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Review Article

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From the whole to its parts – A systematic analysis of affordances for learning part-whole-relations in digital apps

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ABSTRACT

Understanding part-whole relations is crucial in early mathematics education. However, both analogue and digital learning environments often lack systematic approaches to foster part-whole understanding. With the rising popularity of educational apps, it is essential to evaluate how they implement learning of part-whole relations. Accordingly, this review aims to evaluate whether educational math apps provide a systematic approach to learn part-whole relations. Therefore, we first developed a framework for evaluating apps with a focus on opportunities for learning part-whole relations. Second, we applied this framework to evaluate n = 18 apps. Results indicated that none of the reviewed apps implemented a systematic approach to learn part-whole relations from hands-on to more abstract compositions/decompositions including number triples up to ten. In contrast, the automating of number compositions/decompositions is most frequently targeted. These findings underscore the importance of selecting educational apps carefully and integrating principles from effective learning environments and digital learning research into future app design.

Keywords: part-whole relations, compositions/decompositions, learning environments, mathematical education

INTRODUCTION

Teacher (showing an egg carton with six eggs):

"How many eggs are in there?"

Student (Sophie, 5 years):

"One, two, three (counts by one-to-one mapping then looks at the other three eggs), hm, I think there are six eggs!"

In this example, the student quickly grasps the cardinality of six eggs by applying part-whole understanding. This means she knows that a quantity of six eggs can be composed of different subsets such as three and three. Interestingly, such part-whole understanding[1] was observed to be a significant predictor of later mathematical development (Kilpatrick et al., 2001; Kullberg & Björklund, 2020). Thus, reflecting that "probably the major conceptual achievement of the early school years is the interpretation of numbers in terms of part and whole (relations)" (Resnick, 1983, p. 114). Accordingly, part-whole understanding is one of the most fundamental milestones in early mathematical education (Hunting, 2003). Due to its relevance, mathematics education is highly interested in how to facilitate the development of part-whole understanding by investigating learning environments in elementary school mathematics. In this vein, Lenz and Wittmann (2023) recently evaluated which opportunities for learning part-whole relations can be found in current mathematics textbooks by reviewing twelve different 1st grade textbooks used in the state of Baden-Württemberg, Germany. Following a qualitative content analysis, they concluded that only four of these textbooks provide a systematic approach to learn part-whole relations from the beginning.

[1] Considering part-whole relations, different terminologies can be found in the literature. In this article, when referring to part-whole relations between numbers as Resnick explained them, we will basically use the term part-whole relations and, when referring to the children's perspective and insights, we will use the term part-whole understanding.

Importantly, however, as the number as well as the use of educational apps is steadily increasing (Statista, 2024), it is worthwhile evaluating whether the results from this textbook analysis generalize to digital apps. As such, the aim of this article is to answer the question whether current educational apps provide a systematic approach to learn part-whole relations.

Therefore, we will first introduce the concept of part-whole relations, how it develops and, based on a literature review, collate principles relevant for learning environments[2] that promote part-whole understanding. Additionally, we will elaborate on how the potential of educational apps can be evaluated referring to the framework developed by Outhwaite et al. (2023b).

Based on this, we will then

(1) propose a new framework for evaluating educational apps with respect to opportunities for learning part-whole relations and

(2) apply this framework to systematically evaluate whether as well as in which ways part-whole understanding is promoted by current educational apps.

The focus of this systematic evaluation will be on apps available in German reflecting expertise with the educational system and affiliations of the authors. Nevertheless, as the market for educational apps for Germany develops not substantially different than all over the world (Baule et al., 2024), our methodology may well be valid to evaluate educational apps in international contexts. To achieve this, we developed the framework as a tool for practitioners to evaluate opportunities for learning part-whole relations in any existing app. Thus, our article contributes to extend the evaluation of mathematical apps “beyond academic circles into the teaching profession” as suggested by Larkin et al. (2019, p. 59).

THEORETICAL BACKGROUND

In her seminal article on early mathematical development, Resnick (1983) argues that developing part-whole understanding enables children to perceive numbers as compositions/decompositions of other numbers. In other words, children need to understand that all numbers can be composed of/decomposed into smaller numbers. In turn, this enables them to understand that, for instance, 6 (as a whole) can be composed from 2 and 4 (i.e., as its parts). Building on that, children can solve problems such as, for example, $2 + 4 = ?$; $6 - 2 = ?$; $6 - 4 = ?$; $2 + ? = 6$, or $? + 4 = 6$. Thereby, more elaborate solution strategies drawing on number relations become possible. In the following, the development of part-whole relations is reviewed before collating principles for learning part-whole relations based on a review of the respective literature.

From Protoquantitative Knowledge to Numerical Part-Whole Understanding

Key components of children’s part-whole understanding is their awareness of number compositions/decompositions as well as their ability to interpret relations between the whole and its parts. Such part-whole understanding is thought to develop based on children’s so-called informal protoquantitative knowledge (Resnick, 1983, 1989, 1992), which is typically acquired before entering school. This includes comparing quantities represented through real objects and operationalizing them with comparison words (such as more, less, and much). Furthermore, protoquantitative knowledge includes understanding increase/decrease processes within quantities, for example, through adding or removing objects (Resnick, 1992).

In particular, it is assumed that young children initially learn about part-whole relations before being able to quantify the respective magnitudes and relations numerically. However, only when they become able to quantify magnitudes by corresponding numbers (for instance through counting) and apply protoquantitative knowledge to these numbers, they develop numerical part-whole understanding. They then start to think about numbers as compositions/decompositions of other numbers (Resnick, 1989). Furthermore, children understand that numbers are additive and, thus, can interpret relations between triples of numbers (with one being composed of the other two, for example $6 = 4 + 2$: Resnick, 1983).

In accordance with Resnick’s (1983) descriptions, models of numerical development, such as the ones by Krajewski and colleagues (Krajewski & Schneider, 2009; Schneider et al., 2013) or Fritz and Ricken (Fritz & Ricken, 2011; Ricken 2009), specify the progression from protoquantitative to numerical part-whole understanding to occur on different levels. Both models consider informal protoquantitative knowledge as crucial for further mathematical development (Weißhaupt & Peucker, 2009) and underline the importance of children’s understanding of numbers in terms of part-whole relations (Fritz et al., 2014). From this, it becomes clear that understanding part-whole relations is seen as central to learning arithmetic (see also Kilpatrick et al., 2001) as it enables children, for example, to understand additional mathematical concepts such as the commutative or the complement principle (Ekdahl, 2019), but also additive compositions (Sarama & Clements, 2009). Based on these, more advanced arithmetic skills such as flexible calculation strategies can be developed and mathematical problem-solving becomes possible (Resnick, 1983, 1989; Sarama & Clements, 2009).

Importantly, it was observed that students who mastered part-whole understanding also performed better in their further mathematical development (Ennemoser & Krajewski, 2007). In contrast, students who have not yet understood part-whole relations tended to use inflexible counting strategies and, thus, tended to develop mathematical learning difficulties more frequently (Gersten et al., 2005; Häsel-Weide, 2016). Therefore, mastering part-whole relations seems central in early mathematics education (Gaidoschik, 2007; Young-Loveridge, 2002).

[2] The term learning environment in general relates to various physical spaces, contexts and cultural settings where students acquire knowledge (Bates, 2015). In this article, using the term learning environment, we are referring explicitly to learning environments that promote part-whole understanding.

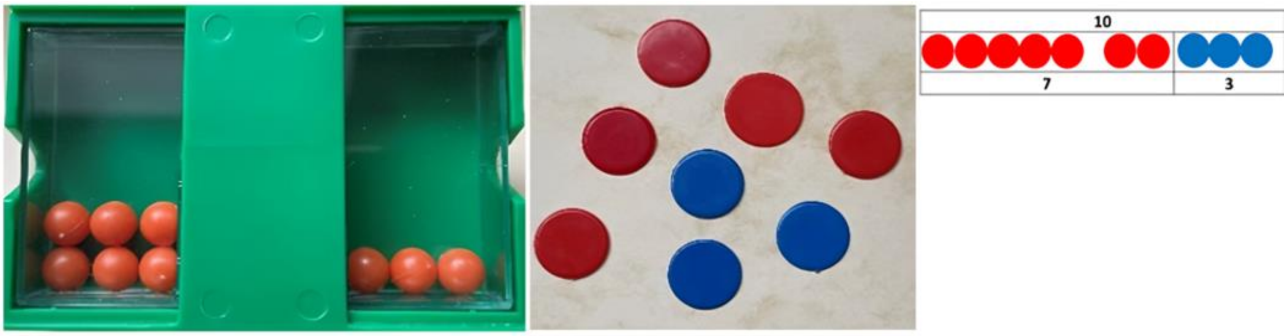

Figure 1. Left: Shaking box; Middle: Reversible plates; & Right: Structured notation of decompositions found (Source: Authors' own elaboration)

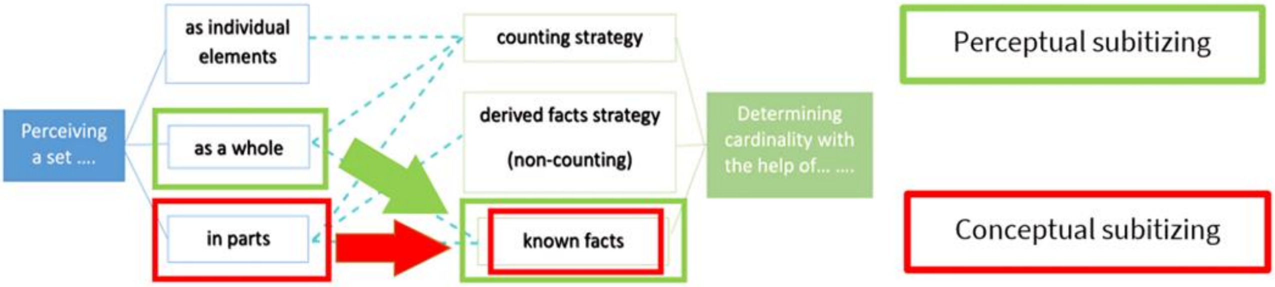

Figure 2. Perceptual and conceptual subitizing in the process of perceiving sets of elements and determining their cardinality (Source: Authors' own elaboration, adapted from Sprenger & Benz, 2020)

Accordingly, it is important to think about how learning environments that aim at supporting the acquisition of part-whole relations might look like. In other words, it is important to ask which principles should be considered when designing or evaluating such learning environments. However, to the best of our knowledge, there is currently no theoretical approach that systematically collects such principles. Therefore, we conducted a literature review on the acquisition of part-whole understanding according to Resnick's (1983) definition (e.g., Björklund et al., 2021; Ekdahl, 2019; Fischer, 1990; Hunting, 2003; Kullberg & Björklund, 2020; Langhorst et al., 2012; Sophian & McCorgray, 1994; Sprenger & Benz, 2020; Wartha et al., 2019; Young-Loveridge, 2002), to collate principles for learning environments that aim to promote part-whole understanding.

Principles for Learning Part-Whole Relations

To enable children to understand number compositions/decompositions as well as to interpret relations between the whole and its parts, learning environments need to provide specific tasks (Björklund et al., 2021). These tasks should encourage children to work out compositions/decompositions on their own on an enactive level (Björklund et al., 2021). Additionally, they should help children acquire part-whole understanding on an iconic level (Kullberg & Björklund, 2020), as well as automate and apply these decompositions on a symbolic level (Padberg & Benz, 2021; Wartha et al., 2019).

Learning environments should progress systematically from hands-on to more abstract representations (Gersten et al., 2005) to support children's development from protoquantitative to numerical part-whole understanding. This means they should provide opportunities for different levels of actions: enactive, iconic, and symbolic (Bruner et al., 1971). Hence, to include these actions when separating the whole from its parts and vice versa, children have to be encouraged to physically manipulate concrete or iconic quantities. For example, they can compose/decompose all quantities in the number range up to ten by acting with material such as shaking boxes or reversible plates (Padberg & Benz, 2021, see **Figure 1**, left and middle). Furthermore, writing down all compositions/decompositions found in a structured table using symbolic digits may further motivate them to systematically work out decompositions of the whole by dividing it into different subsets (see **Figure 1**, right) (Wartha et al., 2019).

Furthermore, it is important to provide tasks that initiate a step-by-step development of number compositions/decompositions and facilitate a non-counting use of visual materials to support children's acquisition of part-whole relations (Padberg & Benz, 2011; Wartha et al., 2019). This might be achieved, for example, by providing structured visual representations of quantities (e.g., the ten-strip). Moreover, recent evidence suggests an association between children's visual structuring ability, which reflects their ability to discern and use structures in visual representations of quantities, and their part-whole understanding (Lüken, 2012; Young-Loveridge, 2002). In this context, two different processes can be distinguished: perceptual as well as conceptual subitizing (Sprenger & Benz, 2020, see **Figure 2**). Perceptual subitizing means the very fast and more or less automatic perception of small quantities, up to three or four elements (Sarama & Clements, 2009). In contrast, conceptual subitizing refers to the process of breaking down a set into subsets/in parts to infer its quantity. For instance, in **Figure 3** it is possible to infer the quantity of six in parts (5 + 1) (Sprenger & Benz, 2020). In line with this, Björklund et al. (2020) observed that using a systematic approach to facilitate children's part-whole understanding, based on tasks using structured visual representations, led to a significant increase in children's use of strategies. These strategies included perceiving a set as a whole/in parts and determining the respective cardinality based on known facts.

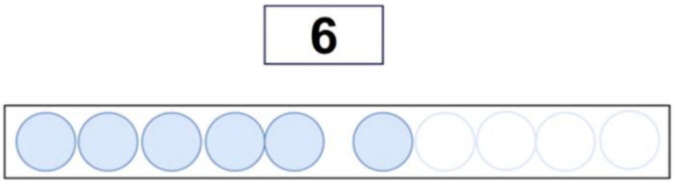

Figure 3. A ten-strip as an example of a structured visual representation (Source: Authors' own elaboration)

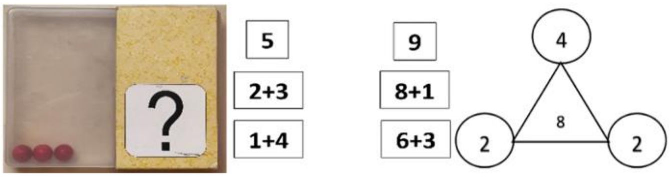

Figure 4. Left: Shaking box with one part hidden; Middle: Matching number triples; & Right: Solving number triangles (Source: Authors' own elaboration)

Accordingly, to incorporate an iconic level and, thus, to support the acquisition of part-whole understanding, learning environments not only have to use any visualizations, but structured visual representations of quantities. These representations should enable children to perceive both, the whole as well as its parts (Kullberg & Björklund, 2020). An example of such a structured visualization is the ten-strip, that facilitates children's experience of numbers as patterns of subsets (see **Figure 3**). Besides, to further discourage counting based strategies and encourage the use of conceptual subitizing to support the formation of structured mental representations (Lüken, 2012), these structured visual representations might be presented briefly.

Additionally, to also include the symbolic level and, thus, to focus on automating number compositions/decompositions, learning environments need to provide tasks that show only one part of the respective number triple. For example, when one part of the whole in the shaking box is hidden (see **Figure 4**, left), children need to produce results automatically to solve the task (Padberg & Benz, 2011) by drawing on their mental representations of structured visualizations of quantities (Lüken, 2012). Furthermore, to focus on applying number decompositions, children have to connect these symbolic number triples with addition and subtraction tasks, for instance, by matching number triples to the respective task or solving number triangles (see **Figure 4**, middle and right) (Padberg & Benz, 2011).

To summarize, learning environments that aim to promote part-whole understanding have to provide a structured step-by-step development of number compositions/decompositions at the levels of non-symbolic/iconic quantities as well as symbolic digits (Kullberg & Björklund, 2020). In particular, learning environments have to encourage children to systematically work out compositions/decompositions of sets on an enactive and iconic level, to acquire them on an iconic level, as well as to automate and apply number compositions/decompositions on an iconic and symbolic level (Björklund et al., 2020; Kullberg & Björklund, 2020; Padberg & Benz, 2011; Wartha et al., 2019).

To conclude, based on these principles, the following questions might be derived to evaluate learning environments:

- systematically work out compositions/decompositions of sets:
  - Are there tasks that encourage children to make their own number compositions/decompositions on an enactive and/or iconic level (e.g., by using shaking boxes, reversible plates, plug cubes, static finger patterns…?) and to write down their results with iconic and/or symbolic notation?
- acquire number compositions/decompositions:
  - conceptual subitizing: Is conceptual subitizing possible (e.g., by using structured visualizations such as the ten-strip)?
  - systematic compositions/decompositions: Is there a decomposition of all numbers in the number range up to 10, considering all triples and using structured visual representations?
  - brief visual presentation: Is it possible to present visual sets briefly?
- automate number compositions/decompositions:
  - Are there tasks showing only one part of the specific number decomposition on an iconic or symbolic level?
- apply number compositions/decompositions:
  - Do children have to use number triples to solve different tasks (e.g., addition or subtraction tasks)?

How these principles/questions might be considered when evaluating the educational value of mathematical apps, for example by incorporating them within existing frameworks to evaluate educational apps, will be shown in the following section.

METHODOLOGY

How to Evaluate the Educational Value of Mathematical Apps

In 2022, there were more than 455,000 educational apps available from Google Play and/-or the Apple App store, which was almost twice as many as in 2014 (Wylie, 2023). This increasing number underlines that educational apps are becoming more and more important in children's education, including primary school children (Larkin et al., 2019). Thus, they are becoming increasingly important in formal and informal teaching and learning environments all over the world (Kim et al., 2021; Kolak et al., 2021). Due to the sheer number of educational apps, as well as the fact that many apps on the market seem to be of low-quality (Larkin & Calder, 2016), it is important for teachers to evaluate whether a particular app is suitable for their intended learning goal (Hirsh-Pasek et al., 2015).

As this seems to be quite a complex endeavor to be done, it is of crucial importance to use tools, such as explicitly designed frameworks (Hirsh-Pasek et al., 2015; Walter & Schwätzer, 2023) to support these evaluation processes (Ladel & Kortenkamp, 2016). Reviewing national as well as international literature, it becomes obvious that there are different frameworks (e.g., Highfield & Goodwin, 2013; Hirsh-Pasek et al., 2015; Larkin et al., 2019; Outhwaite et al., 2023b; Walter & Schwätzer, 2023) that might be applicable to evaluate mathematical apps. Nevertheless, to our knowledge, there is no framework focusing explicitly on evaluating whether an educational app is useful to promote part-whole understanding.

Therefore, we will develop a framework that allows us to evaluate whether and how part-whole understanding is promoted by a particular app based on the framework suggested by Outhwaite et al. (2023b). As the so-called artifact-centric activity theory (ACAT), is considered to be remarkably useful as a theoretical underpinning in this context (Larkin et al., 2019), we will embed Outhwaite et al.'s (2023b) framework within the context of ACAT.

To stimulate sustainable learning processes when using apps or other digital artifacts, children need to actively interact with the digital medium (Hirsh-Pasek et al., 2015). Moreover, these interaction processes should be embedded within different contextual factors, such as, for example, classroom rules, user-dependent prior knowledge or the user group (Larkin et al., 2019). ACAT helps to understand these interactive processes by considering different components of these processes within three sections along a main axis reflecting children's internal adoption and external demonstration of the intended object (see **Figure 5**) (Ladel & Kortenkamp, 2011, 2016; Larkin et al., 2019). In our case, this main axis would comprise part-whole related activities to reach the intended mathematical learning goal.

The upper right triangle comprises different design principles as well as the incorporation of *feedback* within the artifact, whereas the lower left triangle pertains to the use of artifacts in different social contexts (Larkin et al., 2019). Accordingly, we looked for a framework to fit ACAT. In doing so, we considered Outhwaite et al.'s (2023b) framework, which explicitly focuses on analyzing the educational value of mathematical apps, to be very useful as a starting point to further develop a new framework that focuses explicitly on promoting part-whole understanding.

One reason for this decision was that Outhwaite et al. (2023b) considered and incorporated several existing frameworks and approaches for the evaluation of educational apps when developing her framework. As such, she integrated bottom-up frameworks that draw on and evaluate different app features such as *feedback* or *instructions* (Herodotou, 2021). Moreover, she also considered top-down frameworks incorporating cognitive theories of development and learning (Hirsh-Pasek et al., 2015). Thereby, she derived three categories for evaluation: *type of app*, *mathematical content*, and *design features* to classify educational apps (Outhwaite et al. 2023b).

Type of app separates five different types of apps which differ in their targeted user activities and intended learning goals: *practice-based* apps*, constructive* apps*, productive* apps*, game-based* apps, and *parent-based* apps (Outhwaite et al., 2023b). Outhwaite et al.'s (2023b) second category reflects the *mathematical content* of the apps addressing four areas of mathematical development: number representation and relationships, counting, arithmetic and shape, patterns, and measurement. In Outhwaite et al.'s (2023b) framework these categories can be scored on a categorical scale (present or not present). Finally, with respect to *design features*, Outhwaite et al. (2023b) derived the following five app *design features*: *feedback*, *levelling*, *social interaction, task instructions*, and *meaningful learning and problem-solving*. Again, these categories can be scored on a categorical scale (present or not present). To better understand what is meant by these *design features*, several impulse questions are provided (e.g., What kind of *feedback* is provided: motivational, explanatory, or both?).

Moreover, these three categories correspond and can be matched directly to the three sections of the ACAT: *Type of app* might be embedded within the lower left triangle, the *mathematical content* can be integrated into the main axis, and the *design features* reflect the upper right triangle (see **Figure 5**, Source: Ladel & Kortenkamp, 2011). Consequently, as it will be outlined in more detail in the following, we adopted and adapted Outhwaite et al.'s (2023b) framework to create a new framework for evaluating whether current apps provide a systematic approach to learn part-whole relations.

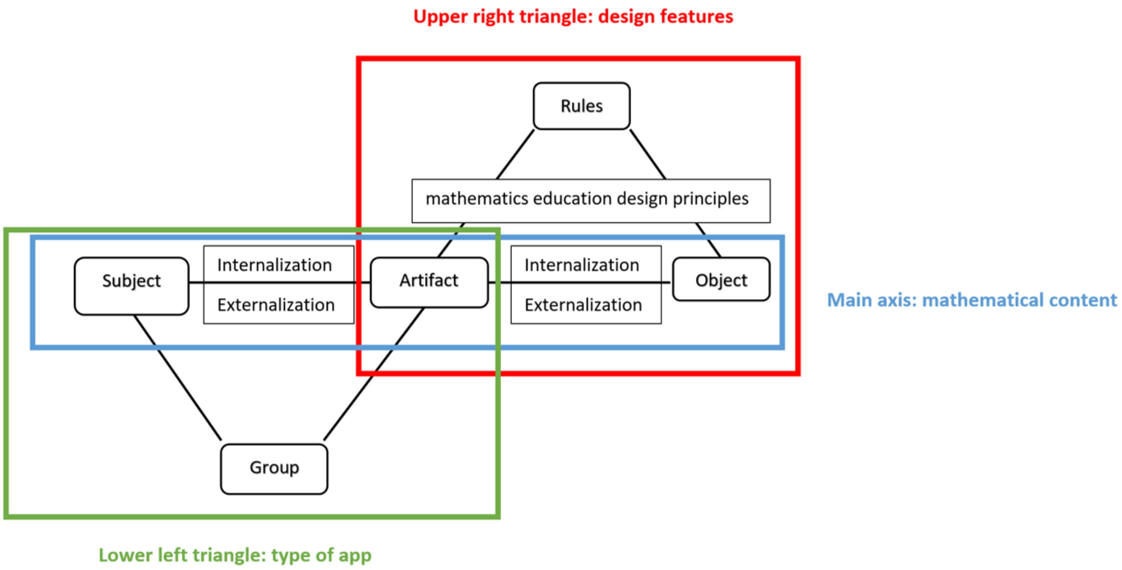

Figure 5. Outhwaite et al.'s (2023a) three categories within the ACAT (Source: Authors' own elaboration, Adapted from Ladel & Kortenkamp, 2011, p. 3)

How to Evaluate a Systematic Approach for Learning Part-Whole Relations in Educational Apps

To evaluate whether current educational apps provide a systematic approach to learn part-whole relations, Outhwaite et al.'s (2023b) three categories *type of app, mathematical content* and *design features* were systematically revised and specifically adapted with the aim to create a new framework that allows evaluating apps with a specific focus on learning part-whole relations (see **Table 1**).

Table 1. Framework for evaluating whether current apps provide a systematic approach to learn part-whole relations (Example: Funexpected Mathe Lernspiele–App)

Type of app					
Practice-based (these apps support learning processes mainly through targeted repetition tasks that might be solved individually by the child)	Constructive (these apps foster an active exploration of new mathematical knowledge)		Productive (these apps encourage children to produce and present their own solutions)	Game-based (these apps incorporate plots and other gamification elements)	Parent-based (these apps provide ideas and further information providing ideas for topic-related actions)
				x	
Mathematical content					
Systematically work out compositions/ decompositions of sets	Acquire number compositions/decompositions			Automate number compositions/ decompositions	Apply number compositions/ decompositions
Are there tasks that encourage children to make their own number decompositions on an enactive and/or iconic level (e.g., by using shaking boxes, reversible plates, plug cubes, static finger patterns …?) and to write down their results with iconic and/or symbolic notation?	conceptual subitizing: Is conceptual subitizing possible (e.g., by using structured visualizations such as the ten-strip)?	systematic compositions/ decompositions: Is there a decomposition of all numbers in the number range up to 10, considering all triples and using structured visual representations?	brief visual presentation: Is it possible to present visual sets briefly?	Are there tasks showing only one part of the specific number decomposition on an iconic or symbolic level?	Do children have to use number triples to solve different tasks, e.g., addition or subtraction tasks?
App design features					
Feedback	Levelling	Social interaction	Task Instructions	Meaningful learning and solving problems	
Is explanatory/corrective *feedback* (e.g., "that's not quite right, because …") and/ or motivational *feedback* (e.g., "well done", "carry on!") provided?	Is individual *levelling* possible (e.g., through different levels of difficulty that can be personalized)?	Is *social interaction* encouraged (e.g., through an in-app character communicating with the child)?	Can *task instructions* be repeated by the child?	Are mathematical skills practiced within a real-life context?	

Regarding the *type of app*, Kay and Kwak (2018) underline that the *type of app* chosen should be determined based on the intended learning goal or on the intended student activity and, thus, whether the focus is, for instance, on practicing or on constructing new mathematical knowledge. Since there are numerous different apps available on the market differing in many aspects such as user-activities or learning goals, it is considered useful for teachers to be made aware of these different types (Kay & Kwak, 2018; Kim et al., 2021). Hence, we fully adopted Outhwaite et al.'s (2023b) first category *type of app* distinguishing between *practice-based, constructive, productive, game-based* and *parent-based* apps. To make it easier for teachers to select the appropriate option and to provide contextual information, we added a short description for each *type of app*, as follows (Outhwaite et al., 2023b):

- *practice-based:* Practice-based apps are designed to support learning processes mainly through targeted repetition tasks that might be solved individually by the child.
- *constructive:* Constructive apps foster an active exploration of new mathematical knowledge.
- *productive:* Productive apps encourage children to produce and present their own solutions.
- *game-based:* Compared to practice-based apps, game-based apps additionally incorporate plots and other gamification elements.
- *parent-based:* Parent-based apps provide ideas and further information for parents or other caretakers providing ideas for topic-related actions.

Accordingly, when evaluating an app using our framework, the most suitable option should be ticked. Obviously, as these types of apps might not be exclusive, it is possible to select several options. Nevertheless, including this category in our framework helps to make teachers aware of the different types and the associated student activities as well as the intended learning goals.

When evaluating educational apps, it is important to consider current models of mathematical development as the theoretical underpinnings of the respective apps (Hirsh-Pasek et al., 2015). Thereby, as our idea was to develop a framework that allows us to explicitly evaluate the aspect of learning part-whole relations, Outhwaite et al.'s (2023b) second category *mathematical content* was adapted, as follows. Instead of addressing Outhwaite et al.'s (2023b) four areas of mathematical development, we included the questions derived above to explicitly address whether learning environments promote part-whole understanding. To reflect whether a respective app is useful to learn part-whole relations, we added a three-point scale reflecting whether the respective principle is considered to be present (black filling), given sometimes (grey filling) or not present (no filling) (see the example of the app *Funexpected Mathe Lernspiele* in **Table 1**). Ergo, when evaluating the *mathematical content* of an app using our framework, it is necessary to go through and solve all tasks of the app that might foster part-whole related competencies. In addition, it is essential to reflect and answer each of these questions by ticking the most appropriate option. In this context, the respective principle is considered to be present (black filling), when it occurs in all of the evaluated tasks, for example, when all part-whole related tasks use structured visual representations. Likewise, the respective principle is considered to be given sometimes (grey filling), when, for example, structured visual representations occur in some, but not in all selected tasks. Similarly, the respective principle is considered to be not present (no filling), for example, when structured visual representations are not used at all in the selected tasks.

Furthermore, we adopted Outhwaite et al.'s (2023b) five *design features* for our framework: *feedback, levelling, social interaction, task instructions, meaningful learning and solving problems*. To make it easier for teachers to select the appropriate option and to provide contextual information, we added a specific question for each design feature, as follows:

- *feedback:* Is explanatory/corrective *feedback* (e.g., "That's not quite right, because …") and/ or motivational *feedback* (e.g., "well done", "carry on!") provided?
- *levelling:* Is individual *levelling* possible (e.g., through different levels of difficulty that can be personalized)?
- *social interaction:* Is *social interaction* encouraged (e.g., through an in-app character communicating with the child)?
- *task instructions:* Can *task instructions* be repeated by the child?
- *meaningful learning and solving problems:* Are mathematical skills practiced within a real-life context?

To evaluate whether the respective design feature is present or not, we, again, applied the three-point scale described above distinguishing whether a feature is present, given sometimes or not present. Similarly, the point on *task instructions*, for example, shall only be considered to be present, when all *task instructions* of the selected tasks can be repeated by the child. Comparably, they need to be considered to be given sometimes, when only some *task instructions* might be repeated by the child or to be not present, when no *task instruction* might be repeated by the child.

Again, as these *design features* might not be exclusive, it is possible to select more than one option. Nevertheless, including this category in our framework helps to sensitize teachers to the different *design features* and associated student activities as well as the intended learning goals.

In sum, applying the second category of our framework should help teachers to evaluate whether as well as in which ways part-whole relations may be promoted by a given app. Moreover, applying the first and the third category helps teachers to become aware of the intended student activities when using the app. From this, consequences concerning the planning and realization of lessons may be derived (e.g., Is it possible to level tasks and, thus, to provide individual learning paths? Do I need to give all the app instructions or is it possible to let the children work on some tasks on their own? Is the app intended to be used for practicing or rather for constructing new knowledge?). Considering the example of the app *Funexpected Mathe Lernspiele* (see **Table 1**), it becomes obvious that the app does not promote possibilities to systematically work out compositions/decompositions of sets. In particular, children are mainly encouraged to apply number compositions/decompositions. This means that the app is only suitable to promote part-whole understanding on the symbolic level. Accordingly, the app does not provide a systematic approach to learn part-whole relations as described above. Furthermore, an acquisition and an automation of number compositions/decompositions is provided only partially because the respective categories (e.g., conceptual subitizing or brief visual presentation) are considered to be given either sometimes or not present at all (see **Table 1**).

In the following, we will first describe the app selection process before depicting the results of our evaluation based on our new framework.

Do Current Educational Apps Provide a Systematic Approach to Learn Part-Whole Relations?

According to Larkin et al. (2019), two different approaches to evaluate educational apps can be distinguished: large scale app reviews or focused evaluations of small, pre-selected app samples. However, a critical limitation of the first approach is the huge number of apps on the market that would need to be looked at. Furthermore, large-scale evaluations often lack a theoretical or methodological foundation (Baccaglini-Frank & Maracci, 2015). Hence, we decided to conduct an evaluation of a smaller, pre-selected sample of apps. Additionally, by applying our new framework, we, thus, implemented a theoretical and methodological foundation in a way that might be adopted by practitioners worldwide.

Thus, as a starting point for our evaluation it was necessary to identify apps to be considered for systematic evaluation, and to define inclusion as well as exclusion criteria. As regards inclusion criteria, we defined the following: A current app is classified as an interactive software that can be used on a smartphone or tablet device and that is currently available free-of-charge, or which at least provides a free-trial version. An *educational* app in this context provides tasks that might foster part-whole understanding (e.g., tasks that encourage children to work out compositions/decompositions on their own).

As regards exclusion criteria, we excluded paid apps for several reasons. First of all, research indicates that free apps tend to have survival rates that are twice as high as paid apps (Lee & Raghu, 2014). Second, paid apps cannot be considered to be better per se (Kolak et al., 2021). Additionally, paid educational apps are more expensive than other apps, which may lead teachers to choose free apps (Dubé et al., 2020). Moreover, paid apps only make up a small part of the market (Falmouth, 2020; Paulsen & Klöß, 2023). Thus, we consider free-of-charge apps are more likely to be selected by teachers.

Accordingly, we systematically searched the German apple and android app store using various topic-related search terms including: Teil-Ganzes-Verständnis, Zahlzerlegung, Blitzblick, part-whole, verliebte Zahlen, Mengen Kindergarten, Zahlen Kindergarten, Rechnen Kindergarten, Zählen Kindergarten, Mengen 1. Klasse, Rechnen 1. Klasse and Mathematik 1. Klasse. This process led to the identification of n = 58 apps. Subsequently, n = 40 of these apps were excluded for the following reasons: n = 14 (duplicate search hits), n = 5 (not available in German), n = 1 (app not available anymore), n = 9 (part-whole relations were not addressed at all), and n = 11 (apps did not offer a free trial version). Consequently, n = 18 apps were considered in our systematic analysis concerning part-whole understanding (for more information see supplementary material).

RESULTS

To evaluate the respective apps, we installed all selected apps on a smartphone or tablet device. We then went through these apps twice and solved all tasks that might foster part-whole understanding. Subsequently, we applied our framework and systematically evaluated: the *type of app*, the *mathematical content* and app *design features*. Concerning the *mathematical content* of the apps, it can be concluded that none of the evaluated apps provide a systematic approach to learn part-whole relations. The results are presented in more detail below.

Type of App

Closer evaluation of the n = 18 apps revealed that n = 12 apps can be classified as practice-based, whereas n = 6 were categorized as game-based. None of them were categorized as constructive, productive, or parent-based (see **Appendix A**).

Mathematical Content

Evaluating the *mathematical content* of the respective apps is key to answer the question whether current educational apps provide a systematic approach to learn part-whole relations. Accordingly, the results are described separately for each of the four categories considering illustrative examples.

Systematically Work Out Compositions/Decompositions of Sets

Our evaluation revealed that n = 0 of the reviewed apps offer tasks that encourage students to work out compositions of/decompositions of sets on an enactive or iconic level, for example by using shaking boxes or reversible plates.

Acquire Number Compositions/Decompositions

Conceptual subitizing

Overall, n = 2 apps systematically use structured visual representations that enable students to practice conceptual subitizing (e.g., such as shown in **Figure 6**, middle). Another n = 7 apps only sometimes use structured visualizations which means that in some cases students may count rather than subitize the respective subset of the whole (e.g., such as shown in **Figure 6**, left). Additionally, n = 9 apps do not provide structured visual representations at all, as they mainly use symbolic notations, for example, such as shown in **Figure 6**, right.

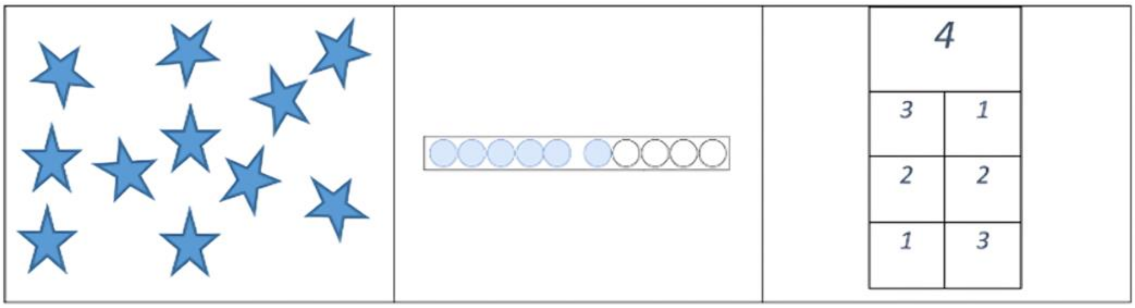

Figure 6. Left: Unstructured visual representation; Middle: Structured visual representation; & Right: No visual representation (Source: Authors' own elaboration)

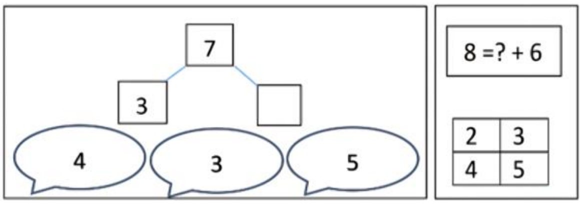

Figure 7. Left: Example of a number triple showing only one part & Right: Children have to use known number triples to solve the task (Source: Authors' own elaboration)

Systematic compositions/decompositions

Our evaluation revealed that n = 1 app provides number decomposition tasks for the numbers 5, 6, and 10. This means that this respective app makes use of some number decomposition tasks in the number range up to 10. In total, n = 17 apps do not use number decomposition tasks that represent the respective subsets by structured visual representations. This result seems surprising, as, n = 7 apps were evaluated to sometimes use structured visualizations. However, a closer look indicated that these structured visualizations are mainly used when children have to connect given quantities with the corresponding digit.

Brief visual presentation

Only n = 1 app provides the opportunity to present visual representations briefly so that students are encouraged to identify the respective cardinality without counting. In turn, this means that n = 17 apps do not provide the possibility to choose between different presentation durations of their stimuli.

Automate Number Compositions/Decompositions

Overall, n = 7 apps provide some tasks that show only parts of the respective number triples at least on the symbolic level (see **Figure 7**, left), whereas n = 11 apps do not use this mode of presentation.

Apply Number Compositions/Decompositions

In n = 9 apps children are encouraged to use known number triples to solve other numerical tasks, like arithmetic problems (see **Figure 7**, right). In contrast, n = 9 apps do not provide tasks that encourage students to explicitly connect number triples to the respective arithmetic problem (see **Figure 8**).

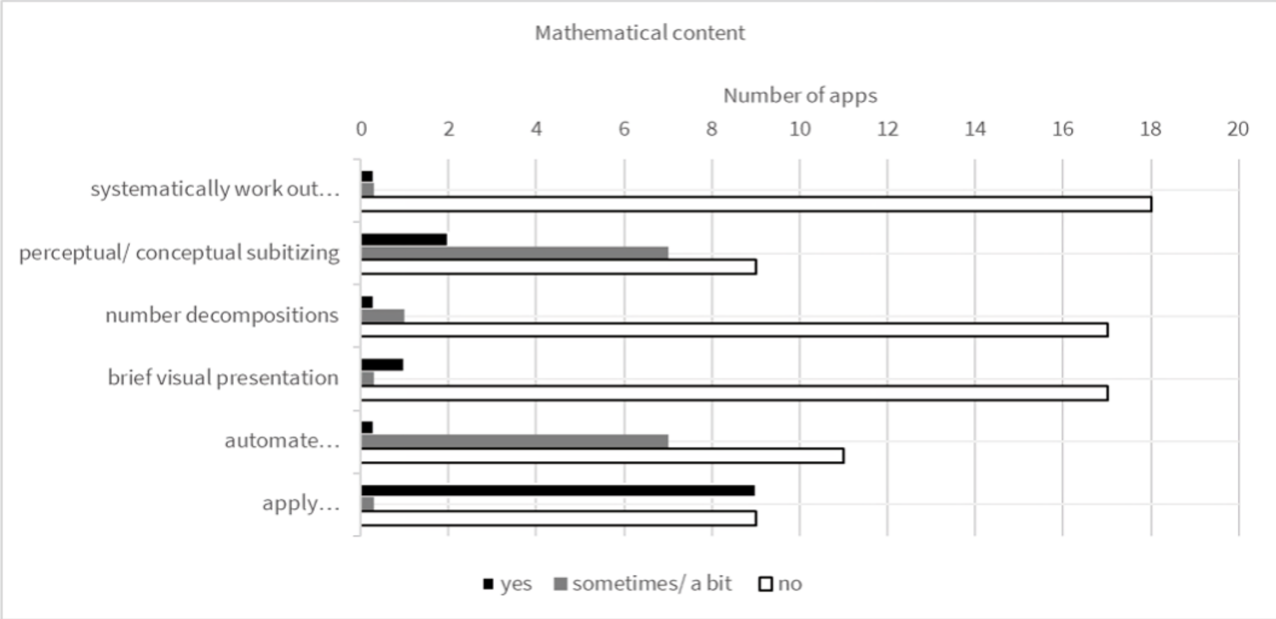

Figure 8. Mathematical content–Bar chart (Source: Authors' own elaboration)

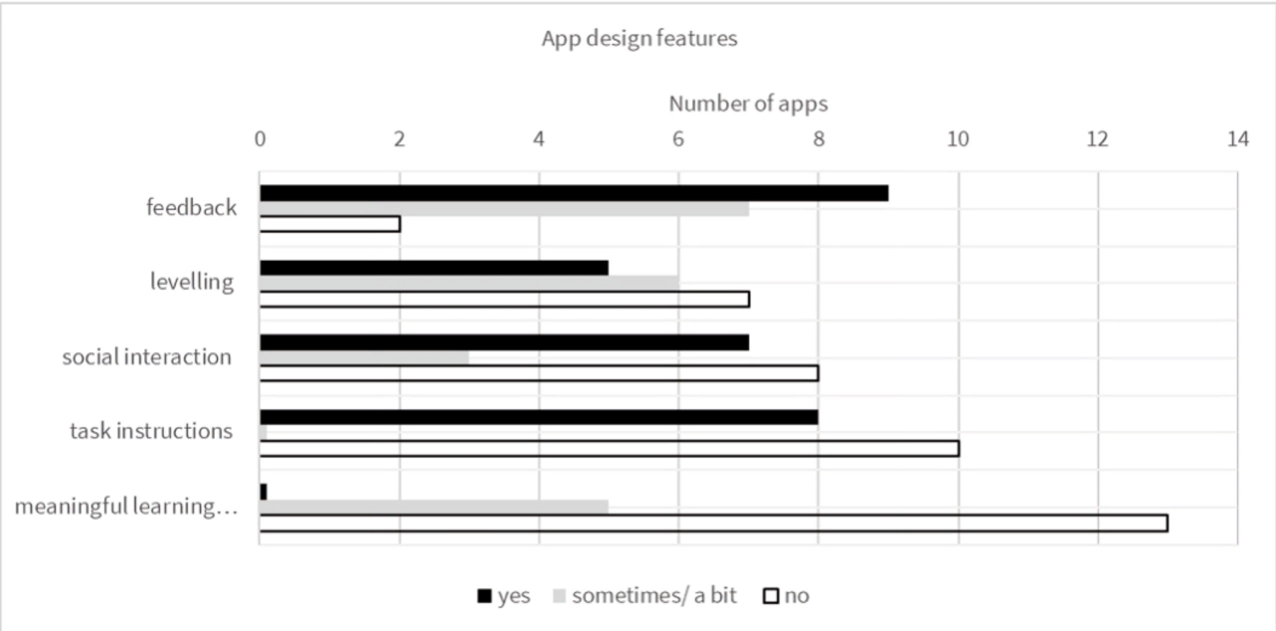

Figure 9. App design features–Bar chart (Source: Authors' own elaboration)

App Design Features

With respect to *design features* of the respective apps, the following results were obtained applying our evaluations routine (see **Figure 9**).

Feedback

Overall, n = 9 apps provide either motivational or explanatory feedback systematically after each task so that the user gets immediate feedback on all learning activities. In total, n = 7 apps provide either motivational or explanatory feedback at least sometimes. Additionally, n = 2 apps do not provide any feedback at all (see **Figure 9**).

Levelling

In sum, n = 5 apps realize several levels of difficulties that can be selected by the user, for example the teacher, to allow personalization to the respective child (see **Figure 9**).

Additionally, in n = 6 apps it is sometimes possible to individually select tasks by the user and, thus, sometimes allow personalization to the respective child. In contrast, n = 7 apps do not offer the option of consciously selecting different levels of difficulty, nor do they allow the user to specifically select tasks.

Social interaction

A total of n = 7 apps encourage social interactions with an in-app character who, for instance, repeatedly introduces the respective task, provides feedback or models the learning activity (see **Figure 9**). Additionally, n = 3 apps encourage social interactions with an in-app character at least in some tasks. However, n = 8 apps do not allow for any social interaction.

Task instructions

Of the reviewed apps, n = 8 provide explicit instructions that can be repeated when demanded by the child (see **Figure 9**). Whereas n = 10 of the apps either do not provide auditory task instructions at all or do not offer the opportunity to repeat these instructions for better understanding.

Meaningful learning and problem-solving

Only n = 5 of the apps embed mathematical skills practice within a real-life context, for instance through math stories (see **Figure 9**). In contrast, in n = 13 apps mathematical skills are primarily trained in isolation.

DISCUSSION

The aim of this article was to evaluate whether current educational apps provide a systematic approach to learn part-whole relations. To answer this question, we

(1) proposed a new framework for evaluating educational apps with respect to opportunities to learn part-whole relations and

(2) applied this framework to systematically evaluate whether as well as in which ways part-whole understanding is promoted in current educational apps (n = 18).

Results indicated that the respective apps are either practice or game-based with respect to the *type of app*. This is consistent with the results reported by Outhwaite et al. (2023a) and corresponds with other more comprehensive analyses indicating that mobile learning environments predominantly emphasize practice-based tasks over constructive or productive tasks (Crompton et al., 2017; Highfield & Goodwin, 2013; Papadakis et al., 2018).

Considering our results regarding mathematical content, it became evident that current apps do hardly provide the opportunity to systematically work out compositions/decompositions of sets, as well as the possibility to acquire or apply number compositions/decompositions to real world contexts. By contrast, automating number compositions/decompositions is most frequently targeted. This finding is in line with the results of the textbook analysis by Lenz and Wittmann (2023). Moreover, it is also consistent with the observation that a systematic introduction and acquisition of number compositions/decompositions is often neglected whereas the automation of number compositions/decompositions is often introduced early and overemphasized in textbooks (Wartha et al., 2019).

Regarding app *design features*, we found that four of the considered features, namely *feedback*, *levelling, social interaction* and *task instruction* were found to be at least partially provided by about half of the apps evaluated. However, meaningful learning and problem-solving (for example through tasks that practice mathematical skills within a real-life context), are only realized very rarely (see also Outhwaite et al., 2023b).

Taken together, the answer to the question whether as well as, if so, in which ways part-whole understanding is promoted in current educational apps is straightforward: Part-whole understanding is not promoted in a structured way from enactive over iconic to symbolic representations as described above. Instead, it is trained in ways that emphasize automation and applying number triples on the symbolic level. As such, it can be said that current educational apps do not provide a systematic way to facilitate children's development from protoquantitative to numerical part-whole understanding.

This, as described briefly in the following, might be realized by developing apps that build on and consider not only principles for learning environments as collated above. Instead, these apps should also consider current evidence on digital learning environments.

Kay and Kwak (2018), for example, found out that different *types of apps*, namely *practice-based, constructive, productive* and *game-based* apps, can enhance children's learning outcomes. This means, future apps that try to promote possibilities to systematically work out compositions/decompositions of sets should incorporate more constructive tasks by encouraging children to actively explore different number compositions/decompositions. Additionally, an app that aims to facilitate acquisition of number compositions/decompositions should build on theoretical approaches that focus on mathematical content and clear learning objectives (Bang et al., 2023; Kim et al., 2021; Outhwaite et al., 2023b). For example, this might be realized by providing tasks that use structured visualizations throughout and offer opportunities to systematically compose/decompose all numbers in the number range up to 10. Considering digital learning environments, this might be further supported by combining different visual and dynamic representations (Highfield & Goodwin, 2013), for example by implementing the possibility to dynamically rotate visual representations. Thus, digital tools might enable different methods of teaching. In order to further implement the application of number compositions/decompositions, it is important to develop apps that consider meaningful learning and problem-solving tasks (Hirsh-Pasek et al., 2015) that encourage children to practice number triples in association with different addition and subtraction tasks within a real-life context.

Limitations

It needs to be noted that there are some limitations to be considered when interpreting the present results. First, we only focused on and considered the German app market. Nevertheless, as the market for educational apps for Germany develops similarly to the global market (Baule et al., 2024), our new framework might be used to evaluate educational apps in international contexts. Furthermore, our evaluation framework should help sensitize the mathematics education community to the fact that the possibilities for facilitating part-whole understanding by educational apps should be critically reflected upon and evaluated in other national and international educational contexts. To do so, our framework might be used as a practical guideline.

Moreover, it needs to be considered that we evaluated only a small sample of pre-selected apps. However, in addition to the argued strengths of such an in depth evaluation of a smaller sample of pre-selected apps (Larkin et al., 2019), we also found that our results are well in line with previous international research on educational apps. This clearly argues against our findings being biased by the number and language of apps considered. Additionally, current apps are steadily changing, and the number of apps is constantly increasing (Papadakis et al., 2018). Therefore, this study can only be seen as a snapshot. As such, staying up to date regarding whether current apps provide a systematic approach to learning part-whole relations, it is important to apply frameworks, such as the one developed above, to evaluate future apps.

Considering the development of our framework, it needs to be noted that we primarily built it on the base of the framework by Outhwaite et al. (2023b). However, as this framework not only was developed considering several other frameworks, but it can also be embedded well within the ACAT, Outhwaite et al.'s (2023b) framework seems to provide a very suitable starting point. However, these considerations emphasize the fact that, no matter which one is chosen, all frameworks constantly need to be reflected whether they are useful to assess the intended learning environment. Moreover, as the apps might change constantly, the respective evaluation tools need to be revised equally.

Finally, the collation of principles for learning environments that promote part-whole understanding might not be exhaustive. Thus, it is important to constantly review part-whole related literature.

CONCLUSIONS

The current study set off to evaluate whether current educational apps provide a systematic approach to learning part-whole relations. Our findings suggest that current educational apps do not provide a systematic approach to learn part-whole relations. Importantly, this aligns well with previous findings indicating that educational apps do not yet exploit the full potential of digital learning of part-whole relations. Instead, they often attempt to transfer analogue teaching and learning environments one-to-one into digital learning environments (Crompton et al., 2017; Hirsh-Pasek et al., 2015). To change this, it is important that interdisciplinary teams consisting of different disciplines, such as, for example, software developers, mathematicians, teachers, graphic designers, students, parents or psychologists work together to co-design and develop future educational apps. Thereby, bringing together practical and theoretical expertise should benefit the development of digital learning environments that follow well-researched theoretical underpinnings and are practical to use (Messiha et al., 2023).

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APPENDIX A

App Selection

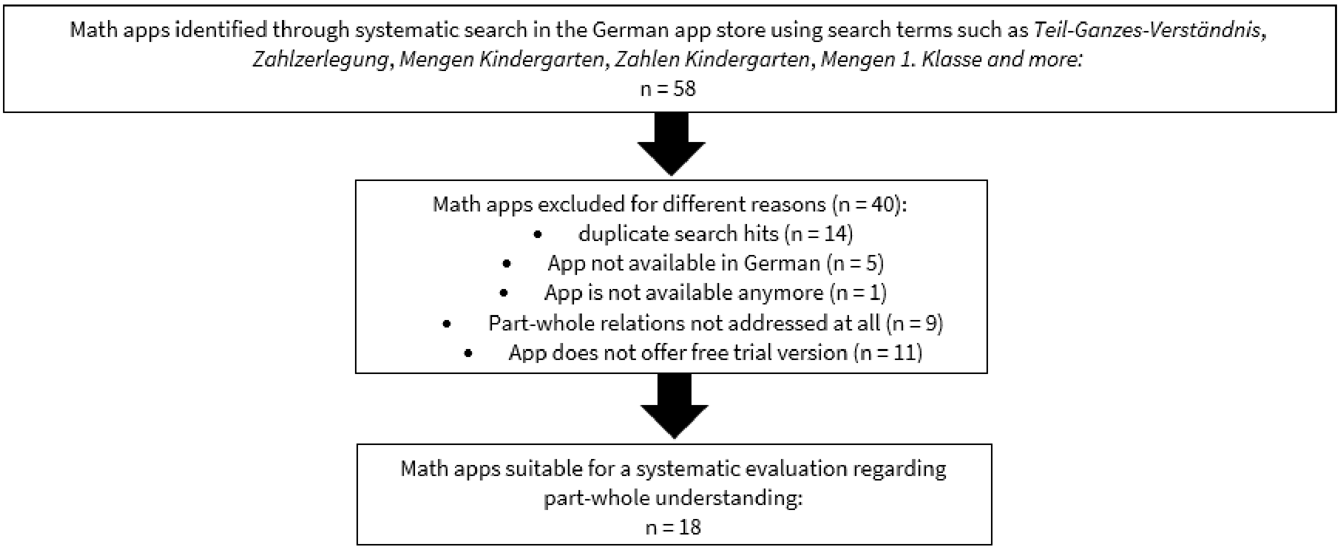

Table A1.

Search history: App store research, 27th July			
Search term	Apps found	App included	App excluded for the following reason
Teil-Ganzes-Verständnis	No results		
Zahlzerlegung	No results		
Blitzblick	No results		
Part-whole	No results		
Verliebte Zahlen	1. Funexpected plus	‣ Yes	
Zahlen Kindergarten	2. Mein Montessori ab 3 Jahren	‣ Yes	
	3. Intellecto Kinder Lern Spiele	‣ Yes	
	4. Malen Spiele Kinder Zeichnen		• Part-whole relations not addressed
	5. Lernspiele Spiele für Kinder		
	6. Kinderspiele zählen Spiel ab 5	‣ Yes	
	7. Zahlen Spiele für Kinder ab 3	‣ Yes	
	8. Zeichnen: Malen für Kinder 2-6	‣ Yes	• Part-whole relations not addressed
	9. Malen für Kinder		• Part-whole relations not addressed
	10. Gute-Nacht Caillou		• Part-whole relations not addressed
	11. Conni Zahlen 1-10		• No free trial version
Rechnen Kindergarten	12. Rechnen lernen–die Mathe App	‣ Yes	
	13. Zahlen Spiele für Kinder ab 3		• Duplicate search hit
	14. Vorschule Lernspiele Einkaufen		• No free trial version
	15. Rechnen Kinder Happytouch		• No free trial version
	16. Conni Zahlen 1-10		• Duplicate search hit
	17. Lazuli 4+ Mathematik Lernspiel		• No free trial version
	18. Lazuli 5+ Mathematik Lernspiel		• No free trial version
	19. Capt`n Sharky Erste Zahlen		• No free trial version
	20. Lazuli 6+ Mathematik Lernspiel		• No free trial version
	21. Erstes Zählen, erstes Rechnen		• No free trial version
Zählen Kindergarten	22. Mein Montessori ab 3 Jahren		• Duplicate search hit
	23. Intellecto Kinder Lern Spiele		• Duplicate search hit
	24. Malen Spiele Kinder Zeichnen		• Duplicate search hit
	25. Lernspiele Spiele für Kinder		• Duplicate search hit
	26. Kinderspiele zählen Spiel ab 5		• Duplicate search hit
	27. Zahlen Spiele für Kinder ab 3		• Duplicate search hit
	28. Zeichnen: Malen für Kinder 2-6		• Duplicate search hit
	29. Malen für Kinder		• Duplicate search hit
	30. Gute-Nacht Caillou		• Duplicate search hit
	31. Conni Zahlen 1-10		• Duplicate search hit
Mengen 1. Klasse	32. Zahlenhäuser rechnen lernen	‣ Yes	
	33. Montessori Zahlen für Kinder		• No free trial version

Table A1 (Continued).

Search history: App store research, 27th July			
Search term	Apps found	App included	App excluded for the following reason
Rechnen 1. Klasse	34. Mathe Land Lernen Kopfrechnen	Yes	
	35. Grundschule 1. Klasse	Yes	
	36. MiniMax Mathe	Yes	
Mathematik 1. Klasse	37. Scoyo–die Lernapp	Yes	
	38. Elevate–Brain Training		Part-whole relations not addressed
	39. Sofatutor	Yes	
	40. Mathespiele Klasse 1/2/3/4		Part-whole relations not addressed
	41. König der Mathematik	Yes	
	42. Kopfrechen Trainer	Yes	
	43. Kopfrechnen und Mathe: FastMath		Part-whole relations not addressed
	44. Mathe Land: Lernen Kopfrechnen		Part-whole relations not addressed
	45. Math Learner: Mathe Einfach	Yes	
	46. Kopfrechnen und Mathe Lernen		Part-whole relations not addressed
	47. Funexpected Mathe Lernspiele		Duplicate search hit
	48. Lernspiele für Kinder 4		Duplicate search hit
	49. Conni Mathe 1. Klasse		No free trial version
	50. Anton	Yes	
Further apps (known by the authors or recommended to the authors)	51. Das Zahlenbuch	Yes	
	52. Blitzrechnen 1-4		No free trial version
	53. MaiKe App (only for Android)		Not available any more
	54. Gracie & Friends Jungle Gym		Not available in German
	55. Gracie & Friends Treasure Bubble		Not available in German
	56. Gracie & Friends Breakfast Time		Not available in German
	57. SoGaBee's Math Facts Fun		Not available in German
	58. Todo Number Matrix		Not available in German

App Evaluation

Table A2.

App	Type of app				
	Practice-based	Constructive	Productive	Game-based	Parent-based
Funexpected plus				x	
Mein Montessori ab 3 Jahren	x				
Intellecto Kinder Lern Spiele				x	
Lernspiele Spiele für Kinder				x	
Kinderspiele zählen Spiel ab 5	x				
Zahlen Spiele für Kinder ab 3				x	
Rechnen lernen–die Mathe App	x				
Zahlenhäuser rechnen lernen	x				
Mathe Land Lernen Kopfrechnen				x	
Grundschule 1. Klasse				x	
MiniMax Mathe	x				
Scoyo–die Lernapp	x				
Sofatutor	x				
König der Mathematik	x				
Kopfrechen Trainer	x				
Math Learner	x				
Anton	x				
Das Zahlenbuch	x				

Table A3.

App	Mathematical content focusing on learning number compositions/decompositions					
	Systematically work out compositions/ decompositions of sets	Acquire number compositions/ decompositions			Automate number compositions/ decompositions	Apply number compositions/ decompositions
	Are there tasks that encourage children to make their own number compositions/ decompositions on an enactive and/or iconic level (e.g., by using shaking boxes, reversible plates, plug cubes, static finger patterns …?) and to write down their results with iconic and/or symbolic notation?	***Conceptual subitizing*** Is conceptual subitizing possible (e.g., by using structured visualizations such as the ten-strip)?	***Systematic compositions/ decompositions*** Is there a decomposition of all numbers in the number range up to 10, considering all triples and using structured visual representations?	***Brief visual presentation*** Is it possible to present visual sets briefly?	Are there tasks showing only one part of the specific number decomposition on an iconic or symbolic level?	Do children have to use number triples to solve various tasks, e.g., addition or subtraction tasks?
Funexpected plus						
Mein Montessori ab 3 Jahren						
Intellecto Kinder Lern Spiele						
Lernspiele Spiele für Kinder						
Kinderspiele zählen Spiel ab 5						
Zahlen Spiele für Kinder ab 3						
Rechnen lernen–die Mathe App						
Zahlenhäuser rechnen lernen						
Mathe Land Lernen Kopfrechnen						
Grundschule 1. Klasse						
MiniMax Mathe						
Scoyo–die Lernapp						
Sofatutor						
König der Mathematik						
Kopfrechen Trainer						
Math Learner						
Anton						
Das Zahlenbuch						

Table A4.

App	App design features				
	Feedback	Levelling	Social interaction	Task Instructions	Meaningful learning and solving problems
	Is explanatory and/or motivational *feedback* provided?	Is individual *levelling* possible?	Is *social interaction* provided?	Can *task instructions* be repeated by the child?	Are mathematical skills practiced within a real-life context?
Funexpected plus					
Mein Montessori ab 3 Jahren					
Intellecto Kinder Lern Spiele					
Lernspiele Spiele für Kinder					
Kinderspiele zählen Spiel ab 5					
Zahlen Spiele für Kinder ab 3					
Rechnen lernen–die Mathe App					
Zahlenhäuser rechnen lernen					
Mathe Land Lernen Kopfrechnen					
Grundschule 1. Klasse					
MiniMax Mathe					
Scoyo–die Lernapp					
Sofatutor					
König der Mathematik					
Kopfrechen Trainer					
Math Learner					
Anton					
Das Zahlenbuch					