to solve problems in level 3. Therefore, at level 3, a deeper understanding of concepts is required when solving such problems. Students must be able to construct meaning from multiple representations. Level 4 involves problems that require not only a well-connected conceptual understanding of a concept but also formal notation, for example, when giving a mathematical proof.

Rach and Ufer (2020) stated that students' prior knowledge must be at least level 3 to be successful in analysis. The contribution of their model is that the level of students' prior knowledge can predict success in analysis, and universities can provide good study advice and support for starting students.

Ancheta and Subia (2020) performed a study to identify students' misconceptions in calculus specifically algebra using a pre-calculus algebra test. They analyzed students' errors and the underlying reasons for misconceptions and concluded that students did not master the basic concepts and laws in algebra. They also stated that the underlying reasons for those misconceptions were the lack of conceptual knowledge and retention of the subject matter. Furthermore, they concluded that teachers did not pay enough attention to identify students' errors and to stimulate retention learning.

Ancheta (2022) conducted a quantitative and qualitative analysis to identify misconceptions and skills deficits in pre-calculus subjects. He used a pre-calculus test and conducted interviews with students and with teachers and found that students have misconceptions about the basic concepts in algebra, trigonometry and analytic geometry. Furthermore, he stated that error analysis is an effective tool for understanding students' mathematics errors and can benefit lecturers.

Recently, Domondon et al. (2023) performed a study on high school students learning basic calculus. The results can serve as a tool for mathematics teachers to understand students' gaps in basic calculus. They recommend performing a qualitative analysis of the errors committed in calculus courses.

In a recent study, Segura and Ferrando (2023) differentiate between strategic and mathematical errors. Strategic errors occur when an invalid strategy is chosen due to incorrect reasoning and will not lead to a solution. Mathematical errors are errors that occur during the mathematical procedure. In their 4^{th} research question, the authors investigated the relation between learner flexibility and mathematical errors. They found that the level of flexibility correlated with the number of mathematical errors made. Learners with high flexibility made fewer mathematical errors.

According to Rach and Ufer (2020), many studies have been performed on prior knowledge and study success, but to our knowledge, there is no research on the impact of prior knowledge in solving calculus A exam questions. Our study aims to contribute to this gap in literature. In our research, we investigated the prior knowledge needed for all the problems of the calculus

A exam, and we distinguished between basic and advanced prior knowledge. We considered elementary algebra and trigonometry as basic prior knowledge, namely calculations involving fractions, exponents, and radicals; special products; tangents of special angles; and trigonometric identities and properties of standard trigonometric functions. The topics: factoring cubic equations, absolute values, determining the slope of a line, rules of differentiation, laws of logarithms, knowledge of extremes and the sign table, interpretation of the derivative, and similarity of triangles are considered advanced since they call for more intricate computations and concepts. Furthermore, we determined the stage in the solution, where prior knowledge errors were made and categorized three stages: near the beginning, referred to as stage 1; near the end, referred to as stage 3; and somewhere between the beginning and the end, referred to as stage 2. The stage can be determinative for one's ability to solve the problem. This implicates that the chance for a correct solution increase when the needed prior knowledge is more towards the end of the solution. When prior knowledge is required at the beginning of the solution, it is crucial for the student to have that knowledge because otherwise, he cannot even start solving the problem. Consequently, he will fail the whole question.

MATERIALS & METHODS

According to Ancheta (2022) and Domondon et al. (2023), error analysis is an effective tool for understanding and interpreting students' errors. Classification is important for a decent interpretation of errors. In his work, Radatz (1979) categorized errors into five classes. His third category, errors due to deficient mastery of prerequisite skills, facts, and concepts, is the focus of our research. Many researchers have developed a special test for their research on prior knowledge. (e.g., Ancheta & Subia, 2020; Rach & Ufer, 2020). In this study, we did not design a special test to assess prior knowledge because, to investigate the impact of prior knowledge on students' poor results, it is useful and necessary to analyze students' performance on the exam. We opted to analyze a previous calculus exam of another lecturer.

We investigated prior knowledge errors made by students taking the calculus A exam of 2019. The participants were 82 1st year engineering students (18-22 years old). These students had studied mathematics and physics as part of their preuniversity education. The exam consisted of nine problems, some of which were divided into subquestions (for a total of 13 questions). A total score of 100 points could be earned, corresponding to a grade of 10. A grade of at least 5.5 was needed to pass the exam. In the answer key of the exam, the step-by-step solution was given along with a score for each step. Consequently, the score given to prior knowledge could be determined.

First, the exam sheets were divided between two researchers, and the analysis was performed independently, but in cases of doubt, we discussed the areas of uncertainty until we came to a conclusion.

To address the main research question, we first determined the required prior knowledge for each exam problem and listed the topics. Simple arithmetic operations and other types of elementary knowledge were excluded from this error analysis. The results are shown in **Table 1**.

Table 1. Required prior knowledge categorized in basic & advanced for main topics

Main topic	Required basic prior knowledge	Required advanced prior knowledge
Proof with mathematical induction	Calculations with fractions & exponents	
Complex numbers	Special products	Factoring & solving cubic equations
	Tangent of special angles	
Limits	Trigonometric identities & properties of trigonometric functions	Absolute values
	Calculations with fractions	
	Calculations with radicals & exponents	
Derivatives & applications		Determining slope of a line
		Laws of logarithms
		Knowledge of extremes & sign table
Related rates		Interpretation of derivative
		Similarity of triangles

Errors involving required prior knowledge are referred to as prior knowledge errors. To determine which prior knowledge errors students made, it was meaningful to first calculate the percentage of students who made errors in the identified prior knowledge of **Table 1**. From the percentages, we can determine which prior knowledge errors students made in the different subjects and whether they were basic or advanced errors. The percentage of students who made prior knowledge errors was calculated in two ways, as shown in **Table 2**.

Table 2. Percentages of students with prior knowledge errors calculated in two ways

Main topic	Prior knowledge errors	Type of prior knowledge errors	Percentage of students with prior knowledge errors	
			Not all students included (%)	All students included (%)
Proof with mathematical induction	Calculations with fractions & exponents	Basic	45	39
Complex numbers	Special products	Basic	14	12
	Tangent of special angles	Basic	23	17
	Factoring cubic equations	Advanced	49	41

Table 2 (Continued). Percentages of students with prior knowledge errors calculated in two ways

Main topic	Prior knowledge errors	Type of prior knowledge errors	Percentage of students with prior knowledge errors	
			Not all students included (%)	All students included (%)
Limits	Absolute values	Advanced	32	29
	Trigonometric identities & properties of trigonometric functions	Basic	37	33
	Calculations with fractions	Basic	25	22
	Calculations with radicals & exponents	Basic	24	17
Derivatives & applications	Determining slope of a line	Advanced	5	5
	Laws of logarithms	Advanced	66	52
	Knowledge of extremes & sign table	Advanced	29	23
Related rates	Interpretation of derivative	Advanced	68	54
	Similarity of triangles	Advanced	66	52

In the first way, we excluded students who did not complete the solution and students who had an incorrect approach (4th column of **Table 2**); the exclusion differed by question. This strategy provides a more realistic picture since we are not able to determine whether these students lacked prior knowledge. In the second way, we included the work of all 82 students so that we could compare this percentage with that of the students who correctly solved the problem (see 5th column of **Table 2**). Furthermore, we identified the impact of prior knowledge on solving exam problems; we will refer to this as the impact of prior knowledge on the solution. The results are presented in **Table 3**.

Table 3. Impact of prior knowledge

Main topic	Required prior knowledge	SPK	PPK	IRK (%)
Proof with mathematical induction	Calculations with fractions & powers	2	5 out of 10	50.0
Complex numbers	Special products	2	1 out of 8	12.5
	Tangent of special angles	3	1 out of 8	12.5
	Factoring & solving cubic equations	1	4 out of 10	100
Limits	Absolute values	1	2 out of 6	100
	Trigonometric identities & properties of trigonometric functions	1	2 out of 6	100
	Calculations with fractions	1	2 out of 6	100
	Calculations with radicals & exponents	2	4 out of 8	50.0
Derivatives & applications	Determining slope of a line	1	1 out of 8	100
	Laws of logarithms	2	2 out of 6	83.0
	Knowledge of extremes & sign table	1	4 out of 6	100
Related rates	Interpretation of derivative	2	7 out of 14	50.0
	Similarity of triangles	1	7 out of 14	100

Note. SPK: Stage of prior knowledge; PPK: Points given to prior knowledge; & IRK: Impact of required prior knowledge on solution

To solve a calculus A problem, a certain amount of prior knowledge, which might vary per question, is needed. Prior knowledge can be required near the beginning, near the end or in another stage of the solution. We obtained the points attributed to prior knowledge and the stage of the solution, where prior knowledge is required from the answer key. In mathematics, it is sometimes possible to continue solving a problem even if one makes a mistake at the beginning or somewhere else. The lecturer, in some cases, considered students' mistakes, evaluated further computations and gave points to some calculations. For the accuracy of the study this was also considered when determining the impact of prior knowledge on the solution, but as mentioned before, was not always possible and had less influence. The main aspects to calculate the impact were the stage and the score given to that step. The impact was calculated as percentages. Thus, the impact of prior knowledge depends on several aspects and can vary per question. If we only consider the stage and the score and, for example, four points out of ten are attributed to prior knowledge but the prior knowledge is required near the beginning of the solution, then the impact of prior knowledge on the solution of this exam question is 100%. Without the required prior knowledge, the student cannot start solving the problem correctly and, consequently, fails to solve the problem. If the prior knowledge in this example is needed near the end of the solution, the impact would be 40% (4/10). The impact of prior knowledge diminishes in later stages of the solution process.

For a good overview, we constructed **Table 4**. In **Table 4**, we presented all the previous information from **Table 1**, **Table 2**, and **Table 3** and included a column with the percentages of students who correctly solved the exam (sub)questions to draw conclusions. While calculating these percentages, we neglected small arithmetic errors. The results are presented in **Table 4**.

Table 4. All tables combined

Main topics	Table 1 Required prior knowledge	Table 3 IPK (%)	Table 2 PPK (%)	Exam question	PS (%)	Interpretation
Mathematical induction	Calculations with fractions & exponents	50.0	39	Mathematical Induction	59	Impact 50%, prior knowledge errors 39%, max 61% correct answer but 59% had correct answer, so, 2% due to new knowledge
Complex numbers	Special products	12.5	12	Calculations with complex numbers	69	Percentages of both required prior knowledge were determined separately 69% correct answer, so, 31% errors due to one or both prior knowledge topics or due to new knowledge
	Tangents of special angles	12.5	17			
	Solving cubic equations in R	100	41	Solving cubic equations	47	Impact 100%, prior knowledge errors 41%, max 59% correct answer but 47% had correct answer, so, 12% due to new knowledge
Limits	Absolute values	100	29	Limits with absolute value function	29	Impact 100%, prior knowledge errors 29%, max 71% correct answer but 29% had correct answer, so, 42% due to new knowledge
	Trigonometric identities & properties of trigonometric functions	100	33	Limits of trigonometric functions	50	Impact 100%, prior knowledge errors 33%, max 67% correct answer but 50% had correct answer, so, 17% due to new knowledge
	Manipulations with fractions	100	22	Limits with fractions	70	Impact 100%, prior knowledge errors 22%, max 78% correct answer but 70% had correct answer, so, 8% due to new knowledge
	Calculations with roots & exponents	50.0	17	Determine horizontal asymptotes	50	Impact 50%, prior knowledge errors 17%, max 83% correct answer but 50% had correct answer, so, 33% due to new knowledge
Differentiation & applications	Determine slope of a line	100	5	Determine slope of a line using implicit differentiation	53	Impact 100%, prior knowledge errors 5%, max 95% correct answer but 53% had correct answer, so, 42% due to new knowledge
	Laws of logarithms	100	52	Logarithmic differentiation	24	Impact 100%, prior knowledge errors 52%, max 48% correct answer but 24% had correct answer, so, 24% due to new knowledge
	Knowledge of extremes & sign tables	33.0	23	Extreme values	54	Impact 33%, prior knowledge errors 23%, max 77% correct answer but 54% had correct answer, so, 23% due to new knowledge
Related rates	Interpretation of the derivative	71	54	Related rates	13	Percentages of both required prior knowledge topics were highest in this question & prior knowledge errors were determined separately 13% correct answer, so, 87% errors due to one or both prior knowledge topics or due to new knowledge
	Similarity of triangles	100	52			

Note. IPK: Impact of prior knowledge per exam question; PPK: Percentage prior knowledge errors (all students included); & PS: Percentage of students with correct answer

RESULTS

In this section, we present our findings and address the research questions.

First research question: What prior knowledge is required for each topic of the exam?

Our first question is addressed in **Table 1**. In **Table 1**, we list all the required prior knowledge of the exam, which includes basic algebra and trigonometry, interpretation of the derivative, knowledge of extremes and sign tables, and similarity of triangles. It can also be seen which of the topics are considered basic and advanced.

Second research question: Which prior knowledge errors do students make in the calculus A exam?

We reviewed students' prior knowledge errors to determine in which topics the errors were made. The percentages presented in the 4th column of **Table 2** show that advanced prior knowledge errors are between 5% and 68% and that basic prior knowledge errors are between 14% and 45%.

When all students were included, as presented in the 5th column of **Table 2**, the percentages differed considerably. Students' advanced prior knowledge errors were between 5% and 54%, and basic prior knowledge errors were between 12% and 39%. The percentages in the 5th column give a distorted impression because we cannot say with certainty whether the students who had been excluded had the required prior knowledge. These students did not complete the solution or had a completely wrong approach. However, including all students was necessary for comparison with other data.

Third research question: What is the impact of prior knowledge on solving each exam question?

The impact of prior knowledge on solving exam problems is shown in **Table 3**. First, we examined all the cases, where prior knowledge was required in stage 1 and further computations could not be considered. This means that students could not earn any points if they lacked prior knowledge or started with an incorrect approach; therefore, the impact in these cases was 100%. We found 7 cases, where the impact was 100%.

In the other cases where the impact was less than 100%, it is necessary to discuss every case individually because we have to consider several factors, such as the stage, the points given to the prior knowledge and the possibility of evaluating further calculations.

To determine the impact of prior knowledge on solving the question about mathematical induction, where the required prior knowledge involved calculations with fractions and powers, we reasoned, as follows: Prior knowledge was required in stage 2 of the solution, and five points out of ten were given to this step. Students who did not master calculations with fractions were unable to complete the solution, and therefore, the impact of prior knowledge was 50% (5/10).

Next, we computed the impact on the solutions of the topic of complex numbers. There were two subquestions about complex numbers, where the impact was less than 100%. The required prior knowledge for the first question was special products. Special products were required in stage 2 of the solution, and one point out of eight was assigned for this step. The answers of students who lacked prior knowledge and therefore calculated incorrect complex numbers could still be evaluated, and the remaining points could be earned. Thus, students could score seven out of eight points, and therefore, the impact of prior knowledge was 12.5% (1/8). For the second question, the prior knowledge was tangents of special angles, required in stage 3 of the solution, and one point out of eight could be earned. Students who lacked prior knowledge simply lost one point, and thus, the impact was 12.5% (1/8). Stage 3 is the last step, and thus, further computations cannot be considered.

Afterward, the impact on the solution of the topic of limits for the case, where the impact was less than 100% was calculated. The required prior knowledge was calculations with radicals and exponents, which was required in stage 2. Students could score four points out of eight points for this step. If a student was not proficient in radicals and exponents, it was not possible to perform further calculations. Therefore, the impact of prior knowledge is 50% (4/8).

Furthermore, the impact on the solution of the topic differentiation and applications is determined. The required prior knowledge was the ability to apply the laws of logarithms. This knowledge was required in stage 2 of the solution and was two points out of six. Students who could not apply these laws could earn a maximum of one point out of six. Further computations could not be evaluated; therefore, the impact was 83% (5/6).

Finally, the impact on the topic related rates is examined. This topic requires a deeper understanding of the differentiation of functions. The required prior knowledge was in stage 2 of the solution and was seven points out of fourteen. Without that knowledge, it was impossible to complete the solution, and further calculations could not be evaluated. The impact was 50% (7/14).

All the relevant information from the previous tables has been combined in **Table 4** to help draw conclusions and discuss results regarding prior knowledge. The interpretation of the findings is also determined and presented in the last column.

We will demonstrate how **Table 4** should be interpreted with an example. The 3rd column shows that the impact of prior knowledge on the solution of the problem involving mathematical induction is 50%. This means that a student who lacks the required prior knowledge loses 50% of the obtainable points for this problem. If we consider all students, the 4th column shows that 39% did not have the required prior knowledge. Therefore, at least 39% of the students lost 50% of the obtainable points for this problem. This means that at most 61% of the students can answer the question completely correct. The last column shows that 59% of the students solved the question correctly. The difference of 2% is due to errors in the new subject matter.

DISCUSSION & FURTHER RESEARCH

We investigated the role of prior knowledge in solving calculus A exam problems. We found that prior knowledge has a strong impact on solving some problems. Several studies on prior knowledge have been conducted, but the impact of prior knowledge on solving exam problems has not been investigated.

Rach and Ufer (2020) distinguished four levels of knowledge. From our analysis, we determined that not all prior knowledge was at the same level. Therefore, we distinguished the required prior knowledge into basic and advanced. The levels of prior knowledge that we identified falls into Rach and Ufer's (2020) categories one and two. The basic prior knowledge corresponds with the knowledge of level one of Rach and Ufer (2020). The advanced knowledge can be identified with the knowledge of level two of Rach and Ufer (2020), where it is required to represent a known concept.

Segura and Ferrando (2023) distinguished between strategic and mathematical errors. They considered strategic errors, which result from the selection of an invalid strategy, as very serious, as it will not lead to a solution. Similarly, we considered the stage in which prior knowledge is required as crucial. If a student makes a prior knowledge error or lacks the prior knowledge necessary at the beginning of the solution, then this error is more serious than a prior knowledge error in another stage.

In some cases, the influence of prior knowledge is in a stage, where the student cannot start solving the problem. Consequently, it is no longer possible to determine whether the students mastered the new subject matter. Prior knowledge will always be needed to learn new subject matter, but the challenge is creating exam problems, where the impact of prior knowledge on the solution is not too high. Otherwise, whether students have achieved the learning objectives of the course cannot be determined. On the other hand, lowering the impact of prior knowledge is not always possible, and moreover, we expected that students should know the basics of mathematics.

In this exam, problems about absolute values, logarithmic differentiation and related rates had the lowest percentages of students with the correct solution. The impact of prior knowledge on the solutions of these three problems was 100%.

Remarkably, 52% of the students lacked the required prior knowledge for the logarithmic differentiation problem even though this prior knowledge involved very simple rules such as addition, subtraction and other simple logarithmic calculations. As a consequence, only 24% of the students correctly solved the problem.

The problem of related rates is a point of concern, as only 13% of students solved this problem correctly. This exam question was a word problem and required a problem-solving approach, where the student first had to convert the practical problem into a mathematical model and then apply the required prior knowledge. Whether a lack of prior knowledge caused the students to be unable to correctly solve the problem could not be easily determined. We suspect that a lack of modeling skills caused the students to be unable to find the correct solution. This subject can be further explored in follow-up research to identify possible causes and find solutions for this problem.

Our investigation revealed that students lacked very elementary prior knowledge, which led to the failure of some of the exam problems.

CONCLUSIONS

Students' poor results in first-year calculus courses are common worldwide. In this paper, we performed an error analysis on students' exam sheets focusing on prior knowledge. The study of prior knowledge errors is important, because students who lack the necessary prior knowledge are unable to solve problems correctly.

First, the necessary prior knowledge for each exam question was determined. We then calculated the percentages of students who made these prior knowledge errors. Students performed poorly on the exam questions about absolute values, logarithms and related rates. We noticed that some errors were in basic algebra and trigonometry, although we assumed that the students had already mastered this type of prior knowledge.

We also determined the impact of prior knowledge on solving the exam question. We found that the stage at which prior knowledge is needed in the solution is crucial for determining this impact. In seven topics, the impact of prior knowledge was 100%. Exam questions should preferably be constructed in a way that minimizes the impact of prior knowledge.

We concluded that students had insufficient prior knowledge on some topics, and this aspect requires attention. However, we only investigated the exams for one year. To gain even more insights, we could have analyzed an exam from another year and compared the results of the two exams. Another limitation was that we could not interview students. Interviews could help better establish the causes of the student errors. Notably, it would be better to consider whether students took the calculus A exam for the first time or did a resit. This would provide more information about the prior knowledge of new students. The analysis was done anonymously, which made it impossible to consider this

Nevertheless, this research contributes to a better understanding of the prior knowledge of high school students when entering technical universities.

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Ethical statement: The authors stated that, in this research the exam sheets of a Calculus A course were analyzed. The Anton de Kom University of Suriname doesn't have an Ethical Committee. The authors further stated that, in order to still comply with the usual ethical guidelines, they took the following into account during the research: Permission to use students' exams sheets was given by the dean of the faculty; No student names are mentioned anywhere to ensure student anonymity; No photos of students' work were used because handwriting could be recognizable; Safe storage of the exam sheets has always been ensured. The aim of this study was to analyze errors in order to make improvements in students' education. So, there was no risk or threat involved in this study.

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