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How to enable teachers to enhance all students' understanding of percentages? A quasi-experimental field trial

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ABSTRACT

Enhancing understanding is crucial for *all* students. Instructional approaches have been developed to achieve this goal, yet little is known about the kind of support that teachers need for their effective implementation. We compare two support conditions designed to enable teachers to enhance students' understanding of the mathematical topic percentages. 51 teachers and their grade 7 classes participated in a quasi-experimental pre-post-test control group design, comparing three groups. (1) Teachers of the material condition received the curriculum material of the MATILDA teaching unit on percentages, with a comprehensive teacher manual on the main principles of the approach. (2) Teachers in the workshop condition received the same materials and additionally participated in professional development (PD) workshops covering key ideas of the approach. (3) The waiting control group used their regular textbooks. Hierarchical regressions were used to compare the effects on students' learning outcome. The analysis reveals that students in both support conditions achieved significantly higher learning outcomes than the control group, i.e., curriculum materials and teacher manuals seem to provide effective support. No additional effect on student outcomes was found for the workshop condition, but a substantially lower teacher dropout than in the material condition. This finding indicates that PD workshops may not be necessary for highly motivated volunteer teachers but can be completed by a wider range of teachers. The findings can inform implementation projects for educational innovations and inspire further research to determine for which teachers a written PD is sufficient and for which face-to-face workshops are more promising.

Keywords: support conditions for teachers, implementation, conceptual understanding, equity

INTRODUCTION

The aim of increasing access to mathematics has been articulated for 200 years, e.g., in the claim "mathematics for all" (Damerow et al., 1984). Over 200 years, school systems gradually broadened their ambitions about *who* should learn mathematics ("from the few to the many") and *what* kind of mathematics should be learned by all (from elementary calculations to arithmetic with understanding to secondary mathematics, Callahan, 2005; Clements et al., 2003). On the institutional level, the goal has largely been achieved in terms of formal access to schooling and unified syllabus for all students. However, in most education systems, the learning outcomes of students from different socio-economic backgrounds, with different language skills, learning disabilities or other different abilities in mathematics are still unevenly distributed (Faragher et al., 2016; OECD, 2016); also in Germany (Baumert & Schümer, 2001; Stanat et al., 2019), where the current study is conducted.

Today, the construct of limited access to mathematics is not only referred formally to students sitting in the classrooms, but also to limited *conceptual access* characterized by limited learning opportunities within mathematics classrooms, in particular by three obstacles:

1. *Limited demands:* Missing learning opportunities for some students due to limited exposure to cognitively demanding, conceptually focused mathematics (Boaler, 2002; de Araujo & Smith, 2022; Faragher et al., 2016; Jackson & Wilson, 2012).
2. *Limited responsiveness to students' diverse abilities:* Limited spread of instructional approaches that offer rich mathematics while at the same time being adaptive to students' diverse abilities and needs, in particular to diverse mathematical prior

knowledge, language proficiency (Moschkovich, 2013), and other cognitive abilities (Blazer & Archer, 2020; Myers et al., 2015).

3. *Unspecific support for teachers:* To date, support for teachers to implement instructional approaches that provide conceptually focused access to rich mathematics for all students do not appear targeted enough to students' diverse abilities (van Mieghem et al., 2020).

In this paper we contribute to the long-term agenda for overcoming these obstacles, by investigating the implementation of an instructional approach that meets the diverse needs of students while providing access to conceptual understanding. We start with an instructional approach for which first evidence for efficacy for overcoming the first two obstacles has already been provided in a small pilot study (Kuhl et al., 2022). This allows us now to investigate the following research question on implementation regarding the third obstacle: *Which kind of support do teachers need to effectively carry out an instructional approach that provides access to conceptual understanding for learners with diverse abilities?*

The results of this study are relevant for implementation research (Century & Cassata, 2016), even beyond the topic of percentages and the ambition of increasing access for all students. By comparing two support conditions for teachers, we follow a general need for research articulated by Cohen et al. (2003) and Leong et al. (2019) how to best close the professional development (PD)-classroom gap and how to best support teachers in realizing innovative approaches.

THEORETICAL BACKGROUNDS

Providing Access to Conceptual Understanding While Considering Diverse Abilities

Mathematics classrooms for students with low socio-economic status, limited language proficiency or learning disabilities have often been shown to restrict students' learning opportunities by offering low-quality teaching focused on procedures rather than conceptual understanding and low cognitive demands (Beswick, 2007; Boaler, 2002; de Araujo & Smith, 2022; Diversity in Mathematics Education Center for Learning and Teaching [DIME], 2007; Jackson & Wilson, 2012, for Germany: Stanat et al., 2019). But all students, including those students struggling in mathematics for various reasons, need the opposite: Cognitively demanding instruction that focuses on understanding basic mathematical concepts (Blazar & Archer, 2020; Charalambous et al., 2018; Myers et al., 2015). Thus, Cai et al. (2020) and van Mieghem et al. (2020) recently called for more research to maximize the learning opportunities of all students.

However, increasing learning opportunities cannot be realized by solely increasing the cognitive demands and strengthening the focus on conceptual understanding, because learning opportunities are also constrained if they do not match students' abilities. For example, if students with low language proficiency cannot participate in sophisticated classroom discourse, they cannot use this discourse as a learning opportunity (DIME, 2007; Herbel-Eisenmann et al., 2011).

Thus, increasing learning opportunities also requires ensuring conceptual accessibility with respect to students' individual preconditions of learning. Psychological research has identified some main characteristics related to successful learning (Hasselhorn & Gold, 2013; Pressley et al., 1989). Pressley et al. (1989) list the following abilities: Prior knowledge, selective attention and working memory, usage and metacognitive regulation of strategies. Prediger et al. (2020) explicitly added language proficiency (previously subsumed in prior knowledge, but also relevant in mathematics learning). Instead of referring to broader background variables (such as immigrant background, gender, or disability), this list focuses those *abilities* which have a direct influence on mathematics learning processes.

For example, accounting for the needs of immigrant students can involve enhancing their language learning. In this case, language proficiency is the relevant ability to refer to, not immigrant background. Referring to abilities rather than demographic backgrounds can help to avoid stereotype threats ("all immigrants have language learning needs") and easily allows the number of students to be increased (e.g., language-responsive classrooms have been also shown to be effective for monolingual students and students with high language proficiency, Prediger & Neugebauer, 2023). Moreover, learning disabilities often (but not always) coincide with limitations in working memory and metacognitive regulation (Alloway et al., 2005; Hasselhorn & Gold, 2013, Veenman et al., 2006). Teaching practices that increase accessibility by considering limited working memory are also useful for students without learning disabilities. At the same time, categories such as learning disabilities can provide an orientation, as these categories are associated with some typical characteristics. However, abilities which have a direct influence on mathematics learning processes should always be observed closely.

Two main practices can be distinguished to ensure conceptual accessibility with respect to each of the six abilities mentioned: enhancement and compensation practices.

- *Compensation practices* aim to "work around weaknesses" (Corno, 2008, p. 3) by helping students work around limited abilities (e.g., lowering language demands for some students or relieving working memory through clearly organized worksheets), this practice is emphasized, e.g., in the universal design for learning (Rose & Meyer, 2006).
- *Enhancement practices* aim to develop abilities by creating targeted learning opportunities. For example, language-responsive teaching approaches do not simplify language demands but promote students' language proficiency by engaging them in rich discussions and offering targeted feedback (Moschkovich, 2013; Walqui, 2006).

As Corno (2008) emphasized, responsive teaching requires combining enhancement *and* compensation practices. A compensation practice for one ability can provide conceptual access to an enhancement practice in another ability, e.g., if student's attention is supported through scaffolds, they can increase their mathematical learning opportunities.

Supporting Teachers' Implementation of Inclusive Instructional Approaches

Several instructional approaches were shown to improve all students' conceptual understanding while ensuring conceptual accessibility by addressing students' diverse abilities (e.g., Blazar & Archer, 2020; Charalambous et al., 2018; see overviews in Scherer et al., 2016; van Mieghem et al., 2020).

However, the success of *implementing these instructional approaches* in school settings is still very limited (Kollosche et al., 2019; Prast et al., 2015; Scherer et al., 2016). At least in Germany, teaching practices for students in non-academic tracks are still shaped by low cognitive demands and a mainly procedural focus (Grünkorn et al., 2020). Similarly in many countries, responding to diverse abilities is not yet part of every teacher's repertoire (Faragher et al., 2016; Prast et al., 2015; Turner et al., 2013; van Mieghem et al., 2020).

More implementation research is needed to find out how teachers can be best supported in providing access to conceptual understanding for all students in mathematics classrooms. Implementation research is defined as "systematic inquiry regarding innovations enacted in controlled settings or in ordinary practice, the factors that influence innovation enactment, and the relationships between innovations, influential factors, and outcomes" (Century & Cassata, 2016, p. 170). Similarly, Penuel and Fishman (2012) call for implementation research "to iteratively refine strategies for improving the implementation effectiveness of interventions" (p. 282). Three main implementation strategies have been developed in the field of implementation research and systematized by Roesken-Winter et al. (2021):

- In the *material strategy,* the focus is on designing and spreading well-designed curriculum materials that support teachers in realizing the chosen teaching approach for a specific mathematics content (e.g., Cohen et al., 2003, Saxe et al., 2001). Leong et al. (2019) emphasized the need for "concretizations" of instructional approaches to be implemented in curriculum materials and artefacts for the classroom (such as the percent bar visualization).
- In the *systemic strategy,* the systemic conditions under which teachers can develop their instructional practices are improved to strengthen the cooperation and ensure sustainability of the innovations (Borko & Potari, 2024), for example lesson studies or establishing professional learning communities (PLCs) are part of systemic strategies. In the case of providing conceptual access to mathematics, multiprofessional learning communities with subject-matter teachers and special education teachers can support the development of inclusive practices (van Mieghem et al., 2020).
- In the *personal* strategy, PD is provided for teachers to promote their expertise and instructional practices, and develop the expertise for well-informed adaptations (e.g., Ball & Cohen, 1996; Desimone, 2009). With respect to raising all students' conceptual access to mathematics, Karsenty (2010) and Beswick (2007) had emphasized the need to unpack the mathematical meanings to develop conceptually oriented teaching practices. Teachers should also be introduced to enhancement and compensation practices to assure curricular and instructional adaptivity (Janney & Snell, 2006).

Ball and Cohen (1996) already emphasized that none of the strategies can work alone. Rather, they need to be combined, especially to bridge the gap between PD and classroom so that teachers really apply what they have learned (Leong et al., 2019). Roesken-Winter et al. (2021) suggested systematically investigate whether PD offers can also be provided in self-learning materials, as the effectiveness of the material support is still unclear.

Whereas the high relevance of the material strategy on the classroom level (i.e., the teacher support by curriculum materials) has been proven empirically (e.g., Leong et al., 2019; Saxe et al., 2001), little is known about the material strategy for teachers' learning. One study in science education indicates that face-to-face PD workshops can have a significant additional effect for teachers' effective implementation of innovative instructional approaches compared to written PD materials (Kleickmann et al., 2016). On the other hand, written PD materials may have a higher degree of concretization, a feature of teacher support that has been shown to be very important in other studies (Leong et al, 2019).

In particular, for our innovation in view, providing access to conceptual understanding for all students, this question about the kind of required teacher support is highly relevant because so far, most PD offers for teachers have been quite unspecific, establishing the general ideas of the instructional approaches, without much concretization,

(a) for specific mathematical topics (as in our case, percentages) and

(b) for the range of abilities to be addressed simultaneously (Prast et al., 2015; Van Mieghem et al., 2020).

Design of a Conceptually Accessible Teaching Unit Aiming at Understanding Percentages

The innovation for which the support conditions will be investigated in this paper is the instructional approach of the MATILDA teaching unit (Pöhler et al., 2018; Prediger et al., 2019) which combines high cognitive demands and a conceptual focus with compensation and enhancement practices to ensure conceptual accessibility with respect to students' diverse abilities. The instructional approach was developed for inclusive mathematics classrooms in heterogenous classes with students of all abilities and backgrounds, including students with special educational needs in mathematics. The general instructional approach is substantiated by the MATILDA teaching unit for grade 7 that focusses on understanding percentages, i.e., grasping the meaning of part-whole relationships in percentages and using informal strategies to determine amount, base and rate in word problems. This topic was chosen as an example because it is of high relevance in all STEM subjects and in practical life, but also often documented to cause difficulties for many students (Parker & Leinhardt, 1995).

The main content trajectory of the teaching unit was taken from van den Heuvel-Panhuizen (2003), it allows enhancing students' conceptual understanding of percentages and draws upon the percent bar as visualization. The example tasks in **Figure 1** show on the one hand the high cognitive demands, on the other hand how conceptual accessibility is ensured with respect to diverse abilities:

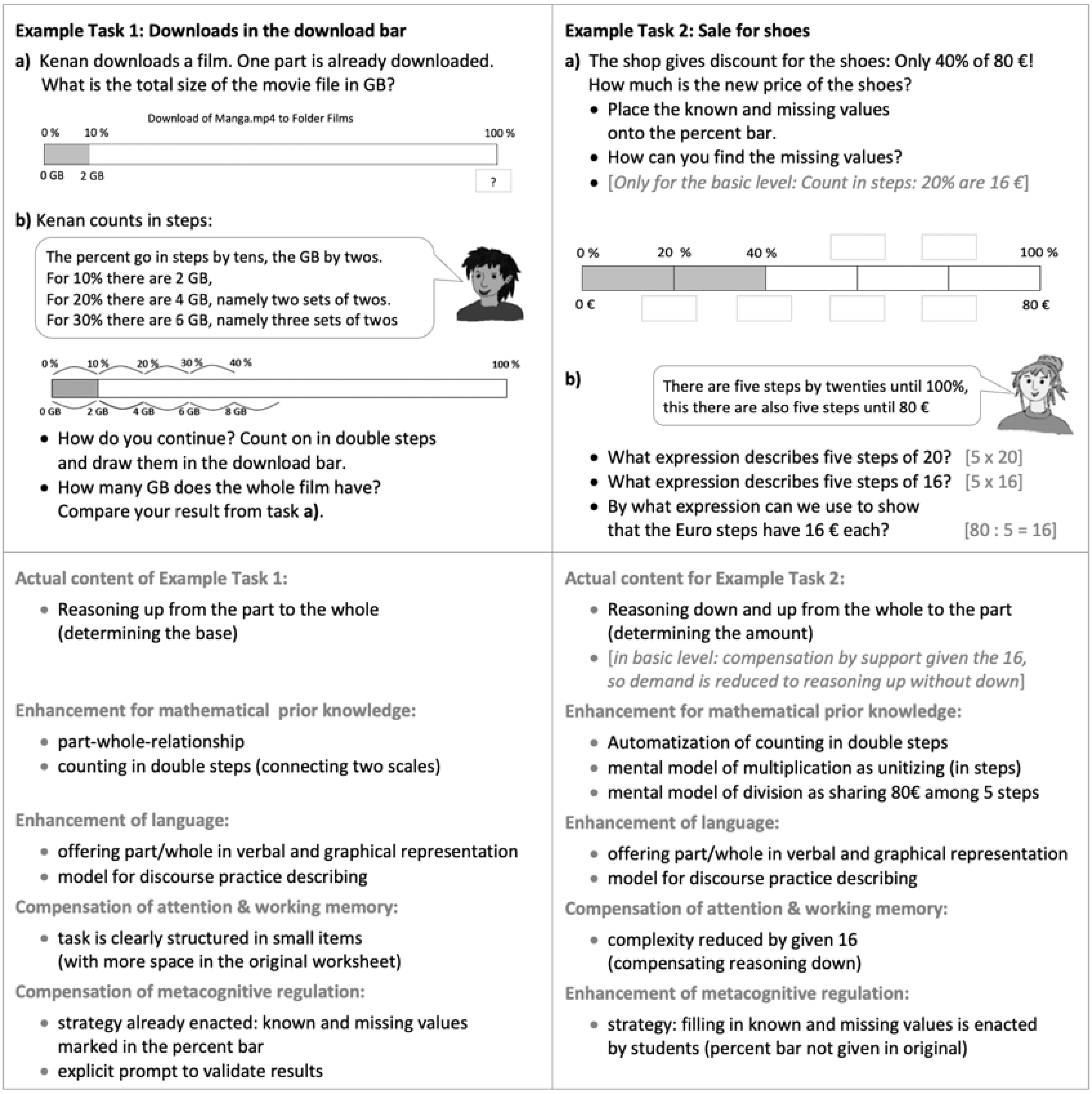

Figure 1. Examples for the enhancement and compensation opportunities for different abilities in the curriculum material of the MATILDA teaching unit on percentages (Pöhler et al., 2018)

- *Language proficiency* is a crucial ability to participate in discourse practices of describing connections and explaining meanings of mathematical concepts such as percent, base, amount and their relations (Morales et al., 2024). Rather than lowering language demands for students with low language proficiency using compensation practices, the material was enriched by incorporating systematic enhancement opportunities for improving students' participation in discourse practices (by repeatedly eliciting and supporting explanations and by sharing meaning-related expressions such as part of a whole, set of in the task 1 in **Figure 1**, Pöhler & Prediger, 2015).
- *Metacognitive regulation* is crucial for the processes of solving percent word problems. Whereas task 1 compensates limited metacognitive regulation by pre-structuring the known and missing values on the percent bar (Corno, 2008), task 2 asks student to conduct the metacognitive strategy on their own (Mevarech et al., 2010).
- Relevant *mathematical prior knowledge* in this teaching unit concerns four basic concepts and four basic skills. Task 1 introduces two of them (part-whole-relationship and, counting in double steps), task 2 two more (multiplication as counting in steps, division as sharing), later tasks provide enhancement opportunities to consolidate and exercise them, and to address them explicitly for all students in joint discussions, e.g., by using them as a basis of argumentation for a justification.
- Compensation practices can reduce cognitive load to make efficient use of all students' (possibly limited) *attention and working memory* (e.g., Kuhl et al., 2021; Wiley et al., 2014). For example, the use of automated skills and recurring visual representations can free up working memory resources (Gathercole & Alloway, 2008). We integrated these aspects in the teacher manual as differentiated adaptation opportunities for specific students (Prediger et al., 2019).

The MATILDA teaching unit was developed in a series of successive steps. First, the focus was on mathematical accessibility and language-responsiveness (Pöhler & Prediger, 2015, with proven effectiveness in Prediger & Neugebauer, 2023). Afterwards, the unit was optimized to ensure accessibility with respect to the other abilities (Prediger et al., 2019). The results of an initial pilot study under highly controlled conditions indicated efficacy of the underlying instructional approach, as classes taught with the MATILDA teaching unit had significantly higher learning outcomes than the control group who were taught with their regular textbooks (Kuhl et al., 2021). Findings from an additional video study show how teachers implement the teaching unit with a wide range of enhancement and compensation practices for each of the abilities (Prediger & Buró, 2024), so it seems reasonable to assume that success might depend on the teachers' support.

Research Questions and Hypotheses

Based on the reviewed state of research, and the presumed opportunities of teacher support provided by curriculum material and written PD material (Cohen et al., 2003; Kleickmann et al., 2016), we refined our research question from the beginning, by keeping the systemic strategy constant for all teachers and examining the effects of teacher support in a material strategy, and the additional effect of PD workshops. Thus, we pursued the following research questions:

RQ1. To what extent can teachers be enabled (through material support and PD) to enact an instructional approach with a given curriculum material that increases students' understanding for percentages more than in the control group?

RQ2. Do the effects on students' learning outcomes differ between face-to-face PD workshops and written PD materials?

As we assume that the MATILDA curriculum material and the two support conditions to have a general positive effect on the development of all students' conceptual understanding of percentages, we investigated the following hypothesis:

H1. Teachers in both support conditions (workshop and material condition) bring their students to higher learning outcomes in percentage understanding than teachers in the control condition who work with regular textbooks.

Based on previous findings we also suspected that the face-to-face PD workshop engages teachers more actively. This resulted in the second hypothesis:

H2. The workshop condition is more effective for students' learning outcomes than the material condition.

METHOD

Research Design and Procedure

To answer our research questions, we designed a quasi-experimental pre-post-test control group study with two support conditions and one waiting control group (see advance organizer in **Figure 2**). We initially recruited 90 volunteer mathematics teachers and special education teachers as well as their inclusive grade 7 classes to participate in our study. Some of the teachers taught in co-teaching tandems, all in heterogeneous groups of students.

Teachers and classes were assigned to three treatment conditions, with two support conditions and a waiting control group serving as independent variables. Their conceptual understanding of percentages served as the dependent variable to address the effect on learning outcomes for all students. In addition, the following individual characteristics of the students were captured

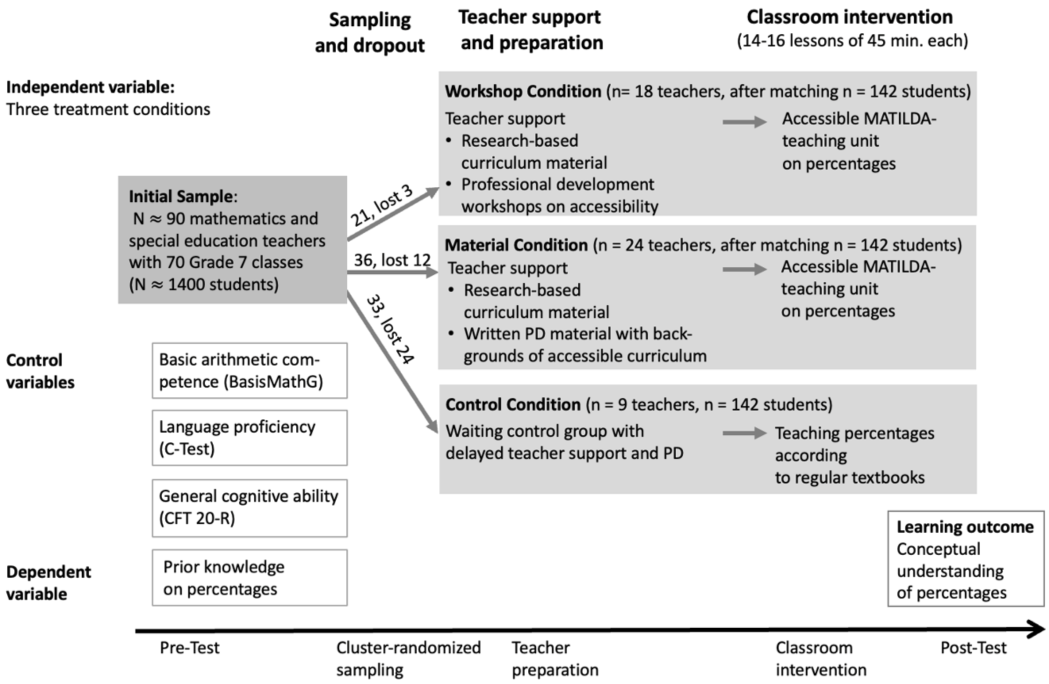

Figure 2. Research design of the quasi-experimental field trial with two support condition and control group (Source: Authors' own elaboration)

Example items for problem type "Find the base"	
Pure format	5 % are 250 €. Find the base.
Visual format	What is unknown here? Find the missing value. 
Text format	Potatoes consist of 75 % water. How much water (in g) is contained in 1000 g potatoes?
Example item for problem type "Find the amount"	
…	
Text format	When buying a new kitchen, Family Mays receives a discount of 250 €, that was 5 % of the regular price. What is the normal price of the kitchen?
Example item for problem type "Find the base after reduction"	
…	
Text format	Mrs. Schmidt pays 30 € for a dress in the summer sale. The dress was reduced by 40 %. How much did the dress cost before?

Figure 3. Items in the percentages test in different problem formats and problem types (Pöhler et al., 2017)

as control variables: basic arithmetic competence, prior knowledge on percentages, language proficiency in German, and general cognitive ability.

Measures

The learning outcomes in percentages served as dependent variable. After the classroom interventions, a standardized post-test was administered to assess conceptual understanding and flexible use of percentages. The items have an open answer format with the following problem types: "find the amount," "find the base," and "find the base after reduction." For each problem type, items varied in three formats: "pure format," "text format," and "visual format," with percent bar representations (examples in **Figure 3**). The test has a good internal consistency, with Cronbach's alpha of .834 (in a sample of N = 1,120 students and 24 items) and was validated in earlier studies (Prediger & Neugebauer, 2023).

For the control variables, the following individual characteristics were measured:

- *Prior knowledge on percentages* was assessed using seven items chosen from the post-test that could have been solved by the students before the intervention. This short pre-test has a satisfactory internal consistency, with Cronbach's alpha = .83 for the post-test (in a sample of N = 1,120 students and seven items).
- Basic arithmetic competence that is relevant prior knowledge for learning percentages (fractions, meanings of multiplication and division, proportional reasoning, etc.) was measured before the intervention by the standardized test BasisMath G6/7 with Cronbach's alpha of .83 (28 items, N = 1,120) and extensive validation studies (Moser Opitz et al., 2016).
- General cognitive ability was assessed using the well-established instrument CFT 20-R. Within our initial sample (N = 1,124 students) the standardized test reaches Cronbach's alpha of .76.
- As language proficiency has often been shown to influence mathematics learning (Stanat et al., 2019), academic language proficiency in German was assessed using a C-test, a widely used, economic, and valid measure that consists of three cloze texts (Grotjahn et al., 2002). It reached Cronbach's alpha of .77 (N = 1,122).
- Furthermore, students' socio-economic status (operationalized by the books-at-home-index), age, gender, multilingual background (operationalized by reported languages spoken at home) and immigrant status (operationalized by own and parents' country of birth) were captured in a self-report questionnaire often used in other studies.

Support Conditions

Three conditions were systematically compared (see **Figure 2**):

- The teachers in the *workshop condition* also formed multiprofessional learning communities and received the same curriculum material and teacher manual as the material group. Additionally, the personal strategy was extended by participation in four face-to-face PD workshops (of 3-5 hours, each) in which principles and general approaches for working with all students' diverse abilities were addressed, tried out and reflected in a sandwich structure (Burkhardt & Swan, 2017). The PD workshops covered instructional practices for dealing with diverse abilities for the percentages as well as other mathematical topics. These included the difference between compensation and enhancement practices, as presented before and their substantiation for the teaching unit on percentages with respect to different abilities (in particular diverse mathematical prior knowledge, language proficiency, attention and working memory, and metacognitive regulation). Within the four workshops, teachers were engaged in PD activities on five main tasks of teaching (Prediger & Buró, 2024): identifying potential demands and noticing students' abilities, setting differentiated learning goals with respect to diverse abilities, compensating limited abilities by scaffolding, enhancing targeted learning goal with respect to the ability, and engaging students in joint learning (Prediger et al., 2020). These tasks of teaching were addressed by PD activities with example tasks, student products, and video excerpts from classrooms. After workshop 2, teachers started to teach the teaching unit on percentages (presented in Section 2.3) for which teaching experiences were collectively reflected.

Table 1. Comparability of three intervention groups of students on cognitive variables and age: Descriptive and results of the ANOVA

		Descriptive statistics				ANOVA		Effect size
		n	M	SD	CI 95%	$F_{(2, 423)}$	p	Eta^2
Basic arithmetic competence	Workshop group	142	-0.08	1.05	-0.26-0.09	0.71	.49	.00
	Material group	142	0.04	1.01	-0.12-0.22			
	Control group	142	0.03	0.93	-0.12-0.29			
Prior knowledge on percentages	Workshop group	142	-0.03	0.98	-0.20-0.12	0.99	.37	.01
	Material group	142	0.09	1.03	-0.08-0.27			
	Control group	142	-0.05	0.98	-0.22-0.10			
General cognitive ability	Workshop group	142	-0.03	1.07	-0.21-0.14	0.13	.88	.00
	Material group	142	0.01	0.96	-0.14-0.17			
	Control group	142	0.01	0.97	-0.14-0.18			
Language proficiency	Workshop group	142	0.02	0.98	-0.14-0.18	0.19	.82	.00
	Material group	142	0.02	1.01	-0.14-0.19			
	Control group	142	-0.04	1.01	-0.21-0.13			
Age	Workshop group	142	12.64	0.68	12.57-12.71	14.49	.00	.04
	Material group	142	13.01	0.79	12.94-13.08			
	Control group	142	12.76	0.72	12.67-12.84			

Note. Main background variables in z-standardized values & CI: Confidence intervals

- The teachers in the *material condition* formed multiprofessional learning communities in their schools with mathematics and special education teachers (systemic implementation strategy according to Roesken-Winter et al., 2021) and were supported by curriculum materials (material implementation strategy according to Burkhardt & Swan, 2017; Roesken-Winter et al., 2021), here for the accessible MATILDA teaching unit (see **Figure 1**). For the personal strategy they received written PD opportunities in a teacher manual which was 53 pages long. This teacher manual described all instructional practices for dealing with diverse abilities of all students, directly tied to the tasks of the curriculum materials that were also discussed in the workshops of the workshop condition, but only in expository form without PD activities. However, as teachers in the material condition were also teaching the teaching unit, a strong activation took place as well.
- The teachers in the *control condition* were part of a waiting control group. The multiprofessional learning communities taught the same contents on percentages with their regular textbooks and established instructional practices. They received no material support for percentages. Due to ethical reasons and as an incentive to participate in the study, teachers in this condition receive the curriculum material and PD material after the study.

Implementation fidelity for the two support conditions (material condition and workshop condition) was controlled using the teachers' diaries with respect to a workbook completion rate of more than 80% (as in Lawrence et al., 2017).

Sample

The initially recruited sample of n = 90 mathematics and special education teachers at schools in socially underprivileged urban areas was assigned to three groups (no school was represented in two groups). However, the sample was reduced as shown in **Figure 2**. Here, the three groups had different levels of dropout:

- In the workshop condition, 21 teachers initially started, 3 dropped out so that only 18 were included in the final sample as their classes completed the teaching intervention as well as pre- and post-test (in total, 263 students completed all tests).
- In the material condition, 36 teachers initially started, 12 dropped out so that only 24 were included in the final sample as their classes completed the teaching intervention as well as pre- and post-test (in total, 358 students completed all tests).
- In the control group condition, 33 teachers were initially recruited, but only the 9 could be included in the final sample that completed the teaching intervention as well as pre- and post-test (in total, 143 students completed all tests).

As we expected a higher dropout in the material condition, we oversampled them in the initial sample. The dropout differences were significant: 14% in the workshop condition and 33% in the material condition (and even more in the control condition). This could be a relevant observation for comparing the implementability of the workshop condition and material condition: The self-learning expectations with only written PD material might not have sufficiently motivated 33% of the teachers to complete their study participation. Besides their great perseverance, the remaining teachers were comparable in teaching experience (varying between 4 and 20 years with medians of 12 for material condition, 11 for workshop condition and 10 for the control group).

A propensity score matching was conducted to re-achieve comparability between all three groups of students after teacher dropout. The matching procedure was administered by calculating the propensity scores using the package "MatchIt" (Ho et al., 2011) for R (R Core Team, 2020). It was carried out based on the following variables: basic arithmetic competence, prior knowledge on percentages, language proficiency and general cognitive ability. Based on the calculated propensity scores the nearest neighbors (Thoemmes & Kim, 2011) were selected. The teachers had no influence on the matching. The sample was reduced by the procedure to n = 142 students per group.

Table 1 shows the descriptive characteristics of the three groups as well as the results of the variance tests ANOVA checking for group differences. The three matched groups did not differ in any of the control variables (basic arithmetic competence, prior knowledge percentages, general cognitive ability, language proficiency; $p > .30$) with the exception of students' age that was significantly higher in the material group ($p < .01$), yet with a small effect size ($Eta^2 = .04$).

Table 2. Comparability of three intervention groups of students on multilingual background, immigrant status and gender: Descriptive and Chi^2 test results

	Workshop group (n = 142)	Material group (n = 142)	Control group (n = 142)	Tests for differences	
				$Chi^2_{(2)}$	p
Multilingual background (in %)	27.5%	34.8%	39.0%	4.32	.12
Gender (% of girls)	33.7%	31.7%	34.7%	1.11	.57
Immigrant status (% of one parent born outside Germany)	28.3%	37.8%	33.9%	6.94	.03

Table 3. Descriptive data for learning outcomes in conceptual understanding of percentages

		Descriptive statistics				ANOVA		Effect size
		n	M	SD	CI 95%	$F_{(2,423)}$	p	Eta^2
Post-test raw score	Workshop group	142	14.13	6.99	12.97-15.29	13.59	< .01	.06
	Material group	142	15.46	7.43	14.23-16.70			
	Control group	142	11.49	4.93	10.67-12.30			
Post-test z-score	Workshop group	142	-0.03	0.98	-0.20-0.12	13.59	< .01	.06
	Material group	142	0.09	1.03	-0.08-0.27			
	Control group	142	-0.05	0.98	-0.22-0.10			

Note. CI: Confidence intervals

As **Table 2** reveals, there were also no group differences for gender or multilingual background, but students with immigrant status were significantly underrepresented in the workshop group according to the conducted Chi^2 tests.

Data and Statistical Analysis

All analyses were conducted using SPSS (26). In the first step, the main effect of group adherence was tested by ANOVA. However, this does not take into consideration students' individual characteristics. Therefore, we calculated linear regressions in which the learning outcomes in the post-test score were entered as the dependent variable to test our hypotheses. The hierarchical regression is useful to analyze the effect of a predictor variable after controlling for other variables by calculating the change in the adjusted R^2 at each step of analysis. This shows the gradual increase in variance for each variable included in the model. We previously centered the metric and categorical predictor variables to estimate the effects in a conclusive manner. The metric variables were centered by z-standardization. To include the categorical variable in the regression, we used simple dummy coding (1; 0).

RESULTS

Effectiveness of the Support Conditions

Table 3 shows the descriptive data for the learning outcomes in conceptual understanding of percentages (in raw scores and standardized z-score used for the later regressions). It was found that teachers in both support conditions bring their students to higher learning outcomes than the control group teachers. The results of the ANOVA indicate a significant effect of group adherence, with a medium effect size ($Eta^2 = .06$).

However, the descriptive data and the ANOVA excludes students' individual characteristics which might impact the learning outcomes. Potentially relevant predictors among students' individual characteristics are included in the stepwise hierarchical regressions in **Table 4** to overcome this restriction.

The variables were included in a theory-based order, starting with the variable that explained the most variance in many studies (e.g., Hasselhorn & Gold, 2013; Pressley et al., 1989), basic arithmetic competence, prior knowledge percentages, general cognitive ability and language proficiency. Further control variables are not listed here as they did not have a significant effect and weakened the models. The models shown in **Table 4** include these students' individual characteristics as level 1 factors and teachers' participation in the different conditions as level 2 factors. Model 3 is the model with strongest predictive power for the learning outcomes on percentage understanding, with an adjusted R^2 of .57.

The standardized regression coefficients β can be interpreted in the role of effect sizes of the predictors. The results show basic arithmetic competence as the strongest predictor for learning outcomes on percentages. In model 3, we have an estimated β of 0.42, indicating that for each positive standard deviation in this predictor variable, the model estimates an increase of the post-test score by 42% of the standard deviation.

Prior knowledge on percentages as well as language proficiency are also significant predictors for the post-test scores with $\beta = 0.27$ and $\beta = 0.18$, respectively.

After controlling these predictor variables on the student level, teachers' participation in the material condition and the workshop condition are both significant predictors for higher post-test scores. Model 3 has a high increase of R^2 by 0.05 compared to model 2, and teacher support in these conditions explained 25%, respectively 20% of one SD in the post-test score ($\beta = 0.25$ and 0.20).

With these results, **H1** can be confirmed, i.e., the implementation of the MATILDA teaching unit was effectively supported in both support conditions.

Table 4. Hierarchical regression for learning outcomes on percentages

Model summary					
Model	**R^2**	**Adjusted R^2**	**R^2 change**	**F**	**df**
1	.48	.48	.48**	197.87**	2,423
2	.52	.51	.04**	113.37**	4,421
3	.57	.57	.05**	92.93**	6,419

Coefficients					
Model	**Predictors**	**B**	**SE (B)**	**β**	**CI 95% (B)**
1	Basic arithmetic competence	0.48**	0.04	0.48**	0.40-0.57
	Prior knowledge on percentages	0.28**	0.04	0.28**	0.20-0.37
2	Basic arithmetic competence	0.40**	0.05	0.40**	0.31-0.49
	Prior knowledge on percentages	0.30**	0.04	0.30**	0.21-0.38
	General cognitive ability	0.03	0.04	0.03	-0.05-0.10
	Language proficiency	0.19**	0.04	0.19**	0.12-0.26
3	Basic arithmetic competence	0.42**	0.04	0.42**	0.33-0.50
	Prior knowledge on percentages	0.27**	0.04	0.27**	0.19-0.36
	General cognitive ability	0.04	0.04	0.04	-0.03-0.11
	Language proficiency	0.18**	0.04	0.18**	0.11-0.25
	Teacher in material condition	0.53**	0.08	0.25**	0.38-0.69
	Teacher in workshop condition	0.43**	0.08	0.20**	0.27-0.58

Note. N = 426; z-standardized values: *p < .05 & **p < .01; CI: Confidence intervals; Dummy variable for workshop condition: Workshop group = 1, Material group = 0, & Control group = 0; & Dummy variable for material condition: Material group = 1, Workshop group = 0, & Control group = 0

Table 5. Hierarchical regression for post-test percentages when comparing only workshop and material condition without control group

Model summary					
Model	**R^2**	**Adjusted R^2**	**R^2 change**	**F**	**df**
1	.63	.63	.63**	243.11**	2,281
2	.66	.66	.03**	135.89**	4,279
3	.66	.66	.00	109.10**	5,278

Coefficients					
Model	**Predictors**	**B**	**SE (B)**	**β**	**CI 95% (B)**
1	Basic arithmetic competence	0.62**	0.05	0.60**	0.52-0.72
	Prior knowledge on percentages	0.28**	0.05	0.27**	0.18-0.38
2	Basic arithmetic competence	0.53**	0.05	0.51**	0.42-0.63
	Prior knowledge on percentages	0.30**	0.05	0.28**	0.20-0.40
	General cognitive ability	0.05	0.04	0.05	-0.04-0.13
	Language proficiency	0.17**	0.04	0.16**	0.09-0.26
3	Basic arithmetic competence	0.52**	0.05	0.50**	0.42-0.63
	Prior knowledge on percentages	0.30**	0.05	0.28**	0.20-0.40
	General cognitive ability	0.05	0.04	0.05	-0.04-0.13
	Language proficiency	0.17**	0.04	0.16**	0.09-0.26
	Teacher in material condition	-0.08	0.08	-0.04	-0.23–0.06
	Teacher in workshop condition	0.62**	0.05	0.60**	0.52-0.72

Note. N = 284; z-standardized values: *p < .05 & **p < .01; CI: Confidence intervals; & Dummy variable for workshop condition: Teacher in workshop group = 1 & Material group = 0

Additional effects of the Workshop Condition

H2 assumes that the workshop condition might be more effective than the material condition. The post-test scores documented in **Table 3** suggest no large difference between the two support conditions, with almost identical standardized post-test scores. For scrutinizing the comparison between the two support conditions, we conducted a second hierarchical regression only with students from the material group and the workshop group, without the control group. The models shown in **Table 5** include the students' background variables as level 1 factors and teachers' participation in the two support conditions as Level 2 factors.

The results of the second hierarchical regression show no further change in R from model 5 to model 6. Here, model 5 (without the level 2 factor) is the model with the strongest predictive power for the learning outcomes on percentage understanding, with an adjusted R^2 of .66. Similar to model 2 in the first regression, basic arithmetic competence, prior knowledge on percentages and language proficiency are significant predictors for the post-test score, whereas general cognitive ability is no significant predictor.

After controlling these individual variables, the workshop condition (i.e., teachers' participation in the PD workshops) shows no significant additional effect on the student's post-test score (β = - .04) compared to the material condition.

DISCUSSION

Summary of the Main Results

The aim of the presented study was to identify the kind of support that teachers need to effectively provide access to conceptual understanding in heterogenous mathematics classrooms, a goal called for by many researchers, but not yet widely implemented (Blazar & Archer, 2020; Prast et al., 2015; Scherer et al., 2016; van Mieghem et al., 2020). Whereas all teachers in our field trial worked in PLCs in their schools (often multiprofessional teams consisting of mathematics and special education teachers), systemic condition alone appears to provide too unspecific support (van Mieghem et al., 2020), as the learning outcomes of the control group were shown to be significantly lower than of those students whose teachers had more support.

In the material strategy (Leong et al., 2019; Saxe et al., 2001), two support conditions were realized. Both relied on curriculum materials for our MATILDA teaching unit (Kuhl et al., 2021; Pöhler et al., 2018), which substantiates an instructional approach that aims to improve learning opportunities for all students by considering their diverse abilities (prior knowledge in mathematics, language proficiency, working memory and attention, metacognitive regulation and strategies), in some compensation practices and consequent enhancement practices (Corno, 2008). The written teacher manual for both conditions, material condition and workshop condition, provided detailed descriptions for suitable instructional practices and the underlying ideas (Prediger et al., 2019), in particular with respect to the core scaffold, the visual model of percent bar (Leong et al., 2019; Morales et al., 2024).

In research question **RQ1**, we tested the **H1** that teachers in material-based support conditions can lead their students achieve learning outcomes in terms of percentage understanding than teachers who taught with the conventional material. Indeed, **H1** could be confirmed as students in the material group and the workshop group significantly outperformed the control group (**Table 3** and **Table 4**). This important result strengthens the empirical base of the material implementation strategy (Burkhardt & Swan, 2017; Roesken-Winter et al., 2021). Given that these results were gained in schools in socially underprivileged urban areas, we see it as an indicator that accessibility was achieved under challenging conditions. Teachers who are supported by the MATILDA teaching unit can lead students to better learning outcomes in conceptual understanding of percentages than teachers with their regular textbooks. This brings us one step closer to the vision of mathematics for all (Clements et al., 2013; Damerow et al., 1984).

The effectiveness of interventions is most often studied under specific controlled conditions (Burkhardt & Swan, 2017). As our field trial was implemented by regular teachers who work in their regular classes, the findings indicate high ecological validity, which is a very positive result on which future work can build.

The support in both support conditions included substantial PD offers for teachers in the written teacher manual. According to Desimone (2009), the content focus of PD offers is one of the most influential features to improve their practice. In particular, the combination of content knowledge and pedagogical content knowledge seems to have a beneficial effect on teachers' knowledge and skills, instructional quality as well as student development. In terms of specific PD content, the PD offers reflected Karsenty's (2010) and Beswick's (2007) call for intense work on unpacking conceptual learning opportunities and the detailed discussion on how to realize enhancement and compensation practices with respect to students' varying abilities (Janney & Snell, 2006).

To pursue research question **RQ2** concerning the form of PD offers in the personal strategy, two different support conditions were realized. Following earlier findings in science teacher education (Kleickmann et al., 2016), we hypothesized in **H2** that the PD offered in face-to-face workshops might lead to better student outcomes than the written teacher manual, in particular because the teachers had more opportunities to actively engage with the new instructional approach in well-designed PD activities (Prediger et al., 2020), because active learning is another critical feature of teachers' PD (Desimone, 2009).

However, the comparison of learning outcomes in **Table 2** and **Table 4** cannot validate our **H2**, as the material group tended to show slightly higher learning outcomes (**Table 5**), yet no significant difference was found between workshop group and material group. Based on these outcomes, we cannot confirm that face-to-face PD workshops are necessary for *all* teachers, even if the dropout rate must be taken into consideration. Apparently, the manual on its own can be a strong support for *some* motivated and committed teachers who have sufficient self-learning capacities to realize quality mathematics instruction from a comprehensive written manual. Their instruction is significantly more effective than the instruction of control teachers working with their regular textbooks, and comparably effective as the instruction of the workshop participants (involving a larger group of teachers with less dropout).

Limitations and Future Research

As in every study, it is important to interpret the findings with caution and to explicitly mention the methodological limitations. One point is the high dropout rate in the material group: Only 14% of the teachers dropped out in the workshop condition, but 33% in the material condition did not meet the high self-learning expectations with only written PD material. One might doubt whether a study should be published in which the control group and one treatment group has such a strong dropout rate.

However, as Karsenty and Brodie (2023) emphasize, such results should be reported as well, as these dropouts are typical effects in many field studies which are too often not transparently reported. They are not only instances of imperfect realization conditions, but an interesting phenomenon of the reality of school implementation processes per se, which should receive more research attention in the future, in particular with regard to teachers' professional satisfaction. Even without these insights into teachers' perspectives, the dropout rate in our study suggests to interpret the null finding in the comparison of outcomes differently: When only written support is provided, even teachers who volunteered for an implementation program (without any time or salary compensation) give up significantly more frequently. The teachers who worked with the material condition until the end of the study may be the most engaged teachers and therefore a somehow selective comparison group. For those teachers with high perseverance, the self-learning PD offers were sufficient. Meanwhile, OD workshops with active engagement continue to be the more promising offer to reach as many teachers as possible (Desimone, 2009).

Our results provide initial insights about conditions to best support teachers in maximizing their students' access. As in every study, it is important to interpret the findings with caution and to explicitly mention the methodological limitations: The dynamics of the dropouts and their impact on students' outcomes could only be discussed with some speculations. In future studies, the interpretative assumptions should be further investigated, in larger samples, more data on teachers' commitment and expertise, and a completely randomized assignment of teachers and perhaps even students.

For this study, we did not check whether the prerequisites of the teachers were comparable, which results in another limitation. On the teachers' side, certain conditions may have interacted with PD and implementation. Here, models could be included that deal with the professional competence of teachers and teacher knowledge, e.g., the conceptualization of Shulman (1986). Frequently in this context, teachers' attitudes, beliefs and motivation are mentioned as factors affecting their behavior (e.g., Beswick, 2007; Chambers & Callaway, 2008; Hall & Hord, 2011). For example, Chambers and Callaway (2008) examined the perceptions of "high and low implementers". They showed that teachers' beliefs influence students' learning. In addition, high implementers are particularly good at overcoming obstacles. Thus, future research should investigate which expertise, attitudes and beliefs teachers require to benefit from PD and effectively implement the instructional approaches underlying the MATILDA teaching unit. The results also indicate that motivational and volitional aspects are crucial.

The effectiveness of interventions is most often studied under specific controlled conditions (Burkhardt & Swan, 2017). As our field trial was implemented by regular teachers who work in their regular classes, the findings indicate high ecological validity, which is a very positive result on which future work can build. However, there are several factors which are important for the successful implementation of instructional approaches such as ours. Gräsel (2010) suggests four major domains which are important for successful implementation in educational systems:

(1) characteristics of the innovation itself (which is highly complex in our case),

(2) characteristics of the involved teachers (which could be more controlled in future research),

(3) characteristics of the individual schools, and

(4) characteristics of the environment and transfer support.

The last point was our main focus, which Gräsel (2010) also emphasizes as extremely important, and points out long-term support must be provided where phases of testing the innovation are combined with phases of reflection. These assumptions should be considered to further increase the success of implementation.

CONCLUSION AND IMPLICATIONS FOR MATERIAL-BASED PD PROGRAMS

Although our findings do not allow generalizations beyond the context of this study (neither beyond the particular population of teachers nor to the population of all students), we have already generated important findings. Also, students from underprivileged classrooms with highly diverse abilities can develop robust conceptual understanding. This is the central finding that raises hope for further programs (Blazar & Archer, 2020).

These outcomes were researched in schools with favorable systemic conditions in which mathematics teachers and special education teachers cooperated well (Borko & Potari, 2024). However, cooperation alone is not enough, as the significantly lower learning gains in the control classes reveal. Thus, our results strengthen the empirical base of the material implementation strategy (Burkhardt & Swan, 2017; Roesken-Winter et al., 2021): Teachers who are supported by carefully designed curriculum materials in which adaptive instructional practices are strongly supported can lead students to better learning outcomes in conceptual understanding than teachers with their regular textbooks. This implies that the design of curriculum materials to include increased conceptual access is an important venue.

The unexpected finding that a written PD manual can at least lead some teachers to equally effective teaching practices as the workshop condition has been extensively discussed in our contexts as we had too few confidence in teachers' self-learning capabilities. At least for a small group of teachers, offering self-learning opportunities can be an important way for future scaled-up PD programs in which workshop conditions can be created for some, and self-learning modules for other teachers.

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