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Modelling the relationship among calculus scholars' beliefs, critical thinking, elaboration, and problem-solving

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ABSTRACT

The aim of this research was to establish a structural relation among calculus scholars' beliefs, self-regulated learning (SRL) strategies, and problem-solving skills related to differential equations (DEs). To identify the relationships between different variables and their impact on DE problem-solving, a correlational study design with an a priori model was established. Three questionnaires were utilized to measure the epistemological and useful mathematics beliefs and SRL of 430 higher secondary school students. Additionally, an evaluation test consisting of five DE tasks was administered. The results demonstrated that there was a strong correlation among epistemological mathematics beliefs, the perceived usefulness of the subject, SRL, and problem-solving. This study confirms ($\beta = .29$, $T = 4.05$, and $p < 0.001$) that students who perceive mathematics as useful tend to improve their problem-solving skills. Similarly, only elaborations ($\beta = .06$, $T = 2.40$, and $p < 0.001$) had shown the mediation role between beliefs and problem-solving. These findings highlight the potential use of these factors in improving students' skills for solving the real tasks. A few important implications were also outlined to promote culture for deep and meaningful learning.

Keywords: differential equation, epistemological beliefs, SRL strategies, problem-solving, SEM

INTRODUCTION

Research in mathematics education is focusing in two dominant arenas including pre-university and university level. The evolution from higher secondary school to higher university levels can be a challenging experience for students. Recognizing the prominent role those differential equations (DEs) play in higher mathematics and scientific fields, there has been a growing focus on preparing students from higher secondary school to university level studies (Quvvatov, 2024; Teschl, 2024). As such, the DEs course at the pre-university level has become a focal point for transitional studies. By emphasizing DE problem-solving skills and providing students with a solid foundation in the subject, pre-university education can help prepare students for the rigors of university-level coursework in mathematics, science, and engineering (García-García & Dolores-Flores, 2021a; McAlinden & Noyes, 2019; O'meara et al., 2017). This, in turn, can enhance their ability to succeed academically and professionally in fields that require a strong understanding of DEs (García-García & Dolores-Flores, 2021b). For this mastery of problem-solving at higher level, role of higher secondary school is very important (Bibi et al., 2024; Guo et al., 2022; Msomi & Bansilal, 2022).

The literature on teaching and learning DEs suggests that there are five primary approaches being used at both the university and pre-university levels. These approaches are algebraic, numerical, graphical, technological, and inquiry-oriented (Aisha et al., 2018; García-García & Dolores-Flores, 2021a). By utilizing these various approaches, educators can provide a diverse and comprehensive learning experience that caters to different learning styles and preferences. Further critical analysis in these directions revealed that the ability of students to solve problem is also significantly affected by context familiarity (Bibi et al., 2018; Tjoe, 2019). Students remained successful while solving physics' tasks as compared to biologically-based problems. In addition to context familiarity, solving non-routine mathematics tasks was also considered. These tasks require students to utilize additional devotion and learning strategies because they are typically unfamiliar, unplanned, and unanticipated (Karatas & Erden, 2017; Polya, 1962). However, due to their complexities, it has been observed that mostly learners avoid to solve non-routine tasks, which can cause significant comprehension issues at higher education levels when they attempt to correlate with real-life problems (Bibi et al., 2019; Duchesne et al., 2019; Jablonski, 2023). Therefore, both routine and non-routine need to be taught by teachers who

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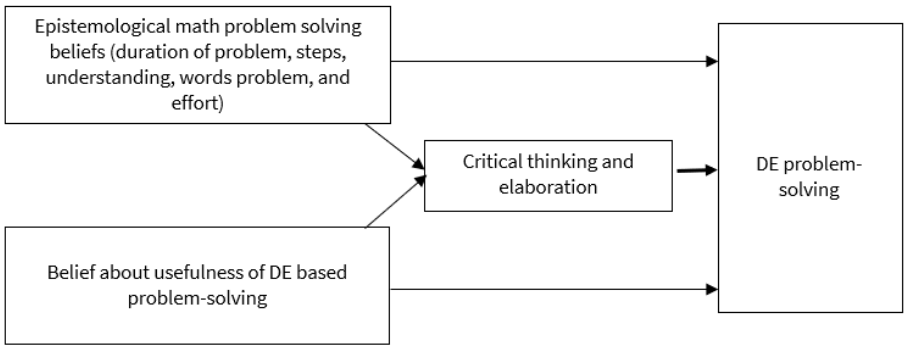

Figure 1. A conceptual framework showing endogenous variables (beliefs) and exogenous variables (SRL) with their constructs (Source: Authors' own elaboration)

are appropriately trained and skilled in order to effectively guide and support their students in their learning. In addition to learning strategies, context familiarity and unfamiliar problem-solving, role of mathematic beliefs were also observed important that contribute to a student's ability to solve DES problems.

Several researchers reported a strong correlation between mathematical performance and mathematic beliefs (Acar, 2024; Muñoz et al., 2024). In this context, epistemological beliefs remained in the central part of numerous studies. Schommer-Aikins and Duell (2005) proposed a model of epistemological beliefs in mathematics, which includes five dimensions of knowledge: source, certainty, control, structure and speed. They found that students who held strong beliefs in these dimensions had better mathematical performance (Schommer, 2019). Students' beliefs about the nature of mathematics and its usefulness were significantly related to their mathematical problem-solving performance (Chen & Turner, 2024; Muis et al., 2021).

Several studies had focused students' beliefs about the usefulness and relevance of learning mathematics (Pedersen & Haavold, 2023; Sintema & Mosimege, 2023). Schommer-Aikins and Duell (2013) also reported that belief in the usefulness of mathematics also have indirect effect. Recently, Sachdeva and Eggen (2023) distinguished useful as "students' beliefs about the relevance of learning mathematics" and importance as the "learners' beliefs about the importance of learning mathematics". Sachdeva and Eggen (2023) had argued theses as prominent predictor to solve mathematical problems. Belief in usefulness increases the level of motivation to work hard towards improving their problem-solving ability and mathematics competence (Sintema & Mosimege, 2023).

Numerous studies also interlinked self-regulated learning (SRL) strategies with mathematics achievement (Ambaryani & Putranta, 2022; Apostol, 2023; Karatas & Erden, 2017; Schommer-Aikins & Duell, 2013). Stockton (2010) emphasized the importance of SRL strategies, specifically metacognition and self-regulation, in mathematics problem-solving. They found that students who were able to monitor and regulate their own learning processes performed better in mathematics problem-solving tasks. Recently, Losenno et al. (2020) claimed that better understanding how several targets of self-regulation work in a concert can be beneficial for better problem-solving (Sachdeva & Eggen, 2021). Overall, literature illustrates that epistemological mathematics beliefs, usefulness beliefs, and SRL had significant effects on mathematics problem-solving and achievement.

Therefore, to test the predictability of this concept, the present work was carried out. A detailed structural equation model (SEM) was developed to explore the relationships between these factors; epistemological and usefulness mathematics beliefs and SRL strategies and their influence on problem-solving. By identifying the direct and indirect effects of these factors, educators can better understand how to design instructional interventions that will help students succeed in DEs and other mathematics courses.

CONCEPTUAL FRAMEWORK

This study has utilized the previously established framework (Aisha et al., 2018; Bibi & Ahmad, 2022). **Figure 1** demonstrates the conceptual framework for the current work. Initially, this framework had been used to access the problem-solving ability of students while solving mathematics tasks. In the previous work, mediation of SRL between goal orientations and DE-based problem-solving was investigated (Bibi & Ahmad, 2022). In this current study, a complete SEM was developed to elaborate mediating role of SRL among epistemological mathematics beliefs, usefulness beliefs and DEs based problem-solving.

Research Objectives

This study aimed at exploring the various factors that affect DE problem-solving at higher secondary school level. For identification of direct and mediating effects, research objectives were established.

1. The first objective was to examine whether epistemological mathematics beliefs, usefulness, and SRL directly affect students' DE problem-solving. This objective aims to identify the direct impact of these factors on students' problem-solving abilities in a distance education setting.
2. The second objective was to examine whether SRL strategies play a mediating role between epistemological mathematics beliefs and DE problem-solving. This objective aimed to investigate whether students' SRL strategies mediate the relationship between their beliefs about mathematics and their problem-solving abilities.

METHODOLOGY

Method

As the aim of this study was to identify the relationships between different variables and their impact on DE problem-solving at the pre-university level, a correlational study design with an a priori model was established (**Figure 1**). Cluster random sampling technique was deemed fit for the current study because any population bias is distributed equally among the selected individuals (Creswell, 2020).

In this study, epistemological mathematics beliefs and usefulness are considered exogenous variables, which means they may have a direct effect on DE problem-solving, but they are not influenced by other variables in the model (Memon et al., 2021). SRL strategies are endogenous variables, meaning they are influenced by the exogenous variables and may also influence DE problem-solving (Jarvela & Hadwin, 2024).

Participants

Research instruments were distributed among 430 participants studying at higher secondary schools in a province of Pakistan. This sample size was a good representation of the population being studied, and it had allowed for generalization of the findings to similar populations. It is also worth noting that the sample was drawn from both government and private institutes, which had added to the diversity of the sample and provided more comprehensive insights into the factors affecting DE problem-solving at the pre-university level in Pakistan.

Research Instruments

Three different types of instruments (questionnaires) along with five non-routine words problems were finalized to investigate different factors of the model. In this study, five non-routine word problems were developed to assess the students' ability to solve DEs in real-world contexts. These problems were carefully designed to incorporate multiple DE components and to relate to everyday life issues (Verschaffel et al., 2020).

The modified Indiana mathematics beliefs scale was used to measure epistemological mathematics beliefs, and it included five dimensions (problem, duration, steps, word problems, understanding, and effort), each consisting of three positively and three negatively oriented items rated on a 5-point scale. The reported reliability of the five subscales was .73. The modified Fennema and Sherman (1976) scale was used to measure usefulness, with a Cronbach's alpha value of .86.

The third instrument "the Norwegian adaptation of the motivated strategies for learning questionnaire (MSLQ) (Pintrich, 1991)" was used to assess SRL strategies. The MSLQ consists of 15 subscales, with six scales measuring motivation and nine scales measuring learning strategies. For this study, only two dimensions, critical thinking and elaboration, were chosen from the learning strategies section. The elaboration subscale consists of six items, while the critical thinking subscale consists of five items. Both subscales are scored on a scale from 1 (not at all true) to 7 (very true). The reported reliability values for the elaboration and critical thinking subscales were .75 and .80, respectively (Duncan & McKeachie, 2005).

Data Collection and Analysis

Responses of 430 participants were critically reviewed, refined and subjected to analysis. data from 430 contributors was collected and used to create a SEM, which was subsequently evaluated.

Exploratory factor analysis (EFA) was used to explain the impacts of the parameters in the relationship. Scree plot analysis was effectively employed to retain the factors. Critically examining the scree plots, one factor structure for each subscale was observed excluding beliefs. Two factors for beliefs were observed. Although, these two factors were explainable. However, there was some uncertainty regarding the uni-dimensionality of the data. It was hypothesized that a general factor might explain the data if it was uni-dimensional (Epler, 2011; Wheeler, 2007). To further evaluate the uni-dimensionality of the data, PCA and other procedures were applied. An iterative process was used to refine the analysis, resulting in a one-dimensional scale. The scree plot supported the existence of only one factor solution. Similarly, for usefulness (second construct), factor analysis results indicated only one factor. And also, for the case of elaboration and critical thinking, results explained one factor structure for each subscale. After EFA, data of 430 contributors was used to create a SEM, which was subsequently evaluated.

RESULTS

In current study, the measurement model with factor loading greater than .7 and Cronbach's alpha .90 had achieved good reliability and convergent validity, respectively. **Table 1** displays overall results of the items. The acceptable values of factor loadings and AVE should be greater than or equal to 0.5, Similarly, CMR should be greater than or equal to 0.7 in order to demonstrate that the observed variables are internally consistent and reliable measures of the underlying factor (Hair et al., 2010). All constructs in the model met the criteria for discriminant validity, which suggests that they are distinct from one another and are measuring different underlying constructs. **Figure 2** illustrates the structural model established for the current study.

Several other assessments including the coefficient of determination (R^2) and predictive significance (Q^2 and q^2) were also performed to evaluate the predictive accuracy and significance of the model. R^2 values, with values of .67, .33, and .19 indicating substantial, moderate, and weak predictive accuracy, respectively (Chin, 2008). Based on guidelines a moderate effect size for the DE problem-solving construct, with an R^2 value of .64 was observed (Guven & Cabakcor, 2013). This means that 64% of the variance

Table 1. Factor loading, CR, and AVE of the measurement model

Construct	Subscales	Outer loadings	Cronbach's alpha	CMR	AVE
Epistemological beliefs for problem-solving	DP	.83	.90	.93	.72
	ST	.85			
	UN	.86			
	WP	.86			
	EF	.82			
Usefulness	B51	.90	.93	.95	.77
	B52	.89			
	B53	.90			
	B54	.86			
	B55	.87			
	B56	.84			
	AV4	.92			
	AV5	.93			
	AV6	.90			
Critical thinking and elaboration	CR1	.77	.81	.87	.58
	CR2	.78			
	CR3	.75			
	CR4	.74			
	CR5	.77			
	EL1	.80	.91	.93	.69
	EL2	.84			
	EL3	.84			
	EL4	.85			
	EL5	.82			
	EL6	.83			

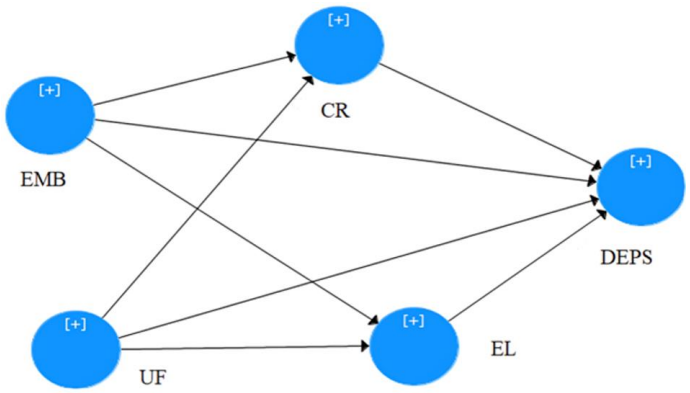

Figure 2. Structural model showing SRL mediating among beliefs and DE problem-solving (Source: Authors' own elaboration)

Table 2. R^2 and GoF results for the established model

Constructs	AVE	R^2	Status	GoF (√(AVE*R^2))
Critical thinking	.58	.188	Small	0.5
DE problem-solving	1.00	.640	Substantial	
Elaboration	.69	.591	Medium	
Usefulness	.77	-		
Epistemological mathematics beliefs	.72			

in the solving DE could be described by the factors of epistemological mathematics beliefs, usefulness, critical thinking, and elaboration. It was also observed that critical thinking and elaboration had weak and moderate effects on the DE task-solving construct, with R2 values of .59 and .19, respectively (**Table 2**).

Table 2 also indicates GoF value .5. The reported cutoff values are; GoF_{small} = .1; GoF_{medium} = .25; GoF_{large} = .36. This suggests that established models had shown a good fit to the current study data (Hoffmann & Birnbrich, 2012). In addition, grounded on the GoF analysis, the model had also a large effect size and showed a good fit to the data. Likewise, the f^2 values are classified as small, medium, and large based on Cohen's (1988) criteria, with cutoffs of .02, .15, and .35, respectively.

Based on the f^2 values, it was found that the influence of usefulness (.118) was closer to a medium effect as well. On the other hand, effects of all other constructs were observed small. For current study, Q^2 is evaluated using cross-validated redundancy that is based on blindfolding procedures (Hair et al., 2013). **Table 3** indicated that Q^2 value for all constructs was above zero, demonstrating significant predictive relevance (Fornell & Larcker, 1981).

Table 3. Q^2 and f^2 values for the established model

Constructs	Q^2	f^2 (critical thinking)	f^2 (elaboration)	f^2 (DE problem-solving)
Critical thinking	.094	-	-	.002
DE problem-solving	.62	-	-	-
Elaboration	.401	-	-	.027
Epistemological mathematics beliefs	-	.035	.154	.02
Usefulness	-	.002	.001	.118

Table 4. Direct paths analysis of the complete model

Route name	B	SE	t	p
Epistemological mathematics beliefs -> DE problem-solving	.124	.057	2.100	.010
Usefulness -> DE problem-solving	.290	.072	4.050	.000
Critical thinking -> DE problem-solving	.035	.037	0.910	.180
Elaboration -> DE problem-solving	.160	.050	3.120	.000

In the second step, a structural model was evaluated to test the hypothesized link within the model. Structural estimates are provided in **Table 4**.

Figure 2 provides a clear and understandable picture of the whole model with direct and indirect effects. The details are provided in the following sections.

To examine the influence of student's epistemological beliefs via the mediatory variable "SRL", structural estimates of the selected route were considered. **Table 5** shows that only elaboration had played mediation role in solving the DE tasks.

DISCUSSION

The results demonstrated that there was a strong correlation among epistemological mathematics beliefs, the perceived usefulness of the subject, and SRL (elaboration only). Considering the mediating effect, it was observed that only elaborations had shown the mediation role between beliefs and problem-solving.

Examining the Impact of Mathematics Beliefs, and SRL on Solving de Tasks: Direct Effects

The findings of the SEM research indicated that all of the variables under consideration significantly affected the problem-solving (**Table 4**). Particularly, Usefulness beliefs had shown highest positive impact.

Usefulness beliefs had shown highest positive impact. It confirms that more a student believed that mathematics is beneficial as it enhances their problem-solving and promotes cognitive depth (Schommer-Aikins & Duell, 2013). Interestingly, when data was compared, public sector respondents were more oriented towards usefulness beliefs and epistemological mathematics beliefs on problem-solving as compared to private sector respondents. Findings of the present study also revealed that male respondents were better in believing about the usefulness of mathematics problem-solving.

Mostly learners declared that learning of DE and related problem-solving is the most relevant mathematical subject because of its usefulness in their daily life. Outcomes of this work were also consistent with previous studies (Pedersen & Haavold, 2023; Sachdeva & Eggen, 2023; Sintema & Mosimege, 2023). Students who perceive mathematics as useful tend to improve their problem-solving skills. Another possible reason might be the belief in usefulness increases the level of motivation to work hard towards improving their problem-solving ability and mathematics competence (Sintema & Mosimege, 2023). Interestingly, Sachdeva and Eggen (2023) distinguished useful as " students' beliefs about the relevance of learning mathematics" and importance as the "learners' beliefs about the importance of learning mathematics". Sachdeva and Eggen (2023) had argued to display the relevance beliefs as extrinsic values and importance beliefs as intrinsic values.

Current study illustrates some valuable findings for teaching and learning of DE course. For example, if a candidate has belief in not being able to solve time consuming problems, he will assume that most of the mathematics problem should be accomplished in less interval of time. On more challenging task, such students totally ignored and moved to another problem. This leads to fixed mindset (Hair et al., 2010; Memon et al., 2021). Therefore, the responsibility of the teacher is to encourage the students in the development of growth mindset and should have the "can-do" policy towards DE containing tasks.

The other possible reason is the social norm like early girls' marriages. About 63% of girls from weak financial backgrounds and even 24% of girls from strong financial backgrounds get married before 18 (Altaf et al., 2023; Malik, Nadeem, & Adil, 2022). In some cases, career choices for women are confined and not same as for boys. Due to lack of opportunities and resources, mostly females do not groom well and could not be able to get chance in decisions making. These reasons strongly affect students' beliefs about learning and problem-solving (Mills, 2001).

Partial Mediation of SRL Between Epistemological Mathematics Beliefs and Solving DE Tasks

SRL strategies were also anticipated to lead to improve the student's epistemological and useful beliefs, and thereby enhancing student's problem-solving. Only elaboration was identified to be a mediator of the epistemological problem-solving beliefs (**Table 5**). However, the indirect effect of epistemological and useful beliefs on problem-solving via elaboration was found to be small ($\beta = .06$, $t = 2.40$, $p < .00$), whereas the direct effect of epistemological beliefs on solving DE tasks was observed noteworthy even after the inclusion of a mediator "SRL strategies" ($\beta = .02$, $t = 2.01$, $p < .01$). Again, in contrast to common belief,

only elaboration has shown partial mediation for DE problem-solving. There might be the existence of some other factors leading to improvement in problem-solving resulting from epistemological and useful beliefs beside SRL strategies. Therefore, researchers have concluded that epistemological beliefs possibly have similar effects on other sets of factors. This parallel effect had caused difference in students' opinion while solving DE based tasks.

The outcomes of this work were also consistent with previous studies (Jansen et al., 2019; Rastegar et al., 2010). Stockton (2010) argued that the association between solving DE tasks and epistemological mathematics beliefs can be enhanced via elaboration. Stockton (2010) also demonstrated a correlation between several indicators of students' SRL strategies and their problem-solving performance. However, the current study's findings indicated that the ability to solve DE problems through critical thinking was unaffected by epistemological mathematics beliefs.

The study by Fadlelmula et al. (2015) provided some support for these findings which asserted that critical thinking had not significantly affected mathematical achievements. Findings of this study had partly contradicted the findings of past literature. This might have appeared for the reason of applying only two strategies (critical thinking and elaboration). Furthermore, results of this work had partly opposed results of Schommer (1990), who had assumed that epistemological beliefs are strongly correlated to SRL strategies and students' achievements. These differences might have appeared for the reasons of conceptual and technical issues resulting from the self-reported measures as illustrated by Muis (2004). Muis (2004) stated that the several studies connecting epistemological beliefs to cognitive, motivational, academic achievement employed self-report measure. As a result, the study's potential to contribute was constrained by technical and conceptual issues.

No Mediation of SRL Between Usefulness Beliefs and DE Problem-Solving

Useful beliefs have significant direct effect of DE Problem-solving. It confirms that more a student believed that mathematics is beneficial as it enhances their problem-solving and promotes cognitive depth (Kandemir et al., 2024; Oh, 2024). However, no mediation effects from elaboration and critical thinking were noticed in relation to useful beliefs. These studies are well align with previous studies (Soltani & Askarizadeh, 2021).

In later studies, Schommer-Aikins and Duell (2013) also reported that belief in useful of mathematics also have indirect effect. Several factors that possibly affect the belief in the usefulness of mathematics for problem-solving can be identified. A possible reason might be the difficulty in measurements of usefulness belief while solving DE based tasks. Another factor could be the limited data availability from a few research studies that have examined this construct "belief in useful of mathematics". Therefore, research studies on usefulness beliefs on DE problem-solving achievement goals persists in its earliest year and are limited. In addition, the mediating role of SRL among beliefs in usefulness of mathematics and solving DE based course was not studied prior to present research. As a result, the findings of the current study regarding the mediating role of SRL between usefulness beliefs and solving DE based tasks were quite novel.

Implications

In present work a detailed SEM was developed to explore the relationships between the several factors; epistemological and usefulness mathematics beliefs, and SRL strategies, and their influence on problem-solving. Findings of this work are highly encouraging. We, therefore, recommend educators and policy makers to implement these factors for effective problem-solving in mathematics.

Current study has proven the prominent role of epistemological and usefulness beliefs for solving challenging tasks. It has also highlighted some valuable conclusions for teaching and learning of DE course. For example, if a candidate has belief in not being able to solve time consuming problems, he will assume that most of the mathematics problem should be accomplished in less interval of time. On more challenging task, such students totally ignored and moved to another problem. Therefore, the responsibility of the teacher is to encourage the students in the development of growth mindset and should have the "can-do" policy towards DE containing tasks.

Similarly, most learners declared that learning of DE and related problem-solving is the most relevant mathematical subject because of its usefulness in their daily life. Students who perceive mathematics as useful tend to improve their problem-solving skills. Belief in usefulness increases the level of motivation to work hard towards improving their problem-solving ability and mathematics competence (Sintema & Mosimege, 2023).

Critical thinking was expected to lead to improve the student's problem-solving. However, in current study, critical thinking had no significant effect. A possible reason might be the engagement of limited set of learning strategies (i.e., critical thinking and elaboration) for current study due to which non-significant value of critical thinking might appear. The other reason might be the self-reporting nature of data that contributed to conceptual and technical issues. For critical thinking, learning environment that actively engage students for real world problem-solving is compulsory (Thornhill-Miller et al., 2023). Besides, critical thinking requires training, practice, time and patience (Jamil et al., 2024; Snyder & Snyder, 2008). Therefore, these outcomes can be fit to the appraisals practices in the Pakistani structures for educations, like grade-focused assessment and also the incorporation of secondary school scores to compute final entry test results. As a result, students might resist to promote critical thinking in their problem-solving.

Limitation

This study is based on quantitative surveys and might have some limitations. In this work, the adapted research instruments were used to assess participants' motivation and their use of cognitive strategies. Questionnaires were based on theories and findings from developed countries. As Pakistan is a developing country, the results might not be consistent with the hypothesis due to changes in resources, infrastructure, teachers and students' abilities. Other limitations may be the limited data, surveys of

a limited range of tasks, strategies, and problem-solving approaches. Therefore, in future, experimental and qualitative parts would be carried out to fill these gaps.

CONCLUSIONS

Based on previous theoretical and empirical studies, current study proposed a hypothetical model to interlink epistemological mathematics beliefs, SRL and problem-solving skills related to DEs. Three questionnaires were utilized to measure the epistemological mathematics beliefs, and SRL. The results demonstrated that there was a strong correlation among epistemological mathematics beliefs, the perceived usefulness of the subject, elaboration for calculus based problem-solving. Similarly, only elaborations had shown the mediation role between beliefs and problem-solving. These findings highlight the potential use of these factors in improving students' DE problem-solving skills and ensuring the learning of DEs is more meaningful and effective.

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Ethical statement: The authors stated that the study was approved by the National University of Modern Languages on 20 August 2019 with approval code 1568696934/R. Ethics/NUML. Participants were well informed about the purpose and requested to give their consent to participate in this work.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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