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Mathematical connections made by preservice mathematics teachers when solving tasks about systems of linear equations

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ABSTRACT

The study reported had the aimed to identify the mathematical connections made by a group of Mexican preservice mathematics teachers (PMTs) when solving tasks on systems of linear equations. We consider a mathematical connection as a true relationship that a person makes between two or more ideas, concepts, definitions, theorems, procedures, representations, and meanings with each other, with other disciplines or with real life. Four PMTs participated voluntarily in task-based interviews, providing the primary data for analysis. Thematic analysis was employed to analyze the data. The findings revealed that the preservice teachers made mathematical connections of procedural, implication, feature, meaning, and different representations. However, the findings also revealed that some preservice teachers faced challenges when attempting to solve the tasks, leading to difficulties in establishing certain mathematical connections.

Keywords: mathematical connections, system of linear equations, task-based interviews, thematic analysis, preservice mathematics teachers

INTRODUCTION

Research in mathematics education focused on the study of the knowledge of preservice mathematics teachers (PMTs) with the aim of successfully managing mathematics learning is not an easy task, however, it is necessary and important (Muñiz-Rodríguez et al., 2020). In this regard, Tribó (2008) suggests that the profile of PMTs should include a solid basis in the scientific area and mastery of specific professional competencies. For this reason, research indicates that the content of initial teacher training programs should be determined by the competencies necessary for their future professional practice (Darling-Hammond, 2006).

Along the same lines, Shulman (1987) states that it is necessary to consider other domains such as curriculum knowledge, educational contexts, and general pedagogical knowledge, which are the core of PMTs knowledge necessary to enhance their practice (Ponte & Chapman, 2008; Rowland et al., 2005). In this research, it is also assumed that it is important for PMTs to be able to make and promote the use of mathematical connections between different mathematical concepts and scientific disciplines to provide better opportunities for students in their learning and understanding of mathematical concepts.

In this regard, Hatisaru (2023) reported that teachers with higher mathematical knowledge for teaching (MKT) tend to make a significantly greater number of connections, compared to those with lower MKT, who focus more on procedural connections. This finding, as also acknowledged by García-García and Dolores-Flores (2021), suggests that the quality and type of connections made by teachers may be linked to their beliefs about teaching and learning mathematics.

Consistent with these ideas, the importance of mathematical connections also lies in facilitating the integration of knowledge and interdisciplinarity, being useful for problem-solving (García-García, 2024) and serving as the basis for achieving mathematical understanding (Coles & Sinclair, 2024; García-García, 2024; García-García & Dolores-Flores, 2021; Hiebert & Carpenter, 1992). Research has helped to understand how students make and use mathematical connections in different contexts and disciplines, highlighting the importance of exploring their structure and function to improve mathematical understanding (García-García, 2024). Accordingly, as Coles and Sinclair (2024) suggest, these connections offer a valuable tool for both researchers and educators.

In recent years, mathematical connections have become a central theme in educational research due to their crucial role in the learning and teaching of mathematics (Font & Rodríguez-Nieto, 2024), so García-García (2024) has reported at least six orientations that research on mathematical connections has taken. However, the literature has highlighted the challenges that both students, PMTs and teachers face when attempting to make mathematical connections (Moon et al., 2013; Radmehr & Drake, 2017). For instance, it has been identified that, in various tasks, including graphing, students tend to persistently rely on algebraic representations, even those with high performance (Dawkins & Mendoza, 2014; Hong & Thomas, 2015). Similar results were reported by Campo-Meneses and García-García (2020) and García-García and Dolores-Flores (2021), who indicated that the most frequent connections are procedural and those of different representations among pre-university students.

Dawkins and Mendoza (2014) suggest that the inadequate use of algebraic techniques can limit students' understanding and their ability to make effective transitions between different representations. Rodríguez-Nieto et al. (2021a) reported that students' understanding is achieved based on the presence or absence of certain mathematical connections that emerge from the mathematical practices they develop. Meanwhile, Li and Fan (2024) emphasized that the structures of textbooks can also influence the way students establish mathematical connections.

Research has highlighted the importance of teachers being aware of the different connections that students can make and adapting their teaching strategies accordingly (Kholid & Dewi, 2024). In this regard, Purnomo et al. (2024) indicated that problem-based learning could provide a structured way to foster deeper and more meaningful mathematical connections between concepts. While Mancilla et al. (2023) suggested that incorporating real-life situations through ethnomathematics in the classroom can enhance the connection between formal mathematics and everyday situations. Similarly, Rodríguez-Nieto et al. (2023) evidenced the need for a more integrated approach that considers both cultural connections and formal mathematics. In this respect, Novo et al. (2019) suggest that it is important to progressively work on conceptual connections in the classroom.

In a recent study with a similar orientation to the present research, focused on the concept of quadratic equations during the COVID-19 pandemic, the most frequent mathematical connections made by PMTs were procedural, feature, and meaning, while the connections of part-whole, modelling, and implication were identified less frequently (Hernández-Yañez et al., 2023). The difference is that in Hernández-Yañez et al. (2023), PMTs who had had face-to-face classes during their first years of university participated, while the participants in the present study mainly received their university courses online during their first years of training, and data collection was done when they had returned to face-to-face classes.

On the other hand, the literature recognizes that systems of linear equations (SLEs) are an important concept in school mathematics across different countries, beginning its study in secondary education (Oktaç, 2018; Turgut & Drijvers, 2021). Its significance lies in the study of more advanced topics in linear algebra, as well as its various applications to a range of problems and contexts (Oktaç, 2018; Rodríguez-Jara et al., 2019; Smith et al., 2022; Zandieh & Andrews-Larson, 2019). However, research on SLEs is limited, although it has been growing in recent years (Oktaç, 2018; Smith et al., 2022).

Research reports that there are calculation errors when solving problems related to SLEs (Fatio et al., 2020; Pulungan, 2019; Wildah, 2024), primarily due to a lack of understanding of the concept, as well as a tendency to solve problems without evaluating whether the information is sufficient to do so (Wildah, 2024). Tatira (2023) highlights that students face challenges when transforming systems of equations into matrices and interpreting the types of solutions, while Pulungan (2019) notes that students struggle to understand mathematical problems that require mathematical modelling and often forget important characteristics of the equations.

The literature has also reported that there are two categories of errors: arithmetic, including three subcategories: operations with integers, operations with fractions, and distributive; and algebraic errors, conceptual and procedural errors (Pérez et al., 2019). According to Fardah and Palupi (2023), the highest number of misconceptions made by PMTs was regarding the definition of SLEs, while the fewest misconceptions were related to the solutions of an SLE. In contrast, Cárcamo and Fuentealba (2019) indicated that students demonstrate problems interpreting solution sets of SLEs.

It has been identified that PMTs have difficulties in identifying the equivalence between systems of equations and in associating situations with mathematical models (Henriques & Martins, 2022). Possibly for this reason, PMTs feel more comfortable solving procedural problems (Kusuma et al., 2021). It is believed that the causes of errors and learning difficulties among students stem from teaching and learning activities in schools (Fatio et al., 2020).

Research focused on the concept of SLEs has also concentrated on studying the understanding of students and PMTs (Ayu & Eko, 2020; Cárcamo et al., 2021; Kusuma et al., 2021; Rodríguez-Jara et al., 2019), as well as designing activities to improve the learning of SLEs by students (Arnawa et al., 2019; Asmi et al., 2021). These studies emphasize the importance of continuing research on this concept due to the difficulties and errors associated with its study in the school context. Furthermore, there are few studies on this concept from the perspective of mathematical connections. Therefore, the aim of this research is to *identify the mathematical connections made by a group of Mexican PMTs when solving mathematical tasks on SLEs*.

Research on mathematical connections has advanced significantly in recent years, but many questions remain unanswered. We consider it essential to identify the mathematical connections used by PMTs in the post-pandemic period since the results could help identify possible deficiencies in their professional training (during and after the pandemic) and seek the development of courses to overcome them. By better understanding mathematical connections, we can design more meaningful and effective learning experiences for all students. Furthermore, considering tasks on SLEs is important because it is part of the curriculum in different countries including Mexico, making it knowledge that preservice teachers must know and use in their future practice (SEP, 2023).

THEORETICAL FRAMEWORK

This research is theoretically grounded in the idea of mathematical connections and their typologies. Therefore, this framework is described, along with the mathematical concept involved in the study.

Mathematical Connections

Businskas (2008) considers a mathematical connection as a true relationship between two mathematical ideas A and B. Instead, Eli et al. (2011) conceive them as the link where prior or new knowledge is used to strengthen or establish the understanding of the relationship(s) between mathematical ideas, concepts, or representations within a mental network. In line with this latter idea, Kenedi et al. (2019) assume that mathematical connections are part of a network of interconnected knowledge with other concepts critical for understanding and developing relationships between mathematical ideas, concepts, and procedures.

Moreover, García-García and Dolores-Flores (2018) define mathematical connection as a process through which a person establishes a true relationship between two or more ideas, concepts, definitions, theorems, procedures, representations, and meanings among themselves, with other disciplines, or with real life. These true relationships become evident in written productions as well as in the verbal arguments provide by a person.

For the purposes of this research, we adopted the position of García-García and Dolores-Flores (2018) because it encompasses the definitions of various authors, in addition to presenting a useful framework for analyzing the productions of PMTs when solving tasks on SELs. Likewise, the following typology of mathematical connections is proposed to be identified in the production of PMTs:

1. *Procedural.* This occurs when a subject employs a procedure (rules, algorithms, formulas, or even the use of a graph) to solve a task involving a mathematical concept (Businskas, 2008; García-García & Dolores-Flores, 2021). It is crucial that this usage be governed by valid arguments. For instance, when someone directly applies and justifies the use of substitution method to find the solution to a SLEs, this person is making this mathematical connection.

2. *Different representations.* These can be of two types: alternate or equivalent representations (Businskas, 2008; García-García & Dolores-Flores, 2021). An alternate representation occurs when a subject can convert (in the sense of Duval, 2017) a mathematical concept using different semiotic registers, and it is an equivalent representation when a treatment (in the sense of Duval, 2017) of a mathematical concept is carried out, meaning it is represented in different forms within the same semiotic register (Businskas, 2008). For example, an alternate representation occurs when a preservice teacher is able to graph the equations composing a SLEs on the cartesian plane; whereas an equivalent representation emerges when they rewrite the equation $ax + by = c$ as $kax + kby = kc$, where *k* is a real number.

3. *Part-whole.* This arises when a subject establishes logical relationships between mathematical concepts, either of generalization or inclusion (Businskas, 2008; García-García & Dolores-Flores, 2021). The former relationship occurs when one mathematical concept is a generalization of another, or when one mathematical concept is a particular case of another, and the latter relationship occurs when one mathematical concept is included in another, or when a concept includes another (Businskas, 2008). For example, when a preservice teacher asserts that the system $2x + 3y = 0, 4x + y = 0$ is a particular case of $ax + by = n, cx + dy = m$ is making this type of mathematical connection.

4. *Implication.* This mathematical connection emerges when one concept depends on another in a logical manner (Businskas, 2008), i.e., when a relationship of the form $P \Longrightarrow Q$ is established. For instance, when a future teacher can establish that if a system of linear equations is consistent and the graphs of the equations overlap, then the system has infinitely many solutions.

5. *Feature.* This appears when someone identifies some attributes, qualities or properties of mathematical concepts that make them different or like others (Eli et al., 2011; García-García & Dolores-Flores, 2021). According to García-García (2024), it may include the verbal or written description of some components of a representation (algebraic, geometric, graphical, etc.) or even some characteristics of a theorem. For example, when a preservice teacher recognizes that in a SLEs, the composing equations are only of degree 1, meaning the maximum degree of the unknowns is 1, they are employing this mathematical connection.

6. *Meaning.* This arises when someone gives meaning to a mathematical concept, that is, what it signifies for them. It includes instances where they offer a personal definition constructed by the subject for these concepts and even when they describe their context of use (García-García & Dolores-Flores, 2021). This typology also encompasses answers where a subject provides a formal definition of mathematical concepts or theorems based on the totality of mathematical characteristics or properties they may possess. This distinction from the previous typology lies in the fact that in this case, the student can indicate all attributes, qualities, or properties, whereas in the feature type, he or she only indicates some. For example, when a preservice teacher defines the solution set of a SLEs as the set of values of the unknowns that simultaneously satisfy the equations of the system, they are employing this mathematical connection.

Systems of Linear Equations

An SLEs is a finite set of linear equations, each with the same variables, where the process of finding its solution set is known as solving the system (Poole, 2011). Additionally, the solution set of an SLEs is the set of all solutions to it, also understood as a vector that simultaneously satisfies each equation in the system. Furthermore, Poole (2011) considers that an SLE is called consistent if it has at least one solution, inconsistent if it has no solution, and two SLEs are called equivalent if they have the same solution sets. Mathematically, the structure of an SLE is of the form:

$$\begin{gathered}a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_n,\end{gathered}$$

where a_{ij} are real numbers denoting coefficients, x_i are variables, and b_j are constants. Taken together, we have a system of m equations with n unknowns.

METHODOLOGY

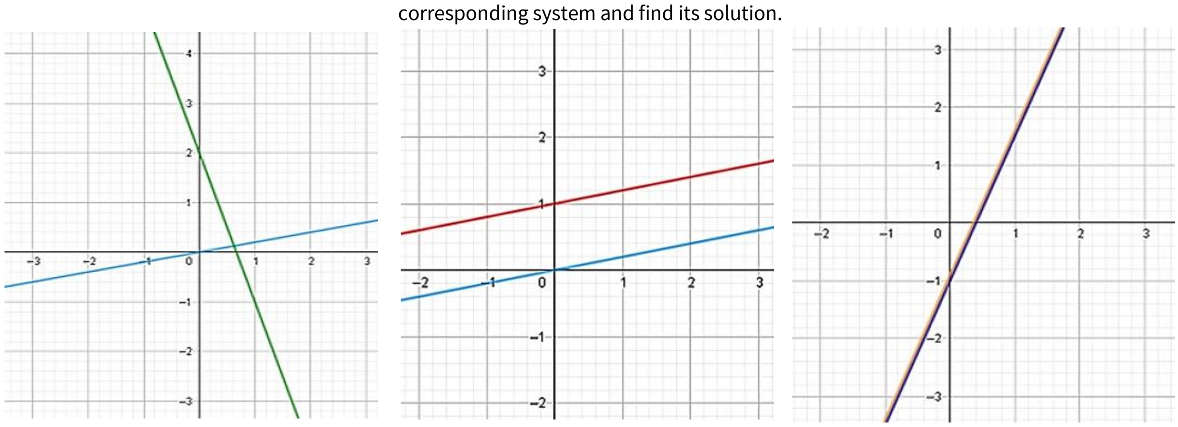
This research adopts a qualitative approach. The method used for data collection was the task-based interview proposed by Goldin (2000), who suggested ten general principles for designing and constructing tasks. For the purposes of this research, the following principles were considered: tasks were chosen to be accessible to participants; tasks incorporated rich representational structures; the interview was explicitly described beforehand; decisions were made in advance regarding what and how much would be recorded; a pilot interview was conducted; and the interview was designed anticipating new or unforeseen possibilities.

Data Collection

For the design of the task-based interviews, activities, and problems on SLEs from the book *Linear algebra: A modern introduction* by Poole (2011) were analyzed. These tasks were then refined to ensure appropriateness, clarity, and the ability to elicit a variety of reflections and arguments from the future teachers as they worked through them. Other tasks were designed by the authors considering the aim of this research. Once the initial instrument with five tasks was designed, it was validated through a pilot test with university-level students and expert validation by a researcher who has worked in the field of mathematical connections.

Once validated, the task-based interview (**Table 1**) was applied to each preservice teacher, with an average duration of 45 minutes. The interviews were recorded in their entirety and after transcribed for analysis along with the written answers of the preservice teachers.

Table 1. Designed and implemented task to the participants

Number	Task
Initial questions	What is a system of linear equations? What is the purpose of a system of linear equations? How do you solve a system of linear equations? What is the solution set of a system of linear equations? When is a system not a system of linear equations ?
1	If you know that the solution of a system is $x = 3 + 2k$ and $y = 2 + 5k$, where k is a real number, how many systems of equations could be formed? Why? Propose a system with that solution.
2	Solve the following problem, providing an explanation for your answer: *Bob and Jim are two good friends who live 420 miles apart. Sometimes they meet for lunch at a restaurant somewhere between their cities. On one occasion, they left their homes at the same time. Both took 4 hours to reach the restaurant. Determine Bob's speed and Jim's speed, given that Bob drives an average of 5 miles per hour faster than Jim.*
3	Observe the following system of linear equations and answer the questions: $x + y = 10$ Does the system have a solution? If your answer is negative, explain why? If is affirmative, indicate how many solutions it has and what they might be? Provide examples of solutions for the system.
4	Indicate what type of system corresponds to each graph according to the classification of system of linear equations. Write the corresponding system and find its solution.  (a) (b) (c)

Context and Participants

The research was conducted at the faculty of mathematics of a university located in southern Mexico. The faculty has been training professionals in the field of mathematics for 43 years and has adequately equipped classrooms for all levels of courses and postgraduate studies. Many of the faculty members hold doctoral degrees, with only a minority having master's degrees. For each course offered to undergraduate students, there are typically two qualified instructors available to teach it. Students have the freedom to choose which professor they wish to take each course with. Additionally, the bachelor's degree in mathematics offers four specializations: basic mathematics (focused on advanced mathematics aimed at generating new mathematical knowledge), mathematics education (training PMTs), computer science (studying the mathematics and its relationship with technological advancements in the field of computing), and applied mathematics (focusing on statistics, probability, and related areas).

The participants consisted of four PMTs (2 females and 2 males), aged between 21 and 25, from the eighth semester of the faculty. All of them were enrolled in the mathematics education program, with grade averages ranging from 7.6 to 9.3 (out of a maximum of 10) throughout their undergraduate studies. Three of the participants had already completed the linear algebra course, while one claimed to have not yet taken it. These four participants represented the entire eighth-semester cohort of the mathematics education program and voluntarily agreed to participate in this research. Hereafter, they will be referred to as PT1, PT2, PT3, and PT4.

Data Analysis

The data were analyzed using thematic analysis, a method that enables the identification, organization, and systematic presentation of information within a dataset. This type of analysis allows the researcher to recognize and make sense of collective or shared meanings and experiences (Braun & Clarke, 2012). Braun and Clarke's (2006, 2012) thematic analysis is typically divided into six phases; however, for the purposes of this research, it was condensed into four phases by García-García (2024), which were utilized in this study.

1. *Phase 1: Familiarization with the data.* This phase involves thoroughly reading the data. It requires multiple readings of the data to grasp the language used by the participants. Taking notes on the data while reading or listening to audio recordings is important to focus the analysis on the subsequent phase, that is, highlighting interview excerpts and written productions where closer attention should be paid due to the presence of a possible mathematical connection. For this reason, at this stage, transcripts were created of all the arguments and justifications offered by the preservice teachers in the video recordings as they solved the proposed tasks. Additionally, the students' written productions were simultaneously observed to identify the relationship between what each preservice teacher wrote and what each he or she argued verbally.

2. *Phase 2: Generate initial themes.* In this phase, themes are conceptualized as mathematical connections. Therefore, to achieve this phase, both the verbal and written productions of the preservice teacher were reviewed to identify *true relationships between two or more ideas, concepts, definitions, theorems, procedures, representations, and meanings among themselves with other disciplines or with real life* (García-García & Dolores-Flores, 2018), that is, the first mathematical connections that the preservice teacher made when solving the proposed tasks. For example, in **Table 2**, is indicated how excerpts from the interview allowed for the generation of a subtheme (specific mathematical connection when solving tasks on SLEs), and this subtheme allowed for the construction of the theme.

Table 2. Example of themes constructed from interview excerpts

Excerpt	Subtheme	Theme
E: What is a system of linear equations? PT2: A system of linear equations consists of two or more equations with two or more unknowns.	A system of linear equations is constituted of two or more linear equations.	Feature mathematical connection
E: Did you know before solving that the system had a solution? PT1: Yes. E: Why? PT1: Because of the intersection point right there [points to the intersection point of the graphs associated with the system of linear equations]. E: Do you think it's necessary for the lines to intersect? PT1: To have a solution, yes.	If the lines associated with the linear equations of a system intersect, then the system has a solution.	Implication mathematical connection

3. *Phase 3: Review and definition of themes.* In this phase, the initial themes identified in the previous phase were reviewed and compared with the verbal and written productions of the preservice teacher. Once we had established confidence in the pattern of meaning conveyed by the preservice teacher in their procedures and arguments, we were able to pinpoint the mathematical connections made by him or her. The definitions of themes were grounded in the preservice teachers' productions and the relationships they constructed.

4. *Phase 4: Report elaboration.* This phase involves a recursive process whereby the developing themes are reviewed in relation to the sub-themes, coded data, and the complete dataset. If any inconsistencies are identified, the sub-themes and themes are refined accordingly. The fourth phase of our analysis consisted of preparing the research report. To achieve this, we chose to provide illustrative examples representing each identified mathematical connection.

To ensure reliability and mitigate the bias of a single researcher in the analysis conducted, triangulation between researchers (Aguilar & Barroso, 2015) was performed in all phases of the analysis.

RESULTS

During the analysis of the collected data, the productions made by the participating preservice teachers have been thoroughly explored from a qualitative perspective, focusing on understanding the mathematical connections they managed to make. First,

Table 3 will be presented, where the number of mathematical connections evidenced in each task by each preservice teacher has been documented. Subsequently, we will explicitly demonstrate how these mathematical connections were identified.

Table 3. Types of mathematical connections identified in PMTs

Preservice teacher	Task	Mathematical connection identified
PT1	Initial questions	Implication
	1	None
	2	Different representations and procedural
	3	Feature
	4	Different representations, procedural and implication
PT2	Initial questions	Feature
	1	None
	2	Procedural
	3	Feature
	4	Implication and procedural
PT3	Initial questions	Feature, meaning and implication
	1	None
	2	Procedural
	3	Feature
	4	Implication
PT4	Initial questions	Meaning and feature
	1	None
	2	Different representations and procedural
	3	None
	4	Different representations and procedural

Mathematical Connections Made by PT1

PT1 made mathematical connections of different representations, procedural, feature, and implication types (**Table 3**). For instance, the implication mathematical connection was evident when he stated that *if a system of equations is not an SEL, then one of the equations in that system is not linear* in the initial questions. For its part, the mathematical connection of different representations was evident when PT1 make a conversion between different representations: from written language to algebraic and symbolic representation (**Figure 1**), and algebraic representation again for solving task 2 (**Figure 2**). The argument provided by PT1 when solved task 2 was:

> I made my representation from Bob to Jim, it's four hundred twenty. So, we don't know the distance (between them) and the restaurant. To get from Bob to 'r' (restaurant) takes a distance of 'a' and from 'r' to Jim, takes a distance of 'b' [points to the drawing].

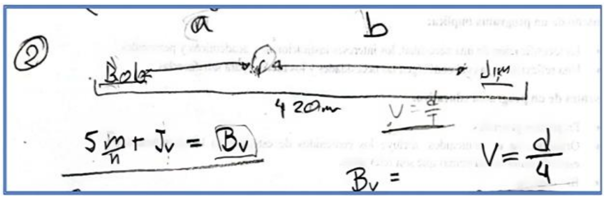


Figure 1. Algebraic and symbolic representation made by PT1 (Source: Field study)

In his attempt to solve task 2, PT1 also made a procedural mathematical connection (**Figure 2**). He used the formula for velocity $v = \frac{d}{t}$ for setting up the SLEs and the substitution method to solve this system (**Figure 2**). PT1's argument was:

> They ask me for Bob's and Jim's speeds, so [...] I understand that speed equals distance over time. I already have the time data, which is four hours. They both took four hours to arrive, so I have my formula which is speed equals distance divided by four.

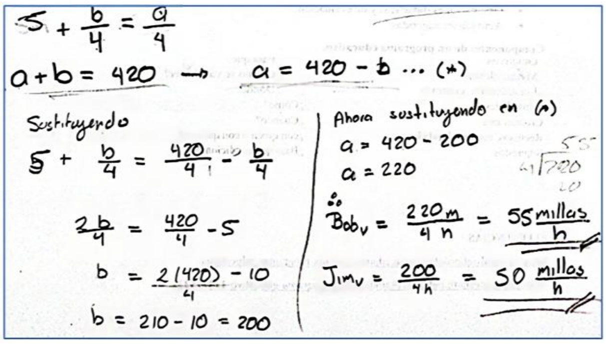


Figure 2. Development of the substitution method applied by PT1 to solve the SEL he built (Source: Field study)

On the other hand, in task 3 and according to PT1's arguments, we inferred that for him, a SLEs consists of two or more linear equations, therefore, the use of the feature type mathematical connection was identified. For its part, in task 4, PT1 initially made the mathematical connection of implication at two moments. First, because when we questioned if the graphs associated with a SLEs provided in section *a* had a solution, he acknowledges that *if the lines associated with the linear equations of a system intersect, then the system has a solution*. In a second moment, it was showed in task 4 section b, when we asked what the solution of the system associated with the graph was, PT1 reacted: *It is evident that they are parallel lines, so being parallel lines, they will never intersect, therefore it has no solution*, that is, for him *if the lines associated with a SLEs are parallel, then the system has no solution*.

In the same task 4, the results allowed us to identify the procedural mathematical connection at different moments. For example, when the PT1 used the point-slope formula $y - b = m(x - a)$ to obtain the equations of the lines for the first system (**Figure 3**) and when he solved the system formed by the substitution method (like task 2). On the other hand, the mathematical connection of different representations was evidenced at two moments, first, when he converted a graphical representation (provided by the task) into an algebraic representation (the SLEs constructed by PT1, see **Figure 3**) and, secondly, when he performed a treatment within the algebraic representation. This latter was evidenced when the PT1 transformed one of the equations that form the system into another equation to apply the substitution method (**Figure 3**). That is, PT1 used equivalent representations.

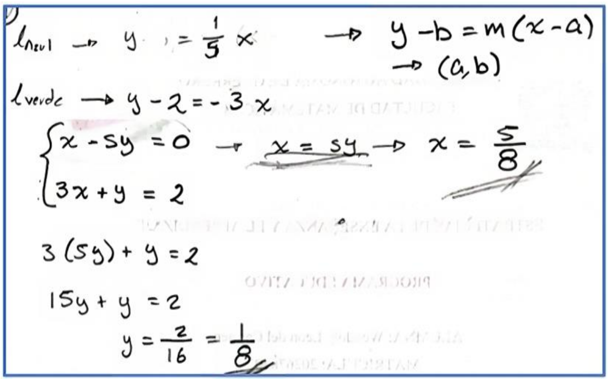


Figure 3. Procedure used by PT1 for task 4 section a (Source: Field study)

Mathematical Connections Made by PT2

The PT2 made mathematical connections of feature, procedural, and implication types in their answers (**Table 3**). Thus, in the initial questions, he demonstrated the use of the feature connection twice. First, when he described that the equations of a SLEs are of degree one. The second occasion, when he explained that a SLEs consists of two or more equations with two or more unknowns. This answer allowed us to identify the feature type mathematical connection because it only partially described what a SLEs represents. The same connection was identified in task 3 because PT2 indicated that a SLEs is formed by two or more linear equations.

In task 2, a connection of different representations was showed when PT2 wrote Bob's speed (*y*) in terms of Jim's speed (*x*) and established the SLEs that allowed him to solve the task (**Figure 4**). In the same task, PT2 used the substitution method to solve the constructed SLEs, providing evidence of the use of the procedural connection (**Figure 5**).

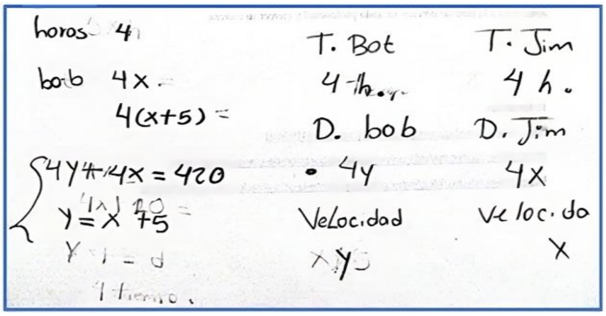


Figure 4. The SLEs established by PT2 to solve task 2 (Source: Field study)

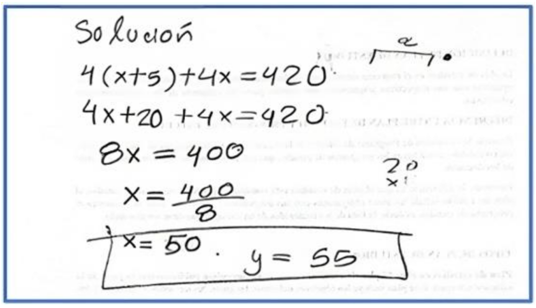


Figure 5. Using the substitution method to solve the SEL that PT2 constructed (Source: Field study)

In task 4, section *a*, when we asked to PT2 if the system associated with the graph has a solution, he replied that, *yes [...] because they intersect*. Like PT1, he considered that *if the lines associated with the linear equations of a system intersect, then the system has a solution*. On the other hand, PT2 argued that *if the graph of a line falls from left to right, then the slope of the line is negative* (task 4, section *a*). Therefore, it was identified that he initially used the implication type mathematical connection in this task.

The previous answer was complemented when PT2 calculated the slope from the graphs of the lines provided in the task using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ (**Figure 6**). To do this, PT2 identified the points of intersection of the lines with the coordinate axes to obtain the equations of the lines. These actions carried out by PT2, in addition to the use of the method of elimination by addition and subtraction (**Figure 7**) employed by him to solve the SLEs that he formed, allowed us to identify the procedural mathematical connection. It is important to highlight that PT2 used the different representations connection together with the procedural connection in all three sections of task 4.

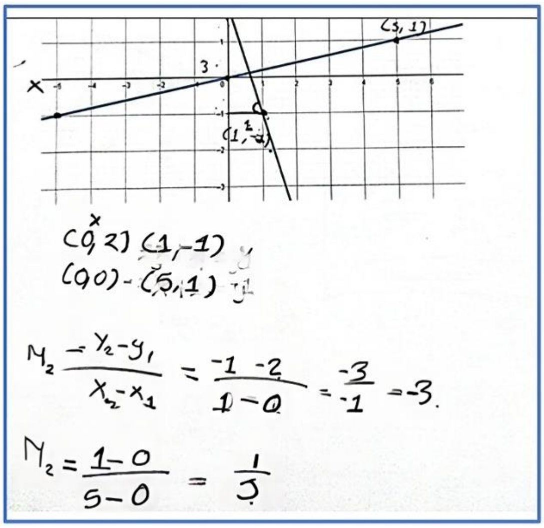


Figure 6. PT2 obtained the slopes of the lines using two points in task 4 section a (Source: Field study)

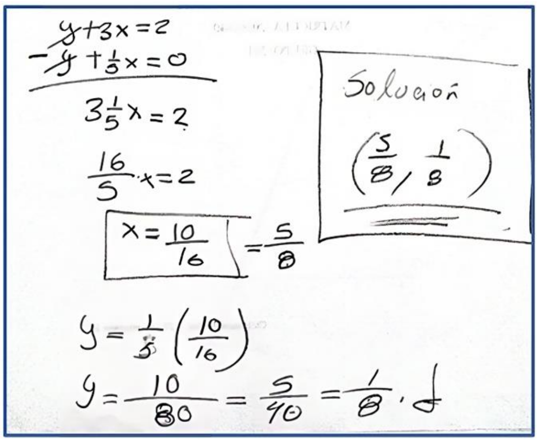


Figure 7. Use of the addition and subtraction method applied by PT2 to solve the SEL in task 4a (Source: Field study)

Mathematical Connections Made by PT3

The PT3 evidenced the use of mathematical connections of feature, meaning, implication, and procedural types (**Table 3**). Regarding this, the feature connection appeared twice in the initial questions. The first moment was when she answered that the equations of a SLEs can have one or more variables when we asked about the characteristics of the equations of a SEL. The second occasion was when she argued that a system is not a SLEs when the exponent of the variables in the equations is more than one.

In the same initial questions, PT3 answered that the solution of a SEL are the values of the variables that make the equality in the equations of the system true. Likewise, at another moment, she expressed that they are all the possible solutions that the equations (referring to the SEL) can have, thus we identified the use of the meaning mathematical connection. The feature mathematical connection was used a third time when, in task 3, the PT3 stated that a SLEs must have more than one equation to be a system.

On the other hand, PT3 used the implication connection twice, first when, in section b of task 4, she indicated that *graphically if two lines do not intersect at any point, then the SEL has no solution*. The second occasion was when, in task 3, she expressed that *if a system of linear equations does not have at least 2 linear equations, then it is not a system of equations*. The PT3 could not solve task 4.

Mathematical Connections Made by PT4

PT4 evidenced the use of mathematical connections of the following types: different representations, procedural, feature, and meaning (**Table 3**). The connection of different representations was identified when, in task 2, she wrote Bob's speed in terms of Jim's speed (**Figure 8**), and the procedural mathematical connection emerged when, in the same task 2, she used the substitution method to solve the SEL that she obtained (**Figure 8**). Below is an excerpt from the interview where PT4 structures a SEL and then uses the substitution method to solve the problem:

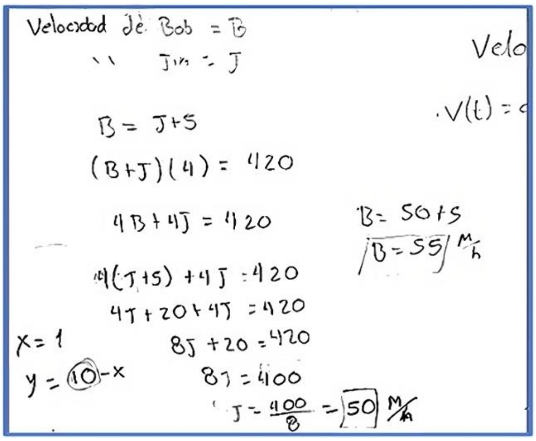


Figure 8. Development of the substitution method applied by PT4 to solve the SEL that she built (Source: Field study)

I: I recommend that you start by establishing your variables [to solve the task].

PT4: The distance is going to be equal to...

I: [...] How can you represent Bob's speed in terms of Jim's?

PT4: So, [writes] $J + 5$.

I: Okay, and what is next?

Subsequently, PT4 reflected on the problem and finally she established the SLEs that solves the task.

On the other hand, the mathematical connection of meaning was identified when PT4 stated that solving a SLEs is about finding the values of the variables [that make the equations true] in the initial questions. In addition, the feature mathematical connection was evidenced when she explained that for a system of equations to be a SEL, the exponent of each equation must be one.

In task 4, the PT4 made the connection of different representations when she wrote the equations corresponding to the given lines in part *a* (**Figure 9**). In the same activity, the procedural connection emerged when PT4 applied the method of substitution to solve the SLEs that she formed previously.

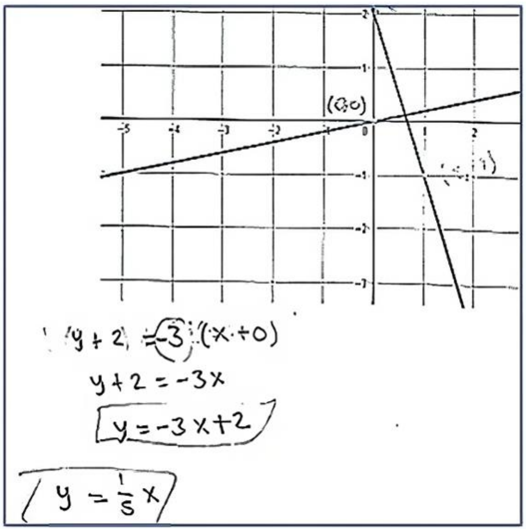


Figure 9. Equations corresponding to the lines given in section a of task 4 (Source: Field study)

DISCUSSION AND CONCLUSIONS

The aim of this research was to identify the mathematical connections made by a group of Mexican PMTs when solving mathematical tasks on SELs.

The results indicate that PMTs make mathematical connections of the following types: feature, procedural, different representations, implication, and meaning (**Table 4**); however, the majority did not employ the expected mathematical connections in each proposed task.

Table 4. Mathematical connections made by the PMTs

Mathematical connections evidenced	Preservice teachers			
	PT1	PT2	PT3	PT4
Procedural	●	●	●	●
Feature	●	●	●	●
Implication	●	●	●	
Different representations	●	●		●
Meaning			●	●

The exploration of mathematical connections made by PMTs during the resolution of tasks on SELs sheds light on several key aspects of their mathematical understanding and pedagogical preparation. The participants demonstrated operational dexterity in solving the SELs using mechanized procedures involved in the various solution methods, which allows us to hypothesize that in their university education, greater emphasis was placed on procedural treatment to the detriment of the conceptual. This resulted in the four participants using the procedural connection, but none made the part-whole connection. This result is consistent with the findings of Jailani et al. (2020) who found that most students had difficulties in making the part-whole connection. This suggests a gap in their understanding of the holistic nature of mathematical concepts involved in the proposed tasks.

In the same vein, while PMTs demonstrated proficiency in making four out of the six expected mathematical connections (**Table 4**), PT1 emerged as the most adept in utilizing these connections, evident in their ability to solve tasks successfully. Notably, PT1's utilization of mathematical connections in an appropriate manner underscores the importance of fostering a robust understanding of connections among mathematical concepts for effective problem-solving, as warned by DeVries and Arnon (2004). These results align with the findings of Jailani et al. (2020) and Hatisaru (2023) who found the predominance of mathematical connections of the implication type and different representations in students and in-service teachers.

In opposition, PT3, despite exhibiting competence in employing the identified mathematical connections, encountered challenges in task solution. This discrepancy highlights the nuanced relationship between the application of mathematical connections and proficiency for solving the proposed tasks. This suggests that successful task achievement requires more than just the recognition of mathematical relationships; it necessitates a deep understanding of how to leverage these connections in problem-solving contexts. This was partly due, as noted by García-García (2024), to their productions being marked using elementary mathematical connections (procedural, feature and implication) primarily. Therefore, it is important to improve the ability of PMTs to make mathematical connections as a powerful way to improve their mathematical understanding, as also recognized by Quilang et al. (2022).

The results are also consistent with those reported by Campo-Meneses and García-García (2020) with university students who more frequently use procedural, implication and feature mathematical connections when solving mathematical tasks. On the other hand, are partially consistent with those reported by Hernández-Yañez et al. (2023), where it was found that PMTs, when working with tasks on quadratic equations during the COVID-19 pandemic, more frequently evidenced the use of procedural, feature, and meaning mathematical connections, while less frequently using part-whole, modelling, and implication connections.

Likewise, are partially consistent with the findings of Rodríguez-Nieto et al. (2021b) who found that the most frequently used connections are procedural, different representations, and part-whole by PMTs.

The above findings underscore a trend in mathematics teacher training to prioritize the procedural, which could influence their approach to teaching SELs in the future. According to Oktaç and Trigueros (2010), this emphasis on procedural fluency may stem from pedagogical practices and instructional materials that prioritize algorithmic proficiency over conceptual understanding. However, the algorithmic handling of SELs to the detriment of a deep understanding of the underlying meanings and concepts could limit the ability of PMTs to help students develop a more meaningful understanding of this mathematical concept (Byerley & Thompson, 2017).

These results showed that the mathematical connections made by PMTs align with those reported by Kusuma et al. (2021), who indicated that preservice teachers are more comfortable solving problems procedurally. Similarly, the results are consistent with those obtained by Kinati and Windyana (2023) and those of Cárcamo and Fuentealba (2019), as many misconceptions were about the definition of a SELs and errors regarding the concept of solution, solution set, and types of solution set of a SELs. On the other hand, the limited incidence of meaning as mathematical connection raises concerns about the depth of future teachers' conceptual understanding and their ability to facilitate meaningful mathematical learning experiences for students (Byerley & Thompson, 2017). This is because the way they understand a mathematical idea is a principal factor in the mathematical understanding that their students manage to develop (Copur-Gencturk, 2015; Thompson, 2013).

Our results allow us to infer that PMTs continue to have difficulties to solve tasks relating to SELs. This leads to the reflection that, in previous levels where the PMTs worked with this concept in the Mexican context, they did not build solid foundations on the concepts and definitions related to SELs. This fact underscores the need to develop interventions aimed at enhancing preservice teachers' ability to make interdisciplinary connections and contextualize mathematical concepts. Professional development initiatives that emphasize the development of mathematical connections and promote transdisciplinary approaches to teaching mathematics could help address these challenges (García-García, 2024). This proposal is important because it aligns with what the official curriculum of Mexico demands, which, among other things, proposes to work on school contents in a transdisciplinary manner and based on projects (SEP, 2023).

Finally, some limitations of this work were the number of participants, as with a larger sample, the results could vary. However, this was due to the low enrolment of students who were training to become teachers at the time of data collection. Future research endeavors should seek to expand participant demographics and explore the efficacy of targeted interventions in enhancing preservice teachers' mathematical understanding and pedagogical practices. Addressing these challenges is crucial for ensuring the effectiveness of mathematics education and promoting mathematical literacy among students.

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Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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