International Electronic Journal of Mathematics Education
2024, 19(4), em0793
e-ISSN: 1306-3030
https://www.iejme.com

MODESTUM

Research Article **OPEN ACCESS**

Impact of school experiences on the beliefs and instructional practices of future mathematics teachers

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Citation: Chandía Muñoz, E., Saadati, F., & Huencho, A. (2024). Impact of school experiences on the beliefs and instructional practices of future mathematics teachers. *International Electronic Journal of Mathematics Education, 19*(4), em0793. https://doi.org/10.29333/iejme/15201

ARTICLE INFO

Received: 10 Jun. 2024

Accepted: 17 Sep. 2024

ABSTRACT

This study examines the impact of cumulative experiences and beliefs during the school career on the pedagogical development of future mathematics teachers in Chile. A sample of 252 pedagogy students from 18 higher education institutions participated in the study, completing a questionnaire that focused on instructional paradigms, teaching experiences, and pedagogical practices. The research employed latent class analysis to identify distinct profiles based on the teaching paradigms experienced by the participants. A logistic regression model further corroborated these profiles. The findings indicate a prevalent inclination towards constructivist beliefs and practices among future teachers, which are significantly shaped by their secondary education experiences, often devoid of a constructivist approach. This study emphasizes the critical role of prior educational experiences in shaping professional competencies in teaching mathematics.

Keywords: initial teacher education, school career, beliefs, instructional practices

INTRODUCTION

The role of the school career in shaping the educational beliefs and practices of preservice teachers (PTs) has been described in different studies (Chandía et al., 2023; Depaepe et al., 2020; Nehls et al., 2020; Öçal, 2021). These studies emphasize the importance of the "school career," which refers to the entire trajectory of experiences–both practical and cumulative–that a PT has undergone throughout their years of formal education. This includes not only the content learned but also the teaching methods encountered, the classroom environments experienced, and the interactions with educators and peers, all of which contribute to the PT's evolving beliefs and practices in education. This can be understood as a social framework that facilitates the acquisition of knowledge and competencies essential for pedagogical development throughout the educational journey. Notably, the experiences of PTs within the structured socio-educational environment, which includes interactions with teachers, peers, and the curriculum itself, have been found to profoundly influence their future teaching methods (Yurekli et al., 2020). Consequently, PTs often replicate the teaching methods they were exposed to during their own education (Ruohotie-Lyhty & Kaikkonen, 2009). This replication suggests that teachers' patterns of classroom practices are often shaped by their own experiences as students, thus highlighting the enduring impact of the school career on teaching approaches.

These patterns are recalled based on the similarity of teaching situations encountered by PTs and the predominant mental associations stored in their secondary memory (Seitz et al., 2023). From a cognitive standpoint, these associative patterns establish connections among socio-affective and cognitive skills, the decisions made, and their outcomes in real-world mathematical teaching and learning scenarios. Within the educational realm, this capacity is recognized as "teacher professional competence" (Blömeke et al., 2022a). In essence, the school career serves as a framework for the development of professional competence in future teachers, shaping their educational perspectives and practices through the assimilation of experiences and cognitive associations gleaned from their educational journey.

This conceptualization provides a way to understand how the cognitive and socio-affective domains of teachers' professional competence relate to future teachers' teaching practices (Wall & Hall, 2016). However, understanding these dynamics does not necessarily entail the ability to explain or justify their inherent complexity. Thus, it cannot be argued that the activated framework of cognitive and affective dispositions in a school situation directly determines the type of strategy future teachers will use, as suggested by Blömeke et al. (2020) and Depaepe et al. (2020). In this framework, beliefs about mathematics, its teaching, and learning emerge as one of the key mediating variables in the literature, as they are an intrinsic part of the memory traces established by PTs in their professional competence initially constructed in their school careers (Blömeke et al., 2022b; Depaepe et al., 2020; Lloyd & Howell, 2019; Seitz et al., 2023).

In the field of mathematics education, beliefs, understood as personal and subjective conceptions about mathematics, its teaching and learning, act as cognitive filters that shape teachers' decisions and actions. These beliefs, which are grounded in prior experiences and individual perceptions, exert a profound influence on the selection of pedagogical strategies, the choice of content, and the assessment of learning. A substantial body of research (Depaepe et al., 2020; Moreira et al., 2020; Yurekli et al., 2020; Zhang et al., 2022) has demonstrated the bidirectional nature of this relationship, underscoring the necessity of investigating and transforming teachers' beliefs to facilitate more effective and equitable practices. In this context, the development of PTs' belief systems is deeply influenced by their lived educational experiences in the school program, including the theoretical and methodological frameworks that underpinned their training and their own teaching practices when they assumed the role of teacher (Lee & Santagata, 2020). Such experiences provide an intuitive and essential foundation that PTs use to develop their approach to effective mathematics teaching. This approach, according to Ariza et al. (2023), translates into how PTs conceptualize the teaching process. Furthermore, Wang et al. (2017) emphasize that this influence is evident in the pedagogical decisions PTs make, which are directly reflected in their teaching practices.

Recognizing the intricate interplay between beliefs and practices, and the profound influence of previous schooling experience, this study investigates the relationships between systematic educational experiences, beliefs about mathematics teaching and the design of mathematical tasks among pedagogy students, focusing on the impact of these factors on instructional practices and professional competence in mathematics teaching. Specifically, our research explores how the school career and established pedagogical paradigms shape these beliefs and practices among PTs enrolled in primary teacher education programs.

LITERATURE REVIEW

Educational Beliefs: A Comprehensive Perspective

In the affective domain of professional competence for teachers, beliefs, attitudes, and emotions are included, which are interrelated (Depaepe & König, 2018), often complicating their separate understanding (Öçal, 2021). Thus, for this study, beliefs are understood as those that involve a relationship between the subject and their experiences. Therefore, a belief is an idea, or even several, about the world or oneself that is perceived as true, which can be conscious or unconscious.

Research on beliefs has established several facts about them, such as

(a) they are formulated at an early age (Pajares, 1992),

(b) they are complex to modify (Jenßen et al., 2020),

(c) they interrelate, forming systems (Moreira et al., 2020), and can be nested,

(d) they are primarily formed by subjects' experiences (Lee & Santagata, 2020),

(e) they are multidimensional (Nehls et al., 2020) and varied within a subject, sometimes even contradictory (Cooney, 2002), and

(f) they affect perceptions, interpretations, as well as the decisions made to solve various situations, such as those encountered in learning or teaching mathematics (Depaepe et al., 2020; Seitz et al., 2023).

In this wealth of information, a new approach has emerged in the literature, coming from the cognitive and neurosciences perspective. Angel and Seitz (2016) affirm that believing constitutes a set of brain functions similar to those involved in cognition and emotion, and they dissociate from the idea that belief solely belongs to the affective domain of a subject's disposition field. Thus, beliefs would be the result of an associative pattern between socio-affective and cognitive skills generated by repetitive ascending and descending processing by specific sensory brain systems, leading to a higher-order meaning of perceived elements, including their attributed emotional value (Seitz et al., 2018).

Thus, beliefs would be suitable for decision-making regarding action selection and event prediction through optimization (Seitz et al., 2023). These authors have argued that the brain could optimize probabilistic beliefs about external variables in a generative model in such a way that, by acting on the world, real sensory data are consistent with the inferred predictions of the model. Inference about events leads to the identification of putative functions of objects (tool use) or people (interaction).

Within the field of mathematics education, research has delineated three categories of belief systems in school and initial teacher education (ITE) settings, but with different latent weights (as illustrated in **Figure 1**):

(1) beliefs about the nature of mathematics (Berk & Cai, 2019),

(2) beliefs about mathematical learning processes, and

(3) beliefs related to the teaching of mathematics (Saadati et al., 2021; Zhang et al., 2022).

These dimensions have been shown to have a significant correlation in both students and teachers. Beliefs about teaching practice not only reflect but also help clarify the underlying conceptions about mathematics and its learning (Lo, 2021; Xue et al., 2022).

In particular, regarding beliefs about the nature of mathematics, two distinct perspectives have been identified in the academic literature: the absolutist or static view and the fallibilist or dynamic view (Berk & Cai, 2019). The absolutist perspective conceives mathematics as an immutable collection of facts, concepts, and procedures that remain constant, universal, and

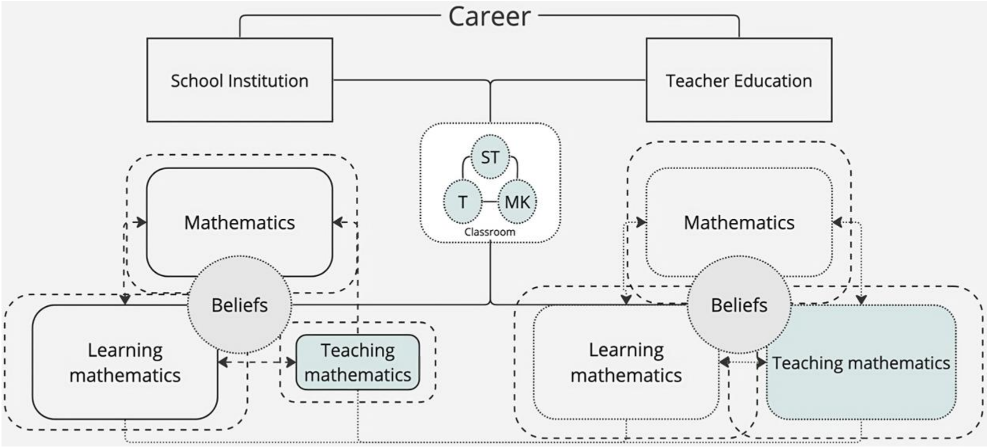

Figure 1. Construction of PTs' belief systems from the school to teacher education institution (ST: Students, T: Teacher, & MK: Mathematics knowledge/mathematics task) (Source: Authors' own elaboration)

culturally neutral. Conversely, from the fallibilist perspective, mathematical entities are considered human constructions that are constantly evolving and influenced by their cultural and temporal context.

Beliefs about learning are related to the role assumed by the subject learning when tackling mathematical situations, which is observed both in the school classroom and in the ITE classroom for PTs (see **Figure 1**) (Xie & Cai, 2021). Along these lines, research finds that pedagogy students, as learners, establish beliefs about themselves concerning mathematics in both scenarios, shaping structures and systems or modifying those already established such as their self-efficacy levels, learned helplessness, motivation, and attributions for success or failure in learning mathematics (Ariza et al., 2023).

Finally, beliefs related to teaching refer to the role played by the teacher, or the PT occupying this role, as well as their pedagogical strategies to facilitate mathematical learning. This involves an assessment of the effectiveness of their methodological approaches (Xie & Cai, 2021). Research identifies two predominant approaches: constructivist and transmissive, which correspond to dynamic and static views of mathematics, respectively. Under the constructivist perspective, mathematical teaching is conceived as an active and meaningful interaction between content, teacher, and students, granting a central role to student experiences within the teaching and learning process. Thus, mathematical learning is considered to mainly arise from proposed activities, classroom interaction, and the teacher's facilitating role.

In contrast, the transmission perspective suggests that mathematical knowledge should be imparted directly and systematically, emphasizing memorization and application of definitions, theorems, and algorithms. In this approach, teaching is directive and teacher-centered, in which the student assumes a passive role as an information recipient. In this view, it is postulated that mathematical learning primarily depends on the student's capacities and intrinsic skills (Depaepe & König, 2018).

These three categories of beliefs begin to be constructed during the school period, including those related to the teaching of mathematics, and continue their development and adjustment in teacher education (see **Figure 1**) (Merk et al., 2017). Therefore, learning mathematics based on specific teaching models results in the initial construction of the teaching identity (Cole et al., 2023), due to the interference of belief systems about the learning and teaching of mathematics that PTs undergo in their school itinerary (Moreira et al., 2020) and in initial education (Curtis et al., 2019) (see **Figure 1**).

Overall, the evolving landscape of beliefs in teacher education underscores the need for further exploration into their stability and influence on pedagogical decision-making (Curtis et al., 2019; Weyers et al., 2024). By delving into beliefs forged in the crucible of schooling, this research seeks to illuminate their enduring impact on teaching practices and educational outcomes.

Relationship Between Experience and Belief About Teaching

Beliefs are strongly influenced by an individual's experiences and fall into two categories: primary and secondary. Primary beliefs emerge from direct experiences, while secondary beliefs are built on the latter, demonstrating the interconnectedness of different belief systems (Moreira et al., 2020). This suggests that primary beliefs about learning and mathematics are fundamental to the development of secondary beliefs about their teaching. Recognizing and identifying these primary beliefs is essential in the school career as they lay the foundation for the formation of secondary beliefs (Cole et al., 2023).

The beliefs that PTs hold about mathematics education are deeply rooted in their previous educational experiences and have a significant influence on their pedagogical practice. According to Sotos (2021) and de Vries et al. (2013), it is the prevailing culture in school education institutions, which includes curricular-disciplinary, learning and teaching domains, that shapes their beliefs about didactics, beliefs that hardly change during ITE. The emphasis on theoretical knowledge in these programs often fails to displace the socially embedded beliefs about teaching with which PTs enter. In addition, personal and non-formal teaching experiences, such as tutoring other students or giving private lessons, play a crucial role in shaping their pedagogical beliefs and methodologies, which, according to Lee and Santagata (2020), are crucial for their professional development as teachers.

The recurring experiences of PTs throughout their education, both at school and in ITE, shape affective and cognitive representations that may be negative or positive. These representations contribute to the formation of attitudes, dispositions and belief systems that manifest in their future teaching practices (Lloyd & Howell, 2019). Although negative experiences during schooling may lead to related dispositions and beliefs, such as anxiety or low mathematical self-efficacy, there is evidence that PT can develop emotional representations that are contrary to those experienced. This phenomenon results from critical questioning of the role and methods of teachers (Jenßen et al., 2020). Research by Jenßen et al. (2020) has shown that both positive and negative experiences shape personal belief systems, confirming that PT begin to form beliefs about teaching in their school career, as observed in **Figure 1**. During this period, students project and adapt their role as learners to that of educators, laying the foundations of their identity as teachers (Cole et al., 2023).

Research Questions and the Significance of the Study

Within this framework, the guiding questions of our research are, as follows:

1. How are PTs' beliefs about mathematics teaching that relate to the design of mathematics tasks classified?
2. How are PTs' instructional practices classified when designing mathematics tasks?
3. To what extent do beliefs about the design of mathematical tasks and teaching paradigms in primary and secondary mathematics education influence the adoption of teaching practices when designing mathematical tasks by PTs?

These questions are designed to deepen our understanding of how educational trajectories shape the initial formation of beliefs about mathematics teaching in future teachers, even before they commence formal teacher education. This complements existing literature that suggests PTs' academic performances not only explain the variance of disciplinary knowledge to be taught (Stürmer et al., 2015) but also affect their ability to analyze classroom situations, commonly referred to as "noticing" (Blömeke et al., 2015; Weyers et al., 2024). Furthermore, this understanding will not only inform the disciplinary learning process, a primary focus of public policies in Chile, but also guide the alignment of PTs' existing models with those intended by Teacher Education Institutions. Given that the graduate profile is influenced by both the initial profile of PTs and the educational proposal provided by ITE programs, recognizing and understanding these influences is crucial (Chandía et al., 2023). Finally, the conceptual approach of this article moves beyond viewing beliefs as purely affective, adopting a more integrative perspective that recognizes beliefs as a network of evaluative knowledge, in line with the perspective of Angel and Seitz (2016).

METHODOLOGY

This study is designed as a mixed-method approach, incorporating descriptive and relational elements. This section explains the methodology in detail.

Participants

A sample of 252 PTs was selected and invited to participate in this study. They were chosen using a stratified random sampling method that considered the geographical distribution in Chile. These students are enrolled in 18 ITE programs, representing half of the programs available in the country, spread across 8 of the 16 Chilean regions. During the data collection phase, participants were in the first or second semester of their training programs. The sample is 20 years old in average, with a standard deviation (SD) of 4.7 years, and a clear female predominance, comprising 88.1% of the total. They voluntarily accepted to be part of the study and were then asked to complete a questionnaire. The questionnaire was administered to the participants during one of their class sessions, with an average duration of 45 minutes.

Instrument

As mentioned above, the data collection instrument used in this study was a questionnaire about the fundamental pedagogical development in mathematics. The questionnaire consisted of a demographic section and four additional sections designed to collect information on: the teaching paradigms that the PTs had experienced during their schooling, experiencing a role as a mathematics teacher, their teaching practices when designing a mathematical task, and their beliefs about teaching mathematics. The following describes in detail how the information was collected in each of these four sections:

1. **Paradigms experienced in primary mathematics education (PEPE) and Paradigms experienced in secondary mathematics education (PESE).** This research investigates the predominant paradigm of mathematics education during PTs' school years, as identified through their self-reporting. The study compares two detailed descriptions of mathematics classes, each representative of the constructivist and transmission approaches. PTs are requested to choose the description that best matches their mathematics teachers' teaching style during their school years. This is in response to the question: "Which of these descriptions most closely resembles how your teacher's taught mathematics during your school period?" The methodology uses critical incidents to trigger participants' reminiscences of their experiences within a context of ecological validity. The PTs' responses are coded for each level of education: primary and secondary, recording the constructivist approach as 1 and the transmissive approach as 0.
2. **Experience in the role of a teacher of mathematics (Rol).** PT was asked if they have conducted formal mathematics classes in school institutions or informally such as private tutoring or teaching peers, given the effect of assuming the role of teacher in teaching situations on belief formation. Responses were recorded dichotomously as either "yes" or "no" under the variable "Rol".
3. **Instructional practice in mathematical tasks design (IPMT).** A simulated professional practice scenario, vignette (**Figure 2**), is proposed as an educational exercise for PTs. The exercise requires PTs to envision themselves teaching second-grade children the algorithms of addition and subtraction, with a focus on problem-solving. PTs are then asked to design a classroom activity that is no longer than 45 minutes. This activity outlines the main task, provides clear

Situation described to student teachers Read the following situation and then respond.

Professor Andrea teaches mathematics classes in 2nd grade and has worked with her students on the addition and subtraction algorithm. In the next class she wants to address the following evaluation indicator with her students: They solve addition and subtraction problems in the range from 0 to 100. The teacher wants to give the student in practice the opportunity to implement an activity that does not last more than 45 minutes (a pedagogical block), in order to observe the implementation of the proposed evaluation indicator. Suppose you are Professor Andrea's intern, and she asks you to design an activity to address the assessment indicator. In addition, he asks you to present what you are going to do before taking the class. To do this, you must complete the following form with your ideas:

- A. Learning activity (or mathematical task) that you will propose to the students. B. In
- the process of interaction with the students, indicate what orientations you will give and what what you expect 2nd grade boys and girls to do.
- C. When the boys and girls approach the mathematical task or activity that you propose and begin to show productions such as representations, procedures or strategies, what will you do with them and with the students.

Figure 2. Vignette used in the questionnaire (Source: Authors' own elaboration)

instructions for the students, and details instructional strategies to address the students' written and oral contributions. This is supported by the fact that beliefs are obtained only indirectly, deducing them from a decision made, as direct responses only account for a part of the belief system (Cooney, 2002). Thus, using a phenomenological approach, inferring from subjects confronting a specific situation to decide, increases the likelihood of understanding the belief system that conditions it (Nguyen & Tran, 2022).

4. **Beliefs about instructional practices.** The last section of the questionnaire was designed as a multiple-choice scale with 18 items about Beliefs about instructional practices in mathematics education. This scale is structured into three dimensions focused on evaluating instructional practices in designing mathematics tasks:
   (a) beliefs about creating opportunities to promote mathematical reasoning through the design and use of mathematics tasks, with indicators related to the generation of strategies and resolution of mathematical problems such as "the tasks I propose will provoke students to look for multiple solutions on their own, without the need for me to intervene" or "the tasks I propose will provoke students to make conjectures or mathematical hypotheses on their own".
   (b) beliefs about interactions that promote mathematical reasoning during the execution of mathematics tasks, with indicators such as "when my students are not able to understand their peers' evidence of thinking, I will ask them questions to guide them" or "I will guide my students so that they can justify and explain to their peers why their strategies work", and
   (c) beliefs about observing and reflecting on students' actions, behaviors, and responses during the resolution of a mathematics task, with indicators such as "if I don't understand my students' productions, I will ask them to find out their mathematical ideas". For more details on the scale indicators, see Chandía et al. (2023).

Data Analysis

As the questionnaire included both qualitative and quantitative data, we conducted both quantitative and qualitative analyses. The multiple-choice scale utilized the beliefs scale, which had been validated through factorial and confirmatory analyses and Item Response Theory using Rasch models analyses as well as ensuring its reliability reported by Chandía et al. (2023).

The data gathered from the IPMT were prepared, coded, and then translated into quantitative data. For coding the responses provided by pedagogy students in the related vignette about their instructional practice in designing tasks, the items from the beliefs scale were used as indicators for coding the corresponding vignette during qualitative analysis (Chandía et al., 2023). Each response was assigned a code indicating the presence (code 2) or absence (code 1) of specific indicators described in the vignette.

Both the beliefs scale and the responses to the vignettes were independently coded by two evaluators. Any doubts or emerging discrepancies were reviewed and resolved by the principal investigator. The Kappa index, calculated over the total data, ranged from .91 to 1, with an average of .94 (SD = 0.06), reflecting high reliability in coding.

Subsequently, latent class analysis (LCA) was applied to both the beliefs scale and the codes derived from the pedagogy students' responses when designing mathematical tasks, to establish predictive profiles and calculate the probability of each student belonging to these classes. Selected responses are quoted to illustrate the statistical findings. These responses have been translated from Spanish to English and slightly reworded to concisely convey the main idea.

Finally, a logistic regression model was developed to examine the relationships between beliefs and instructional practices, incorporating the pedagogy students' previous experience in teaching roles as control variables. The effectiveness of the classification models was comparatively evaluated using Akaike information criterion (AIC) and Bayesian information criterion (BIC) as reference standards. According to Vrieze (2012), lower values in AIC and BIC indicate a better fit of the model. All analyses were performed using statistical packages in the R software (version 4.1.2; R Core Team, 2021).

Table 1. Descriptive statistics of the indicators regarding schooling and teaching experience

	Mean (SD)	Skewness	Kurtosis
Paradigms experienced in primary education	1.31 (0.49)	1.33	1.96
Paradigms experienced in secondary education	1.35 (0.48)	0.63	-1.61
Teaching mathematics experience (role)	1.58 (0.49)	-0.34	-1.89

Note. The type of instruction in primary and secondary education was codified with a 1 if it was transmission type and with a 2 if it was constructivist & teaching experiences were codified with a 1 in the absence of experience and with a 2 for any other level of experience

In the logistic regression model, the profiles of IPMT were taken as the dependent variable. As predictor or independent variables, belief-based profiles (PB), the type of PEPE and PESE, as well as adaptation to the teaching role (Rol), were considered.

$$E(IPMT|\ PICre, PESE, PEPE, ROL) = \Lambda(\beta_0 + \beta_1 PB + \beta_2 PEPE + \beta_3 PESE + \beta_4 Rol + \beta_5 PB{:}\ PESE). \quad (1)$$

In the logistic regression model, the interaction between PB and the type of paradigms experienced in secondary mathematics education stands out. This interaction was the only one that showed a significant predictive capacity among all the possible combinations of independent variables analyzed in the study, so it was decided to add it, taking care of the parsimony of the model.

RESULTS

Schooling and Role

Concerning the types of schooling received by the prospective teachers, we observed an average perception of constructivist paradigms both in primary and secondary education. At the same time, students propose, on average, having had the experience of adopting the role of teacher to teach some mathematics content (see **Table 1**).

Participants' Profiles Based on Beliefs

LCA applied to the "beliefs about instructional practices" in mathematics education confirmed a two-class model fits optimally, outperforming models with three or more classes. This is reflected in the BIC (7,610.84), AIC (7,226.13) indices, and an entropy of 0.83. It is estimated that 73.4% of pedagogy students belong to class 1, while 26.5% are grouped in class 2.

Class 1, referred to as "autonomous for exploration but not for understanding," comprises students who exhibit probabilities greater than 0.5 of preferring instructional practices that enable students to comprehend mathematical concepts or procedures, often using specialized mathematical language. Additionally, they show probabilities greater than 0.8 of holding beliefs that emphasize orienting students to justify and explain their strategies to their peers, although they are able to establish patterns or generalizations on their own and discuss mathematical concepts independently. Students in this class also exhibit a probability greater than 0.7 of adhering to beliefs that promote the constant use of student contributions in the classroom, encouraging the connection and alignment of these contributions with class objectives, provided there is clarity in their understanding.

A crucial aspect of this class is the ambivalence or uncertainty that pedagogy students express to determine the frequency of certain pedagogical practices. Regarding mathematical tasks, there is a probability greater than 0.5 that they express beliefs that their tasks will facilitate children's autonomous exploration of concepts and procedures, the search for multiple solutions, or the posing of conjectures or hypotheses, either almost always or only rarely. In the realm of interactions, significant uncertainty is perceived, with probabilities greater than 0.5, regarding the use of practices that foster reflection and critical thinking in students or guiding them through questions to understand the mathematical thinking of their peers. Additionally, there is insecurity in implementing practices that provide children with the opportunity to use evidence, representations, or diagrams to formulate generalizations, strategies, relationships, and mathematical procedures, as well as to establish connections between concepts, procedures, or representations independently.

On the other hand, pedagogy students grouped in class 2, referred to as "teacher responsible for student autonomy," demonstrate a significant degree of uncertainty across all their beliefs. Students in this class stand out in their practices regarding mathematical tasks, with probabilities greater than 0.5 of rarely or never promoting student autonomy to explore concepts and procedures. This trend is reinforced when reviewing indicators of practices related to mathematical tasks, where the teacher assumes responsibility, showing probabilities greater than 0.7 of adopting such practices in their future mathematics classrooms.

Regarding interactions among the domains of beliefs scale, there is a clear divergence from class 1, as students in this second class show probabilities greater than 0.5 of almost always allowing students to establish generalizations, patterns, and discussions around mathematical concepts and procedures. Regarding the use of student contributions, students in this class exhibit significant indecision in three of the four practices evaluated in this dimension, especially in those that involve connecting procedures, representations, strategies, explanations, and arguments. This is evidenced by probabilities greater than 0.5 of opting for the "almost always" category when responding about their beliefs.

Participants' Profiles Based on IPMT

In the latent class, analysis focused on "IPMT" employed by pedagogy students in designing a mathematical task, it was identified that the two-class model best fits, with BIC indices of 4,286.55, AIC of 4,155.97, and an entropy of 67.06%. According to this model, the probability of a PT being associated with class 1 is 32.94%, while it is 67.06% for class 2.

Table 2. Predictor variables belonging to the latent class of vignette responses (logistic regression analysis results, n = 252)

Variable	β	Standard error	Wald	df	p	Odds ratio
Intercept	-1.85	1.37	-1.35	1	.17	0.15
PB	2.09	0.96	2.16	1	.03	8.08
PEPE	-0.26	0.29	-0.90	1	.36	0.76
PESE	1.51	0.87	1.73	1	.08	4.52
Rol	0.31	0.27	1.11	1	.26	1.35
PB: PESE	-1.31	0.66	-1.98	1	.04	0.26

Note. β: Regression coefficient; Wald: Wald statistic; Chi-square model = 6.83, p < .01; 2 log likelihood = 324.57; Hosmere Lemeshow goodness-of-fit test = 0.13, p = .71; Nagelkerke R^2 = .36; AIC = 324.57; & BIC = 345.74

Class 1, labelled as "tasks to stimulate exploration," is characterized by PTs with over an 80% probability of adopting Instructional Practices that guide students to investigate concepts and procedures. To illustrate the members of this class, here are some examples of responses. For instance, PT no. 152 stated, that these practices are designed to encourage exploration but under the teacher's direction.

> "The guidance I would give is for them to listen to the teacher's instructions to solve the proposed task. What I expect the students to do is that through the instructions, they can solve the problems and if they cannot, they should ask the teacher for guidance to reach the expected result" (PT no. 152).

On the other hand, the pedagogy students classified in class 2, labelled as "tasks to stimulate autonomous exploration," demonstrate with over a 90% probability instructional practices, in the design of mathematical tasks, that promote independent exploration of concepts and procedures by students. This trend is reflected in the decisions made by PT no. 125 and PT no. 118 in their pedagogical approaches.

> "I would give them a two-digit addition problem (45 + 32) for the children to solve on their own using different strategies, the idea is not to use the algorithm, and to write them down on paper. Then, different strategies used by the children are selected for them to explain them afterwards (strategies that do not lead to the answer can be selected to manage error and generate more learning, or other strategies where additive decompositions are made, among others)" (PT no. 125).

> "I would leave a moment for them to solve it on their own first and would monitor in case the children need help. I would expect children to use counting, over-counting, and under-counting of units through pictorial or symbolic representations. I would also expect them to be able to add 10 + 5 and then subtract 15 - 2" (PT no. 118).

Regarding instructional practices related to interactions, the designs of both classes show no significant differences. Both classes exhibit a 50% probability of promoting among students the justification and explanation of the mathematical strategies used to their peers, a practice reflected in the descriptions of PTs 251 and 68. For example, PT no. 251 mentions:

> "... I will ask them to come forward to explain to their peers as a method of reinforcing their learning, thus providing different knowledge, representations, and strategies ...";

while PT no. 68 states:

> "... students must present, communicate, and justify their procedures, concepts, and reasoning logic ..."

Regarding student contributions, PTs from both classes show a 75% probability in their descriptions of integrating their students' productions during the development of classes. An example of this is seen in the description of PT no. 63, which says:

> "I will select those contributions that I consider most relevant to facilitate understanding of the problem to others, and I will decide if there are errors in the students' strategies to address them ... but always trying to move from the concrete to the abstract."

Relationship Among Beliefs and Paradigms Experienced and Instructional Practices

Table 2 presents data indicating that beliefs act as significant predictor variables (Wald (1) = 2.16, p < .05) in the profiles of IPMT, using the profile identified as "tasks to stimulate exploration" as a basis for comparison. According to the logistics regression analysis, if a PT demonstrates belief profiles that hold the teacher accountable for student autonomy, the probability of them designing mathematical tasks that promote student autonomy increases by 8.08 times compared to those who present belief profiles that do not hold the teacher accountable for student autonomy. Assuming all other variables remain constant.

Table 2 also identifies PESE as another positive predictor of instructional practice profiles (IPMT), evidenced by a Wald test result (χ^2 (1) = 1.73, p < .1). A constructivist orientation within PESE enhances the 4.52 times probability (with an odds ratio of 4.52) of a PT adopting teaching practices that foster student autonomy. Moreover, an inverse predictive relationship between PB and the type of PESE has been found (Wald (1) = -1.98, p < .05). PTs who perceived a constructivist paradigm during their secondary training and who held belief profiles that place responsibility on teachers to foster student autonomy are 0.26 times less likely to create mathematical tasks that promote student autonomy. Finally, no significant relationship was found between the adoption of teacher roles and instructional practices in task design, nor with perceived teaching paradigms in elementary education.

DISCUSSION

The findings from this study shed light on the complex interplay between PTs' beliefs, instructional practices, and their instructional practices in mathematics education. The discussion focused on three main findings of this study.

First, the results show a divergent pedagogical orientation among PTs. While Depaepe and König (2018) highlight the intricate relationship between beliefs, attitudes, and emotions, reinforcing the complexity in understanding PTs' instructional orientations, our findings, through a profile analysis based on PTs beliefs, delineate two distinct classes, emphasizing the variability in PTs' instructional orientations. Additionally, Xie and Cai (2021) discuss how beliefs about learning mathematics are shaped by experiences in both school and teacher education settings, further supporting the identification of divergent pedagogical orientations among PTs.

Class 1 demonstrates a pronounced preference for student-centered instructional practices, emphasizing autonomy and comprehension of mathematical concepts. These PTs prioritize encouraging student justifications and active engagement in discussions. However, certain ambiguities in their pedagogical practices suggest a need for clarity and consistency. On the other hand, class 2 PTs exhibit uncertainty across their beliefs, leaning towards teacher-centered practices. They display reluctance to promote student autonomy, extending to interactions within the classroom, indicating potential challenges in fostering critical thinking. This divergence in pedagogical orientations among PTs underscores the complex interplay between beliefs, instructional practices, and their implications for teaching mathematics (Öçal, 2021). Angel and Seitz (2016) shed light on the underlying cognitive processes involved in forming beliefs, suggesting that PTs' instructional practices may be shaped not only by their affective disposition but also by cognitive mechanisms optimizing decision-making. Overall, the observation of divergent orientations among PTs, where class 1 prioritizes student autonomy and comprehension of mathematical concepts, while class 2 exhibits uncertainty and reluctance to promote autonomy, can be supported by Berk and Cai (2019) where delineate distinct perspectives on beliefs about the nature of mathematics, with implications for teaching practices.

Second, similar to beliefs, a two-class model emerged concerning instructional practices in mathematical task design. Class 1 PTs demonstrated a preference for guiding students in exploring concepts and procedures, emphasizing teacher-led facilitation of exploration. In contrast, class 2 PTs exhibited a strong inclination towards promoting autonomous exploration among students, allowing them to independently engage with mathematical tasks. The analysis suggests that pedagogy students who adhere to an instructional practices model based on specific paradigms may exhibit variations in their self-efficacy to teach mathematics. This variability is manifested in the indecision when projecting instructional practices in the classroom, as evidenced by selecting options such as "almost always" or "rarely" when defining models of student interaction or making decisions on how to integrate student contributions in the beliefs scale. This behavior reflects a high degree of critical reflection and questioning of the absolute efficacy of certain pedagogical beliefs. Therefore, the self-efficacy of pedagogy students is influenced by their ability to perceive, interpret, and determine the effective impact of their instructional practices in a real school context (Blömeke et al., 2015; Yang et al., 2019). This potential for reflection allows pedagogy students to question and critically evaluate existing mathematics teaching paradigms based on their practical experience, weighing risks and benefits. Experiencing and reflecting on the teacher role, therefore, shapes self-efficacy, belief systems, and ultimately instructional decisions that a pedagogy student is willing to make when designing context-based mathematical tasks for their future teaching practice (Wang et al., 2017).

However, the logistic regression analysis indicates that PTs with experience in the teacher role may not necessarily promote autonomy. This can be attributed to their experience in the teacher role and its correlation with their ability to analyze, reflect, and have self-efficacy when faced with a teaching situation where instructional practices have not been successful. This finding aligns with the capacities described for expert teachers (Dilekli & Tezci, 2016). Experienced teachers demonstrate a greater ability to analyze classroom situations related to mathematics learning and teaching processes compared to novice teachers (Sherin, 2017). This ability is mainly acquired through classroom experience, where teachers make decisions and observe their effects (Mayer et al., 2017), which clarifies their measure of success. Therefore, formal and informal practices provide opportunities to refine perception, interpretation, and decision-making, modify beliefs and levels of self-efficacy, and restructure frameworks of beliefs, thus affecting instructional practices (Depaepe & König, 2018).

Third, the logistic regression analysis unveiled beliefs as significant predictors of instructional practice profiles among PTs, which is in line with the previous studies (Depaepe & König, 2018). PTs endorsing beliefs centered on the teaching role exhibited a higher likelihood of adopting instructional practices that fostered student autonomy. Furthermore, experiencing a constructivist orientation in secondary mathematics education positively influenced the adoption of student-centered instructional practices. Conversely, PTs exposed to constructivist-oriented paradigms were less inclined to hold teacher-centered beliefs, suggesting a potential alignment between pedagogical beliefs and educational experiences.

The findings underscore the complexity of PTs' instructional orientations, as also identified through a profile analysis based on their beliefs. Regardless of their schooling background or prior teaching experience in mathematics, PTs' profiles of beliefs and instructional practices when designing mathematical tasks were found to align with both constructivist and transmission paradigms. This alignment suggests the reflective capacity of PTs, who critically assess their past educational experiences and anticipate their future teaching roles, shaping their pedagogical beliefs accordingly. These beliefs significantly influence their instructional decisions, even before formal teacher training (Jenßen et al., 2020; Lo, 2021; Weyers et al., 2024).

Additionally, the regression model highlights the pivotal role of secondary education experiences focused on student autonomy in shaping PTs' instructional models. PTs who underwent constructivist experiences during secondary education were more inclined towards teacher-centered instructional models. This suggests that secondary school experiences influence the teaching and learning models of mathematics that PTs construct, establishing valued associative patterns and beliefs in instructional practices (Angel & Seitz, 2016; Lo, 2021).

Interestingly, the regression results also reveal the existence of PTs with instructional practices aimed at promoting student autonomy, despite attributing final responsibility to the teacher. This finding underscores the interplay between beliefs and instructional practices, highlighting that the beliefs alone do not determine instructional practices but interact with other aspects of teacher professional competence.

Furthermore, an inverse interaction between experiencing a constructive formation and practices focused on student autonomous development was observed. PTs who underwent a school formation process under a non-constructivist paradigm were more likely to exhibit beliefs and instructional practice profiles closer to a constructivist paradigm. This indicates the influence of school formation experiences on PTs' belief systems and instructional practices, emphasizing the need for further research to validate these findings and explore the relationship between beliefs about learning mathematics and teaching practices (Lo, 2021; Xie & Cai, 2021).

CONCLUSION

The findings of this study demonstrate the considerable impact of prior educational experiences on the beliefs and pedagogical practices of prospective mathematics teachers in Chile. The responses to the research questions are presented below.

1. **Classification of beliefs about mathematics teaching:** Distinctive belief profiles were discerned among prospective teachers, with a pronounced proclivity towards constructivist pedagogical approaches. These beliefs were primarily formed based on experiences during their school trajectory, where the majority of participants reported exposure to traditional teaching methods, which contrasts with their stated aspirations towards more constructivist practices.
2. **Classification of instructional practices in the design of mathematical tasks:** The pedagogical practices of the participants exhibit a diversity that aligns with their espoused beliefs about teaching. Those who espoused constructivist beliefs tended to adopt more interactive and student-centered pedagogical practices, whereas those who identified with more traditional approaches demonstrated a proclivity for more directive and rote-learning methods.
3. **The influence of beliefs and educational paradigms on teaching practices:** The findings of this study indicate that beliefs about the design of mathematical tasks and the teaching paradigms acquired during primary and secondary education have a considerable impact on the pedagogical practices of PTs. This indicates that despite the theoretical training received in their initial education programs, prior experiences continue to influence their pedagogical decisions and professional identity.

Therefore, the school trajectory of PTs should not only be considered as a selection criterion for initial training programs but also as a determining factor in shaping future pedagogical practices, influencing their cognitive and affective dispositions related to professional competence. This is especially relevant in the context of educational policies in Chile, where the consideration of pedagogical propaedeutic as an alternative route to teacher training is still emerging, and in the face of the intense standardization of teacher training processes as a quality assurance strategy. This approach could underestimate the inherent complexity of teacher training, neglecting the diversity of initial profiles and experiences of the students.

Among the limitations of the study is the possible bias in students' responses in the belief questionnaires, given the lack of measurement of a social desirability unit for contrasting the results. Also, the way of obtaining instructional practices by the mere description of a situation could diverge from what they could present in classroom settings or real teaching situations. Certainly, this can be considered a proxy measure, but as such, it's possible bias should be considered. Finally, the number of PTs responding to the instrument could affect the representativeness of the studied variables.

Author contributions: ECH: study conception, design, material preparation, and data collection; **ECH & FS:** data analysis, and writing the first draft; **AH:** commented on previous versions of the manuscript. All authors have agreed with the results and conclusions.

Funding: This study was supported by FONDECYT 11241550 and ANID/ Basal Funds for Centers of Excellence FB0003.

Ethical statement: The authors stated that the study was approved by the Scientific Ethics Committee of Social Sciences, Arts and Humanities at Pontificia Universidad Católica de Chile, Santiago, Chile on April 20, 2018 (Approval code: 151230001). Written informed consents were obtained from the participants.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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