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How Lithuanian mathematics teachers interpret student thinking at the different career stages

Monika Grigaliūnienė [1*], Aušra Rutkienė [1], Erno Lehtinen [1,2]

[1] Vytautas Magnus University, Kaunas, LITHUANIA
[2] University of Turku, Turku, FINLAND
***Corresponding Author:** monika.grigaliuniene@vdu.lt

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ABSTRACT

This study examines the pedagogical content knowledge (PCK) of Lithuanian mathematics teachers at different stages of their professional careers. The research sample comprised 134 participants ranging from novice teachers with no teaching experience to experienced educators, with the longest career spanning 45 years. The participants were divided into four groups according to the length of their career and the results were analyzed accordingly. While no significant correlation between PCK and career length was found in the overall test results, statistically significant differences were found between them for certain questions. The results make it clear that some teachers have difficulty interpreting students' errors and developing appropriate teaching strategies. This study indicates that further research is needed to deepen the understanding of PCK development throughout teachers' careers and to investigate the factors that contribute to differences in PCK within groups of teachers with similar career lengths.

Keywords: mathematics pedagogical content knowledge, pre-service teachers, in-service teachers

INTRODUCTION

Mathematical skills are undoubtedly beneficial for all students as they are associated with critical and analytical thinking, effective information processing, valid evaluation, objective decision making, creative problem solving and other skills (Trei, 2015) that are important for students' future (Council of the European Union, 2018). This makes it clear why improving students' mathematical skills is an increasingly important goal.

Large-scale research underscores the central role of teachers in ensuring the quality of instruction (e.g., Burroughs et al., 2019; Hattie, 2003). The concept of effective mathematics teaching is often identified as a defining feature of expert educators (Grant, 2022). However, the precise nature of teaching effectively remains challenging to define due to the inherent complexity of the concept (Irvine, 2019; Kini & Podolsky, 2016). Despite this complexity, there is broad consensus that effective teaching encompasses the ability to manage many teaching processes simultaneously, including those directly related to subject matter and others that extend beyond it (Blömeke & Kaiser, 2017).

From a general pedagogical perspective, effective teaching has been associated with the ability to successfully mediate student engagement (Lampert, 2001), adapt to the diverse needs of learners (Tomlinson, 2001), establish supportive learning environments, and manage them effectively (Darling-Hammond & Bransford, 2005), among other factors.

In the context of subject-specific pedagogy, research indicates that possessing knowledge of mathematical content alone is insufficient for effective teaching (Lehtinen, 2008; Loewenberg-Ball, 2000; Oonk et al., 2015). Increasingly, it is recognized that teachers require specialized knowledge that equips them with the ability to provide clear explanations and perceive their subject matter through the lens of their students (Cheredeko & Shahbazi, 2013; Hardre et al., 2014). Such skills are essential for teachers to be able to provide the necessary support to their students (e.g., Baki & Arslan, 2017; Güler & Çelik, 2018; Saran, 2018).

This type of knowledge is unique to teachers because it is a synthesis of pedagogical knowledge (general knowledge of teaching methods) and content knowledge (subject matter knowledge), which means that teachers can organize their knowledge from an instructional perspective and use it as a basis for helping students understand specific concepts (Cochran, 1997). Shulman (1986) distinguished this type of knowledge from content knowledge and general pedagogical knowledge and called it pedagogical content knowledge (PCK). PCK is not the only term to define teachers' specific knowledge (Depaepe et al., 2013) thus some researchers define PCK differently (e.g., Ball et al., 2008; Cochran, 1993). However, all definitions include the same fundament - the recognition/awareness of students' errors and the ability to adapt instructional strategies to students' needs, which is manifested in the teacher's ability to recognize students' errors, understand the underlying cause of students' errors and support their learning through the use of appropriate instructional strategies, and adapt and assess students' learning experiences throughout the instructional process (e.g., Grigaliūnienė et al., in review; Hill et al., 2008, Shulman, 1986, Watson, 2008).

PCK has been identified as a necessity for effective teaching (Depaepe et al., 2013). Many studies establish a link between PCK and teacher competence (Blömeke & Kaiser, 2017) and consider PCK to be an important indicator of effective teaching in terms of student learning outcomes (e.g., Callingham, 2015; Cueto, 2016; Kunter et al., 2013; König & Kramer, 2016). While the current state of research makes it clear that this knowledge is essential for effective teaching, research also shows that some teachers lack PCK (e.g., Depaepe et al., 2018).

Some studies show that pre-service teachers lack PCK, which means that they cannot teach as effectively as more experienced teachers and cannot provide high-quality support to students (e.g., Koçak et al., 2017; Martin & Jamieson-Proctor, 2020). However, the lack of this knowledge is not only evident in pre-service teachers, but also in more experienced teachers (Depaepe et al., 2018). While professional competencies are expected to develop with experience (Berliner, 2005), studies show that teaching skills develop in the first few years of teaching experience, but thereafter not all teachers improve their teaching skills with increasing years of teaching (Brody & Hadar, 2015; Hanushek, 2011; Huang & Moon, 2009; Kraft & Papay, 2014). A review by Grant (2022) states that mere teaching experience is not enough to improve the pedagogical knowledge required for effective teaching, but that deliberate development of skills is needed.

Research has been conducted into how, but also when, the improvement in teaching takes place. Some of the large-scale studies conclude that the average effectiveness of teachers no longer increases significantly after three to eight years (Hanushek, 2011; Hanushek & Rivkin, 2010; Hargreaves & Fullan, 2012) and that teachers improve the most in the first few years. In their study of more than 3,000 teachers, Kraft & Papay (2014) found that different developmental trajectories can be observed after a few years of teaching. Some continuously increase their teaching efficiency, while the development of others comes to a standstill after a few years. The study by Kini and Podolsky (2016) provides evidence of improvement over the years, and similar to Kraft and Papay (2014), they find that the improvement is not monotonic and the trajectory varies from between individuals.

Recognizing and interpreting students' thinking and possible misconceptions is the focus of most studies on mathematics teachers' PCK. Some specific misconceptions or difficulties are particularly emphasized in the studies. A large number of studies refer to students' difficulties and typical misconceptions in learning rational numbers and operations with them (Barmby, 2013; Hourigan & O'Donoghue, 2012; Lannin, 2013, Özdemir et al., 2017; Siegler et al., 2012; Van Hoof et al., 2017). Such as natural number bias that make it difficult to understand the size of fractions and decimals or arithmetic operations with fractions. Many studies address the need for teachers to recognize students' difficulties with arithmetic algorithms, such as errors in division (Charalambous, 2016; Cueto, 2016; De Corte et al., 1991), challenges in understanding the nature of algebra concepts (including functions) (Chang et al., 2016; Demonty, 2018; Hatisaru, 2020; Leinhart et al., 1990; Marban, 2020) and geometry concepts (Alkhateeb, 2018; Baki, 2017; Fukaya, 2022; Kleickmann, 2013; Kleickmann, 2015; Krauss, 2020).

Present Study

This study was conducted in Lithuania, where students' mathematical skills are at an average level compared to other OECD (2015) countries, with national mathematics examinations indicating a decline in mathematical performance (e.g., Lietuvos Radijas ir Televizija [Lithuanian Radio and Television] [LRT], 2020). This situation highlights the serious problems in mathematics education and the need for effective solutions in the hope of improving the situation.

PCK (as defined by Shulman, 1986) of mathematics teachers has not been researched in Lithuania yet. It is essential to capture the state of the knowledge of Lithuanian teachers to enable development of their expertise (PCK can be taught to a certain degree Depaepe et al. (2008).

In this study dealing with mathematics teachers, we focused on the following research questions:

1. *Is there an overall difference between mathematics teachers' PCK at different stages of their career (based on career length)?*
2. *What particular aspects of PCK appear to be more challenging for teachers at the particular stage of their career?*

METHODS

The participants were mathematics teachers in Lithuanian secondary schools and pre-service mathematics teachers. The participants were selected by convenience sampling from schools cooperating with the university and from students studying at the university. The total number of participants was 134, which consisted of 41 pre-service teachers who were currently studying mathematics and 93 in-service teachers who were currently working as mathematics teachers. As many of the student teachers had teaching experience prior to starting their formal teacher training, the analysis in this study was based on the length of time the teachers had been teaching: 1st group–career length up to 1 year (n = 36), 2nd group–early career teachers between 1 and 7 years of teaching (n = 31), 3rd group–between 8 and 30 years of teaching (n = 31) and 4th group–more than 30 years of/ teaching (n = 36).

Participants received a link for the test and had, on average, around two weeks to complete it at the time of their convenience. Data collection lasted two school years (from the beginning of school year 2022/2023 till the end of school year 2023/2024).

All questions were adapted from previously published studies (Buschang et al., 2012; Callingham et al., 2015; Cueto et al., 2016; Depaepe et al., 2015, 2018; Güler & Çelik, 2018, 2019; Kleickmann et al., 2015; Lai & Jacobson, 2018; Lim & Guerra, 2013; McGuire,

Table 1. Open-ended question scoring example

Example question	
Students were asked the following question 'There are ten families in the neighborhood and the average number of kids per family is 2.3. One family with 5 children leaves the neighborhood. What is the average number of children per family now?' One student says, 'I don't know how many children in each family; therefore, I cannot answer.' How would you help him?	
0 points (incorrect)	"I would ask the pupil to think more." & "Make equation."
1 point	"You don't need to know that to answer. Find out how many children all the families have together. Then use the concept of average." & "I would explain in detail how the average is calculated."
2 points (fully answered)	"I would say, 'Can you tell me how many children there were when there were 10 families? And how many children are there now?' Then I would ask him to remember what the average is. We could also use the calculation of his grade average as an example."

Table 2. Descriptive statistics for pedagogical content knowledge questionnaire score and career length

	Min	Max	Mean			Skewness		Kurtosis	
			Statistic	Standard error	Standard deviation	Statistic	Standard error	Statistic	Standard error
PCK score	9	38	24.02	.591	6.839	−.127	.209	−0.929	.416
Career length	0	45	16.84	1.397	16.16	.321	.209	-1.631	.416

2013; Norton, 2012; Sintema & Marbam, 2020; Zolfaghari et al., 2021) in which they were tested with larger samples (see **Appendix A** for an English translation of the full questionnaire).

The questions for the questionnaire were selected based on mathematical topics. The topic of rational numbers has been shown to be difficult for students (e.g., Behr et al., 1984), so most questions focus on decimals (5th, 12th, and 15th questions in **Appendix A**) and fractions (9th, 10th, 11th, 13th, and 14th questions in **Appendix A**). Whole numbers arithmetic was also included in the PCK questionnaire as the foundation on which all later mathematics is built (1st, 2nd, 3rd, and 4th questions in **Appendix A**). In addition, basic concepts of algebra (18th, 19th, and 20th questions in **Appendix A**) were part of the test, along with other questions covering other aspects of school mathematics.

The PCK test consisted of 21 PCK questions that were open-ended (n = 14), multiple-choice (n = 4), and mixed (n = 3). For multiple-choice questions, respondents could only select one of the given options, without the opportunity to explain their answer. Both open and mixed questions required a written response (up to 4,000 characters). On average, respondents took around 35 minutes to complete the questionnaire.

The multiple-choice questions were scored from 0 (incorrect) to 1 (correct), and the open-ended questions were scored from 0 (incorrect) to 2 (fully answered) (see example in **Table 1**).

The mixed questions consisted of a multiple-choice question and an open question, i.e., the respondent first had to choose the correct answer and then explain their choice; the total score ranged from 0 (incorrect or irrelevant choice) to 3 (correct choice and fully elaborated answer).

The questionnaire was pilot tested to determine the suitability of the questionnaire for further implementation. Three volunteers, including two pre-service teachers and one in-service teacher, completed the questionnaire and gave feedback on how the questionnaire could be improved by clarifying some of the questions; no major changes were made before main data collection.

The first author of the paper created scoring guidelines and scored all responses. To control inter-rater reliability the widely applied procedures presented by Gwet (2014) was followed. Another Lithuanian researcher (background as a mathematics teacher) scored about 10% of the responses independently. Of all double-coded open-ended questions, 83% were scored equally, and the remaining 17% were discussed by the researchers until a consensus was reached. Most disagreements concerned a few questions that were discussed in detail. After the discussion, the remaining responses were re-evaluated to adjust the final score to the consensus. Cronbach's alpha of the pedagogical knowledge questionnaire was α = .792, demonstrating high internal consistency and reliability of the questionnaire (Cronbach, 1951).

Descriptive statistics and crosstabulation, (Chi-square and standardized residuals) were calculated using IBM SPPS statistics software, version 29. Standardized residuals greater than 2 and less than -2 were considered large and $p < .05$ was used as threshold levels of significance for Chi-square.

For crosstabulation the PCK-scores of the whole test were classified in four levels (1st level up to 18 points, 2nd level between 19 and 25 points, 3rd level between 26 and 29 points and 4th level–more than 30 points). Cut-off points for PCK levels were determined based on distribution of the PCK scores to form four similar size groups (quartiles).

RESULTS

Descriptives of the study variables are presented in **Table 2**.

The crosstabulation results show that there was no statistically significant overall relationship between career length and PCK knowledge (χ^2 = 887.440 (924), p = .801) (**Table 3**). However, although the overall results do not indicate a statistically significant difference between the groups, the group of teachers with the longest careers had significantly lower frequency than the other groups in the highest (4th) PCK level category (see **Table 3**).

Table 3. Crosstabulation table of teachers' PCK level (PCK score)

Experimental group	1st PCK level			2nd PCK level			3rd PCK level			4th PCK level		
	N	%	Standard response	N	%	Standard response	N	%	Standard response	N	%	Standard response
1	7	25.0	-.2	9	24.3	-.3	10	27.8	.1	10	30.3	.4
2	3	10.7	-1.4	7	18.9	-.5	9	25.0	.2	12	36.4	1.6
3	7	25.0	-.2	8	21.6	-.2	8	22.2	-.1	8	24.2	.1
4	11	39.3	1.3	13	35.1	1.0	9	25.0	-.2	3	9.1	**-2.0**
Total	28			37			36			33		

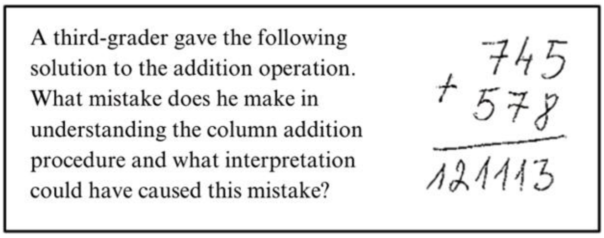

Figure 1. Interpretation of the addition algorithm error (adapted from Norton, 2012).

Table 4. Crosstabulation table of teachers' answers to the first test item

Experimental group	0			1			2		
	N	%	Standard response	N	%	Standard response	N	%	Standard response
1	1	5.6	-1.7	13	28.3	.2	22	31.4	.7
2	1	5.6	-1.6	9	19.6	-.5	21	30.0	1.2
3	4	22.2	-.1	10	21.7	-.2	17	24.3	.2
4	12	66.7	**3.3**	14	30.4	.5	10	14.3	**-2.0**
Total	18			46			70		

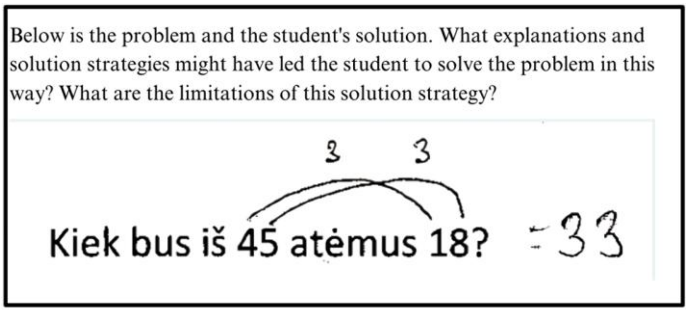

Figure 2. Student's answer to the question 'How much is 45 minus 18?' (adapted from Norton, 2012)

To answer the second research question, we present detailed results of the tasks where there were statistically significant differences between teachers at different stages of their careers. In the first question of the PCK test (**Figure 1**), teachers were asked to comment on students' understanding of column addition. While the error was relatively easy to spot, there were teachers who struggled to point out the underlying interpretation of the student's error (**Table 4**). Teachers were expected to notice pupils struggling to interpret score over 10 in each column.

Chi-square test showed that there was statistically significant relation between career length and PCK scores ($\chi^2 = 22.743$ (6), $p < .001$). According to standardized residual the group of teachers with longest career has significantly more 0 points answers and significantly less 2 points answers compared to the other groups.

Some answers were too superficial to capture students' thinking: 'Adds consecutive numbers' (0 pt), 'Does not know the rule for adding columns' (0 pt). Four teachers refrained from answering this question on the grounds that mathematics teachers do not need to comment on primary school maths: "I will not comment on teaching methodology in primary school" (0 pt), "I will not deal with the methodology of primary school teaching" (0 pt).

Good answers pointed out both errors in thinking and possible teaching instructions, e.g.,

> "The pupil probably didn't understand the teacher's explanation that we don't write the ten in the answer but add it to memory by adding +1 on top. When explaining it to the child, I would divide the numbers into ones, tens and hundreds and start to work out what happens when the column becomes more than ten" (2 points).

The second question (**Figure 2**) invited teachers to comment on pupil's strategy of subtraction. Chi-square test showed not overall statistically significant relationship between PCK and career length groups ($\chi^2 = 11.130$ (6), $p = .084$), however the group with longest career has significantly more 0 points answers compared to other groups (**Table 5**). Teachers were expected to notice pupils subtracting "smaller number from the bigger" and not considering number as a whole consisting both units and tens.

Table 5. Crosstabulation of teachers' answers to the second questionnaire item

Experimental group	0			1			2		
	N	%	Standard response	N	%	Standard response	N	%	Standard response
1	3	11.1	-1.6	15	32.6	.8	18	29.5	.4
2	4	14.8	-.9	12	26.1	-.4	15	24.6	.2
3	7	25.9	.3	8	17.4	-.8	16	26.2	.5
4	13	48.1	**2.1**	11	23.9	-.4	12	19.7	-1.1
Total	27			46			61		

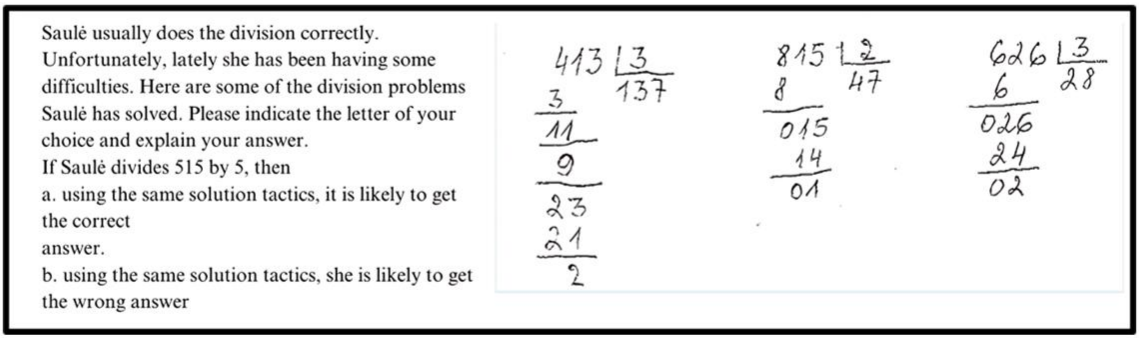

Figure 3. Students' division algorithm solutions (adapted from Cueto et al., 2016)

Table 6. Crosstabulation of teachers' answers: Choice part

Experimental group	0			1		
	N	%	Standard response	N	%	Standard response
1	2	20.0	-.4	34	27.4	.1
2	2	20.0	-.2	29	23.4	.1
3	1	10.0	-.9	30	24.2	.2
4	5	50.0	1.4	31	25.0	-.4
Total	10			124		

Table 7. Crosstabulation of teachers' answers: Explanation part

Experimental group	0			1			2		
	N	%	Standard response	N	%	Standard response	N	%	Standard response
1	8	14.8	-1.7	9	27.3	.0	19	40.4	1.8
2	7	13.0	-1.6	9	27.3	-.5	15	31.9	1.3
3	14	25.9	.4	10	30.3	-.9	7	14.9	-1.2
4	25	46.3	**2.8**	5	15.2	-1.3	6	12.8	-1.9
Total	54			33			47		

Zero-point answers included teachers stating that they don't know, e.g., 'I don't know', ' I don't have an answer', ' I have no idea', ' I don't know, I teach grades 9-12', or other reasons that presumably show no identification of underlying cause for this mistake: ' Often students perform actions without thinking', 'The pupil does not know the rule' and possibly claiming issue with the problem 'No minuend and no subtrahend', also just stating that it is wrong, e.g., 'Incorrect answer. Must be: 27'.

Good (2-point) answers included pointing out possibility of student remembering to 'subtract smaller number from the bigger and/or subtract tens from tens and ones from ones' and incorrectly applying this strategy here.

The third question (**Figure 3**) was about a long division algorithm and a very common pupils' mistake. This question had two parts-test item (choosing the correct answer from the given) and open question (explaining the result). While all groups did relatively well on the choice item (**Table 6**), there were still few that could not identify the pattern in pupil's mistake. Teachers were expected to notice the pattern of pupil's calculation and spot missing 0 in the calculation algorithm.

Chi-Square test showed no statistically significant differences between groups in the choice question ($\chi^2 = 3.194$ (3), $p = .363$), but differences between groups in the explanation question were statistically significant ($\chi^2 = 25.410$ (6), $p < .001$). Classification of open-ended questions is presented in **Table 7**. According to the standardized residuals the group of longest career teachers differ significantly by having more 0 points answers and less 2 points answers compared to the other groups.

Wrong or insufficient explanations included fragmental answers, e.g., 'a. She will get the right answer', 'The answer is b', 'Pupil does not know subtraction', 'Number must be divided consistently', not understanding the problem, e.g., 'solving using 1st strategy will give correct answer, solving using 2nd or 3rd strategy will give wrong answer' and again referring to primary school mathematics as something outside of their specialty, e.g., 'I don't know, I teach grades 9-12.'.

Partially correct (1 point) answers usually included recognizing the mistake, but their reasoning was lacking, for example, "There is a gap in the learning of division, when we move more than one number. Of course, when teaching children to divide, it is important to think logically <...> Students need to be encouraged to think."

"A student tells you that multiplying two numbers always gives a number, is greater than the multiplicands. What will you say to this pupil?"

Figure 4. Fourth questionnaire item (adapted from Kwong et al., 2007).

Table 8. Statistics of the fourth questionnaire item

Experimental group	0			1			2		
	N	%	Standard response	N	%	Standard response	N	%	Standard response
1	6	22.2	-.5	15	22.4	-.7	15	37.5	1.3
2	1	3.7	**-2.1**	14	20.9	-.4	16	40.0	**2.2**
3	6	22.2	-.1	18	26.9	.6	7	17.5	-.7
4	14	51.9	**2.5**	20	29.9	.5	2	5.0	**-2.7**
Total	27			67			40		

"How would you explain student that a-(b+c) and a-b-c are equivalent?" (adapted from Buschang et al., 2012)

Figure 5. Task to explain equivalence of algebraic expressions (adapted from Buschang et al., 2012)

Table 9. Statistics of the sixth questionnaire item

Experimental group	0			1			2		
	N	%	Standard response	N	%	Standard response	N	%	Standard response
1	2	16.7	-.7	20	29.4	.4	14	25.9	-.1
2	1	8.3	-1.1	10	14.7	-1.4	20	37	**2.1**
3	4	33.3	.7	19	27.9	.8	8	14.8	-1.3
4	5	41.7	1.0	19	27.9	.2	12	22.2	-.7
Total	12			68			54		

Full (2-point) answers described in detail the mistake of the pupil and how detailed explanation of long division algorithm could help pupil overcome the mistake.

The fourth question (**Figure 4**) was about were common misconception of product of multiplication being greater than multiplicands. The question (**Figure 4**) asked teachers to formulate a response to student: Teachers were expected to consider a variety of numbers that can be multiplied including negative numbers, zero and rational numbers.

For two points, teachers had to mention at least two different aspects–0 and 1, fractions and integer numbers. Chi-square test showed that there is significant relationship between PCK and career length ($\chi^2 = 26.447$ (6), $p < .001$). According to standardized residuals (**Table 8**), the group with longest career teachers differ significantly by having more 0 points answers and less 2 points answers compared to the other groups, group of early career teachers (1 to 7 years) have significantly less 0 points answers and significantly more 2 points answers compared to the other groups.

Zero-point answers included teachers not answering or giving fragmented answers, e.g., "wrong", "not always" and "what about 0?" as it doesn't explain why zero is mentioned in this case, also "student, you are wrong, here's the example 4*0.5", "no, I will give an example with multiplication by 1".

Incomplete answers show correct understanding, however, answers are wrong from the perspective of stimulating student's growth, e.g., "If it's a primary school student who doesn't know what fractions and negative numbers are, I'll say he's right. For an older pupil, I will tell them to repeat the theory" (1 pt).

Figure 5 shows a task in which participants are asked to explain the concept of equivalence of algebraic expressions to students. They were expected to explain the meaning behind the concept and give examples to illustrate their reasoning.

Crosstabulation and Chi-square test (**Table 9**) showed statistically significant relationship between PCK and career length groups ($\chi^2 = 12.658$ (6), $p = .049$). According to the standardized residuals the early career teachers (group 2) had more 2 points answers.

Good example of full answer:

> "I will give you an example with sweets: We have a certain number of sweets. We need to calculate how much will be left if we: consecutively minus the number of candies we want to give to two friends, add the number of candies we want to give to two friends, and minus the result from the current number of candies I will ask you to come up with two solutions for this problem. From grade 8 onwards, I will be using the multiplication factor of the singular (-1) by the plural (b-c)."

Some answers were lacking clarity and/or explanation, for example, "Minus before brackets", "Because the multiplication of the negative" or only mention one aspect of the question: "I would use specific numbers instead of letters", "I don't know which class the pupil is in and what his experience is". Examples of the insufficient answers include "The problem is missing data", "They are equivalent", "Because answers are the same".

For each of the given situations, write whether you would use it to explain the mathematical operation in class and explain why?

a) 3/5 of the chocolate bar was used to make chocolate mousse. Gustas and Liza ate 1/4 of the remaining chocolate together. What part of the original chocolate bar did they eat?

b) You need 1/4 of the well water to fill the pool. Today the well is only 3/5 full. How much water will be left in the well after the pool is filled?

c) Anna used 3/5 of the red paint and 1/4 of the yellow paint in art class. How much red and yellow paint is left?

d) Neither one of these situations is appropriate.

Figure 6. Questionnaire item asking to pick problem that illustrates given operation and explain their choice (adapted from Depaepe et al., 2015).

Table 10. Statistics of the third questionnaire item: Choice part

Experimental group	0			1		
	N	**%**	**Standard response**	**N**	**%**	**Standard response**
1	13	22.0	-.7	23	27.4	.6
2	18	30.5	-1.2	13	23.4	-1.0
3	15	25.4	.4	16	24.2	-.3
4	13	22.0	-.7	23	25.0	.6
Total	59			75		

Table 11. Statistics of the third questionnaire item: Explanation part

Experimental group	0			1			2		
	N	**%**	**Standard response**	**N**	**%**	**Standard response**	**N**	**%**	**Standard response**
1	15	19.5	-1.3	7	25.9	-.1	14	46.7	**2.1**
2	17	22.1	-.2	4	14.8	-.9	10	33.3	1.2
3	19	24.7	.3	8	29.6	.7	4	13.3	-1.1
4	26	33.8	1.2	8	29.6	.3	2	6.7	**-2.1**
Total	77			27			30		

The student claims that he cannot calculate the area of a parallelogram. What could be the reason for this?

Figure 7. Seventeenth question translated to English (adapted from Kleickmann et al., 2015).

The thirteenth question asked teachers to select a problem illustrating the 3/5-1/4 operation (**Figure 6**). Teachers were expected to correctly evaluate the appropriateness of the tasks and select the second option as the only appropriate one. They were also expected to recognize that other options were not only inappropriate, but simply wrong.

As seen in **Table 10** and **Table 11**, a lot of teachers had issues with this question as many responses were evaluated 0 points. Chi-square test showed no statistically significant relation between PCK and career length groups in the multiple choice question ($\chi^2 = 4.549$ (3), $p = .208$) (**Table 10**), however relationship between groups and PCK in open-ended explanation question was statistically significant ($\chi^2 = 15.959$ (6), $p = .014$). According to the standardized residuals, the 1st group had significantly more 2-point answers and 4th group had significantly less 2-point answer compared to other groups (**Table 11**).

There were a lot of mistakes by identifying correct choice as many respondents stated that none of the choices of problems fits the operation, e.g., "I wouldn't choose any of them. All are wrong.", "None of them is appropriate because the amounts of chocolate, water in a well, swimming pool and jar are not known, and the above operation does not answer any of the questions in the problem".

As a mixed question, it also required teachers to explain their choice. Many teachers, especially the group with the longest career, did not attempt to explain their choice, whether it was right or wrong, e.g., "b", "d", "b is wrong". Fully correct answers included explanations that "the conditions of tasks a and c do not correspond to the given operation. I can only use the condition in problem b to explain the operation", " I would choose option b. Because when we look at the operation, we see that 3/5 is the content (the whole)–what we have at the moment. And from the content we subtract the "something", which is the water needed to fill the pool. Parts a and c are not appropriate, because those examples would imply that 3/5 is part of something".

Seventeenth question (**Figure 7**) asked respondents to notice possible misconception of pupil and mention a teaching strategy to overcome that misconception. While the question did not specify why students could not calculate the area of parallelogram, the expectation of respondents was to think of possible misconceptions rather than environmental circumstances. Teachers were

Table 12. Statistics of seventeenth questionnaire item

Experimental group	0			1			2		
	N	%	Standard response	N	%	Standard response	N	%	Standard response
1	19	42.2	**2.0**	10	19.2	-1.1	7	18.9	-.9
2	1	2.2	**-2.9**	19	36.5	**2.0**	11	29.7	.8
3	11	24.4	.2	10	19.2	-.6	10	27.0	.5
4	14	31.1	.5	13	25.0	-.3	9	24.3	-.3
Total	45			52			37		

expected to understand students' preconceptions about the representation of height in geometric figures and to point out that the height of the figure does not necessarily have to be "inside".

The Chi-square test showed that there was a statistically significant relationship between PCK and career length (χ^2 = 20.265 (6), p = .002). According to the standardized residuals, pre-service teachers (group 1) had significantly more 0-point responses and early career teachers (group 2) had significantly fewer 0-point responses compared to the other groups (**Table 12**).

While the expectation was for responding teachers to think about misconceptions, some referred to situational circumstances, e.g., "The pupil does not have a ruler and cannot measure the lengths of the top and base", "No data given" (0 pt.). Unfortunately, some respondents mentioned that second picture of the question is wrong because height is outside parallelogram, e.g., "second drawing is incorrect", "the height in the second is drawn incorrectly", "the second height is incorrectly drawn" (0 points).

Some answers indicated the pupil's lack of flexibility, however, it did not directly answer the question, e.g., "The pupil does not realize that one figure can be decomposed into other figures which are simpler and whose areas he can already calculate" (1 point).

Fully correct answers mentioned several possible reasons for pupil's struggle to answer, e.g., "Does not know the formula for the area of a parallelogram. Does not have a height and does not know how to find it. Does not understand that the height may not necessarily be inside the figure" (2 points).

DISCUSSION AND CONCLUSIONS

The aim of this article was to provide an insight into Lithuanian teachers' PCK at the different stages of their career.

Regarding the first research question, "Is there a statistically significant difference of mathematics PCK at different stages of their career (based on career length)?". While there were no statistically significant differences on the overall score of PCK questionnaire, some statistically significant differences for separate questions were found. Most of the statistically significant differences were either showing superiority of the 2nd group (early career teachers) or low achievement in PCK tasks of the 4th group (longest career teachers).

Despite popular assumptions (e.g., Kind, 2009) teaching experience wasn't found to be an important predictor in teachers' capability to answer particular PCK questions. Even more so, teachers with the longest career, in the questions where statistically significant differences between groups were found, were the ones underperforming compared to other groups, not the other way around.

This could be explained with the phenomena called arrested professional development (Ericsson, 2018). This phenomenon explains why work experience alone does not improve professional knowledge and notes that intentional effort is needed to improve past the initial improvement during the first few years in the field (Grant, 2022).

Another possible explanation lies in the limitations of the research instrument. It is particularly important in evaluating PCK not only of long career teachers but especially of early career teachers. In this study, early career teachers were found to score above average compared to the other groups on the questions where statistically significant differences were found between the groups. Therefore, it is possible that a high PCK questionnaire score does not capture the essence of teacher expertise (Stigler & Miller, 2018). Early career teachers may lack knowledge of classroom management and teaching scripts that would enable them to better support their students (Wolff et al., 2021), and this may be where a gap between theoretical PCK and PCK in practice appears. Supported also by other researchers (e.g., Pouta et al., 2021), there is a possibility that early career teachers, while able to better verbally explain pupils' difficulties, may not have the skills to actually adapt their knowledge to real life situations.

As for the second research question, "What particular aspects of PCK appear to be more challenging for teachers at the particular stage of their career?" it seems that choice-based questions are easier for teachers at all stages of their career. There were 7 tasks out of 21 in which there were statistically significant differences between participants in the different phases of their teaching career. Most of these questions are either primary school mathematics or early secondary school mathematics level that teachers were expected to answer easily. Issues in explaining early mathematical concepts such as addition algorithm, basic subtraction, etc. were common for teachers with the longest career. In many of these cases, respondents claimed to not being able to answer some questions because it is out of the grade range that they are currently teaching or declined answering them because of that reason. This could be due to their long experience in teaching higher grades. While it is natural to have more knowledge of the topics that teacher is working with, knowledge of fundamental concepts and conceptual mistakes from earlier grades is crucial because some of the pupil's carry their misconception through the years and they always need teachers' support.

Moreover, the situation can be interpreted as very concerning, as it was found that teachers make the same mistakes as their students. For example, in the thirteenth question (**Figure 6**), some teachers claim that none of the situations fit the operation, which is not correct. This indicates that the teachers have difficulty connecting the context of the word problem with the arithmetic operation, which is often the case with the students. In the seventeenth question (**Figure 7**), some teachers incorrectly stated that the height of the parallelogram must be inside the figure to be valid, which is incorrect.

Future studies could look at the manifestation of PCK in real-life situations, whether through lesson observations or video analysis of lessons. This type of research has been conducted in other countries and important results are available for groups of teachers studied. However, no such studies have yet been conducted with Lithuanian mathematics teachers and more in-depth research is needed. In addition, other teacher characteristics have also been found to have an impact on teaching effectiveness, e.g., their motivation or self-efficacy (Darling-Hammond, 2009; Hattie, 2003; Norton, 2019), so these characteristics could also be investigated in the Lithuanian context alongside PCK.

Also, it was found in this study that differences between groups divided on the length of teachers' career were found in some PCK questionnaire questions. It implies possible reasons behind differences in capability of answering those questions. It has been remarked by researchers (Agathangelou & Charalambous, 2021; Shulman, 1986) that PCK is highly dependent on the content to be taught, thus finding these patterns would benefit the state of the research on PCK.

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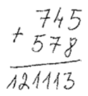
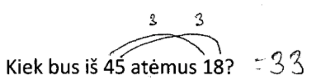
APPENDIX A

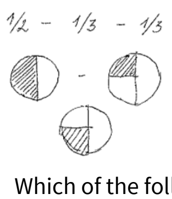
1. 3rd grader carried out the following addition:



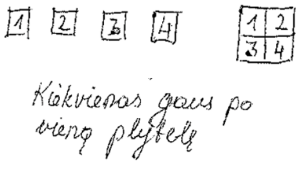
   a. What was his conceptual error and what teaching might have led to that error?
2. Usually Saulė solves math exercises (division) correctly. However, she recently started having difficulties. Below are some exercises solved by Saulė.
   a. If she were to divide 927 by 3:
   b. She will
      i. probably answer correctly if she uses the same procedure.
      ii. probably answer incorrectly if she uses the same procedure.
3. Below there is a task and student's solution to it (text in picture: "How much is 45 minus 18?").



   a. What teaching and strategies might have led to this method? What are the limitations of the method?
4. A pupil tells you that when you multiply two numbers together, the product is always larger than either of the two numbers. How do you respond to the pupil?
5. Below are two tasks for 4th graders. Do you think a 4th grader would consider these two problems equally difficult or one easier than the other? Explain your answer.
   a. Rūta is selling 3 melons už 5€. How much would 9 melons cost?
   b. Giedrius is selling 3 melons už 6€. How much would 9 melons cost?
6. How would you explain to a pupil why *a-(b+c)* and *a-b-c* are equivalent?
7. While you were teaching square roots, you asked your students to order these numbers: $a = 5\sqrt{2}, b = 3\sqrt{5}, c = 5\sqrt{3}$. You noticed, that some of your students gave the response $b = c > a$. What may be the common misconception of those students? What would you do to help them?
8. Imagine you were to introduce pupils to prime and composite numbers. You are wondering, what numbers to choose as examples. Out of the options below, which set of numbers would be best for introducing primes and composites? Explain your choice.
   a. Pirminiai 2, 5, 17. Sudėtiniai 8, 14, 32.
   b. Pirminiai 3, 5, 11. Sudėtiniai 6, 30, 44.
   c. Pirminiai 3, 5, 11. Sudėtiniai 4, 16, 25.
   d. Pirminiai 2, 7, 13. Sudėtiniai 9, 24, 40.
   e. Visi jie vienodai tinkami.
9. Matas was asked to find $\frac{1}{2} - \frac{1}{3}$. He drew the picture below and said, "1 of 2 parts minus 1 of 3 parts is 1 of 3 parts".



   a. Which of the following statements best explains his reasoning?
      i. Matas does not understand partitioning a whole into equal parts.
      ii. Matas understands how to partition a whole into equal parts but does not understand what the parts mean.
      iii. Matas understands how to make parts but does not understand what the whole means.
      iv. Matas is using a fair share approach on a fraction on a fraction subtraction task.
10. Jonas was asked to show and tell how much each person gets if four people share five chocolate bars equally. Jonas's response is shown below (text in picture: "Each will get one bar"):



a. Which statement below best describes John's reasoning.

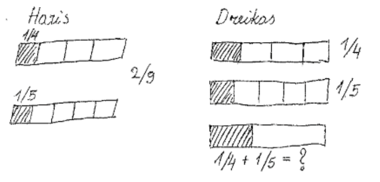
i. Jonas correctly partitions the chocolate bar but does not provide a fraction as an answer.

ii. Jonas correctly partitions the chocolate bar and provides a fraction as an answer.

iii. Jonas incorrectly partitions the chocolate bar and does not provide a fraction as an answer.

iv. Jonas incorrectly partitions the chocolate bar and provides a fraction as an answer.

11. Haris and Dreikas solved for $\frac{1}{4}+\frac{1}{5}$. Their work is shown below.



a. Which statement(s) below appropriately describe these students' reasoning?

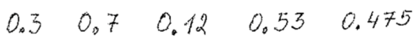
i. Haris referenced the same whole when solving the problem.

ii. Haris had difficulty combining the parts of two different fractions.

iii. Dreikas referenced the same whole when solving the problem.

iv. Dreikas had difficulty combining the parts of two different fractions.

12. In the task of ordering the following numbers from the smallest to the largest 0,53; 0,7; 0,475; 0,12; 0,3, the student provided this as his answer. Write down the presumable student's reasoning and evaluate whether the student's solution is correct.



13. Would you use the situations given below as examples to illustrate "$\frac{3}{5}-\frac{1}{4}=\cdots$"? Explain your reasoning for every situation.

a. $\frac{3}{5}$ of the chocolate bar was used to make chocolate mousse. Gustas and Liza ate $\frac{1}{4}$ of the remaining chocolate. What part of the original chocolate bar did they eat?

b. $\frac{1}{4}$ of well's water is needed to fill up the swimming pool. Today only $\frac{3}{5}$ of the well is filled with water. How much water will remain in the well after the swimming pool is filled?

c. In the art lesson Anna used $\frac{3}{5}$ of her jar of red paint and $\frac{1}{4}$ of her jar of yellow paint. How much paint of each color remains?

d. None of these. Another situation like …

14. Students were asked how many numbers there are between $\frac{3}{8}$ and $\frac{6}{8}$. Some students disagree on what is the correct answer. What answers students with different levels of fraction understanding might have and how you would explain them the correct answer?

15. Invent a word problem for the task of "34,99 · 0,3".

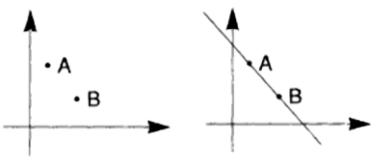
16. As an assignment, students were given a picture of the cone and asked to determine how many different types of cross sections there are. One student draws three circles.

a. Identify the student's limited thinking.

b. How would you help this student understand that there are other different cross-sections?

17. Student claims that he cannot calculate area of parallelogram. What misconceptions he might have?

18. In the picture below, a graph of two points A and B are shown. When a student was asked to give an example of the function that passes through these points, he came up with this solution. When asked, if there is another answer, the student says "no".



a. If you think the student is right, explain why.

b. If you think the student is wrong, how many functions which satisfy the condition can you find? Explain.

19. Jonas is solving algebraic expressions. Below is his solution to several mathematical tasks. If Jonas's solution is incorrect, how would you explain his mistake?

    1. $6(1+4x)+2 = 6(5x)+2 = 30x+2$

    2. $7+5(2+3x) = 7+5(5x) = 7+25x$

20. The task is $\frac{(x+4)}{x+1} = \frac{2x+8}{3x}$. A student makes the following explanation and solution for the problem above: "(2x + 8) is two times (x + 4). Then 3x should be two times of (x + 1). So, 2 (x + 1) = 3x, then 2x + 2 = 3x and x = 2." Which of the following statements are true about this solution?

    a. The strategy used by the student is completely wrong.

    b. The student made proportion reverse.

    c. The result composes the solution set.

    d. Although the strategy is true, it gives a missing solution.

    e. The student made a calculation error but reached the correct answer by chance.

21. Students were asked the following question "There are ten families in the neighborhood and the average number of kids per family is 2.3. One family with 5 children leaves the neighborhood. What is the average number of children per family now?" One student says, "I don't know how many children in each family; therefore, I cannot answer." How would you help him?