

# Elementary mathematics teacher education programs in Greece, Romania, and Spain

Álvaro Nolla <sup>1\*</sup> , Roberto Muñoz <sup>2</sup> , Alexandru Iosif <sup>3</sup> , Lamprini Ananiadi <sup>4</sup> 

<sup>1</sup>Universidad Autónoma de Madrid, SPAIN

<sup>2</sup>Universidad Politécnica de Madrid, SPAIN

<sup>3</sup>Universidad Rey Juan Carlos, SPAIN

<sup>4</sup>Mathemagica: Institut für Nachhilfe, Wernigerode, GERMANY

\*Corresponding Author: [alvaro.nolla@uam.es](mailto:alvaro.nolla@uam.es)

**Citation:** Nolla, Á., Muñoz, R., Iosif, A., & Ananiadi, L. (2024). Elementary mathematics teacher education programs in Greece, Romania, and Spain. *International Electronic Journal of Mathematics Education*, 19(4), em0789. <https://doi.org/10.29333/iejme/15041>

## ARTICLE INFO

Received: 20 May 2024

Accepted: 03 Aug. 2024

## ABSTRACT

As a contribution to the discussion of how to improve the initial teacher training in mathematics of elementary teachers, this work presents a descriptive and comparative study of the courses devoted to mathematics and mathematics education in the education degrees of Greek, Romanian, and Spanish universities. Considering the constraints of each national law and within the Mathematics Teacher's Specialized Knowledge model framework, we analyze the official descriptions of such courses to organize them by their content, to describe their distribution along each degree, and to study the optional and specialization courses in mathematics offered by each university. Data are compared among countries, and across programs within each country, with the aim of including similar studies from other states. Moreover, they are confronted with the National Council on Teacher Quality standards for the mathematics preparation of elementary teachers, showing that, in general lines, GRS data seem not to be in tune with such standards.

**Keywords:** initial teacher training, mathematics, mathematics education, early childhood education, primary education

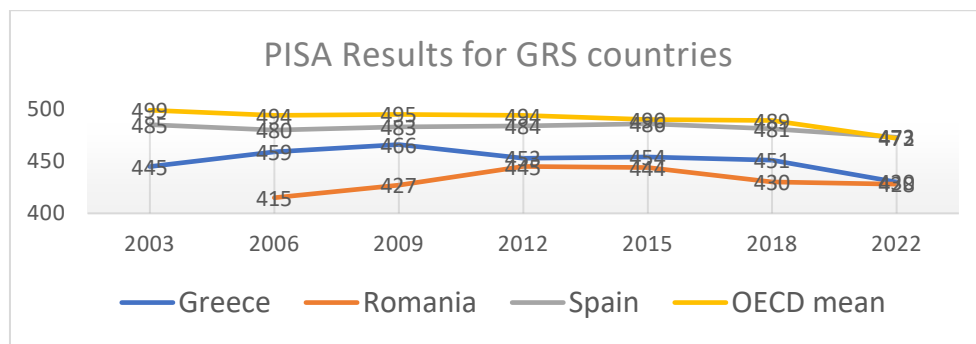
## INTRODUCTION

Teachers need to deeply understand the topics they teach (Ma, 1999). This deep understanding is a path from the knowledge of facts to the recognition of why they are valid, under which circumstances, and how they can be weakened or denied (Shulman, 1986). This is particularly important in mathematics. Elementary mathematics teachers must handle expertly routines, but also go beyond procedures (Ball et al., 2008). They have to solve and pose meaningful questions using such routines, and they also have to recognize the underlying mathematical structures. In addition, teachers should be capable of producing and explaining basic logical arguments, of writing them in a preliminary mathematical language, of being aware of the obstacles and difficulties that can appear in the process, and of appreciating the beauty of mathematics.

In fact, the word mathematics comes from the Ancient Greek μαθηματικός (“on the matter of that which is learned”). The etymology is uncovering that there are unique needs for the courses designed to form people responsible to teach mathematics. Contents to be taught to a future teacher and the way in which they are presented must be carefully chosen to enlighten the task of a teacher, not only showing a particular content-Mathematical Knowledge (MK)-but also the knowledge necessary to teach it-Pedagogical Content Knowledge (PCK)-with a special focus on its mathematical nature (Carrillo-Yañez et al., 2018).

It is therefore important to carefully design quality curricula that equip future teachers with the necessary mathematical competences to develop their future professional work. In this respect, there is a worldwide debate on how elementary mathematics education should be arranged, what contents and skills a future teacher should achieve during his or her initial teacher training (Gasteiger & Benz, 2018; Senk et al., 2012). The instrumental nature of mathematics and the influence of the early leanings on their development in later educational stages suggest that mathematics should play a central role in elementary teacher study programs (Baroody et al., 2019), but the approaches differ from country to country.

Alsina (2020), Nolla et al. (2021), and Méndez Coca et al. (2021) present a picture of initial teacher training in mathematics and mathematics education in elementary education degrees of Spain. They analyzed the number of credits assigned to such disciplines, the contents of the syllabus, their distribution along the curriculum, and the optional or specialization itineraries in mathematics offered to prospective teachers. Their findings show mismatches between the international standards of the *National Council of Teacher Quality* (NTCQ) and the concrete reality of the study programs of those universities, as well as concerns about the low proportion of mathematics subjects in these education degrees and a disconnection between research findings,



**Figure 1.** PISA results evolution of the GRS countries (<https://www.oecd.org/en/about/programmes/pisa.html>)

subject contents and the legislative curriculum. To contribute to the discussion of how to improve the initial teacher training in mathematics of elementary teachers it could be of interest to compare it with other countries (Tatto & Menter, 2023).

In this respect, our main goal is to show a general panorama of the initial teacher training in both mathematics and mathematics education in the universities of Greece, Romania, and Spain (denoted GRS from now on). These three European countries have different traditions, with Pisa results below the OCDE average, and have a highly compatible unit of measurement of their courses, the ECTS, European Credit Transfer System. We deal with the following research questions:

**RQ1.** What role mathematics and mathematics education plays in the curricula of teacher training education degrees in GRS universities?

**RQ2.** Are these formation plans in line with what recognized standards on the preparation of elementary teachers suggest?

For this purpose, we carry out a quantitative analysis of initial teacher training programs in the fields of mathematics and mathematics education in GRS universities for elementary pre-service teachers, and we make a comparative study of the three countries in the context of the international standards proposed by the NTCQ.

Our specific goals are then the following:

- S1.** To provide a description of the mathematics and mathematics education courses in the GRS education degrees. In particular, to present the amount of credits of such courses, their distribution along the degrees, and to organize them by their nature and their content type, as described in Method section.
- S2.** To discriminate whether there is a consensus on a minimum number of credits necessary to guarantee the training of pre-service teachers in mathematics and mathematics education.
- S3.** To confront the GRS data with the standards of the NCTQ report.
- S4.** To determine whether there exists any kind of specialization itineraries in mathematics in GRS education degrees.

## INTERNATIONAL AND LEGISLATIVE CONTEXT IN GRS

Education requirements to become an elementary school teacher usually include a bachelor's degree in primary or early childhood education, or other similar studies. Most institutions use the *credit* as a measure unity of the length of their courses. Hence, the number of credits of subjects related to mathematics and mathematics education, and the type of mathematical content of such courses in a concrete degree could provide a relevant information on the background acquired by its students on this field. The European Higher Education Area (EHEA) has established a common framework for a group of countries<sup>1</sup> in which the credit, European Credit Transfer System (ECTS), is highly compatible. Then, it seems natural to choose some EHEA countries to collect their data and make comparisons.

We consider three EHEA countries, Greece, Spain and Romania, which belong to different cultural traditions: Central European, and western and eastern Mediterranean, and which are rather young democracies: Greece, 1975, Spain, 1978, Romania, 1991. Focusing on the education of pre-service teachers, the requirements to become an elementary school teacher are different: In Spain and Greece, candidates must possess a four-year bachelor's degree in either early childhood or primary education, while in Romania the degree takes three years, and then candidates complete their education in a one-year practice period (see European Education and Culture Executive Agency, Eurydice et al., 2018). Finally, dealing with mathematics, their last results in the 2022 Pisa report<sup>2</sup> are similar and below OECD average (472 points): Greece 430 (position #44/80), Romania 428 (position #45/80) and Spain 473 (position #27/80). The GRS evolution from 2003 is shown in **Figure 1**.

In GS there are two bachelor's degrees in education: One to become primary school teachers (of 6-12 years old students), while the other focuses on the early childhood period (4-6 years old in Greece, 0-6 years old in Spain). They take 4 years and 240 ECTS, and can be studied in public universities and, in Spain, also in private ones. In Spain there is also a non-university vocational training degree that qualifies to work with children from 0 to 3 years of age. In Romania, however, there is a single bachelor's

<sup>1</sup> Cf., [http://www.ehea.info/page-full\\_members](http://www.ehea.info/page-full_members)

<sup>2</sup> Cf., <https://www.oecd.org/pisa/publications/>

degree in education which prepares to be both a primary school teacher (6-11 years old) and an early childhood teacher (3-6 years old). It comprises 180 ECTS and can be studied in either public or private universities.

In Greece, the law 1269/1982<sup>3</sup> structures the degrees of the public universities. According to the provisions of Φ5/89656/B3/13-8-2007<sup>4</sup>, the distribution of credits depends on each department. The content of each curriculum is drawn up by a special committee and certified by the national Agency HQA following the law 4653/2020 (see Section 7 in Eurydice report<sup>5</sup>).

In Romania, the structure of the degree is prescribed in 1/2011<sup>6</sup>, and divided into a basic training module (*Discipline de Dominiu*, with a minimum of 20% of the total credits), a disciplinary module (*Discipline de Specialitate*, min. 50% and *Complementare*, min. 5%) and a practice module (*Practica*, min. 15%). The mathematics subjects are located at *Discipline de Specialitate* module, and it is prescribed the existence of at least two courses: One with mathematics content, and other with mathematics education content.

In Spain, the laws ECI/3857/2007<sup>7</sup> and ECI/3854/2007<sup>8</sup> structure the studies as in Romania: *Módulo de Formación Básica* (min. 25% in Primary; min. 41.7% in Early Childhood); *Módulo Didáctico y Disciplinar* (min. 41.7% in Primary; min. 25% in Early Childhood); and *Practicum* with a minimum of 20.8%. Mathematics and mathematics education courses are also included in the didactic and disciplinary module.

Finally, let us present the official requirements for becoming a fully qualified teacher (see Figure 2.1 in European Education and Culture Executive Agency, Eurydice, 2018). In the three countries a successful graduation is necessary. In Romania, a confirmation of professional competency after graduation, usually after a one-year long period working as a teacher, is also needed. In GS, to work as a public employee, graduates must also pass an examination, which is organized by the Supreme Council for Civil Personnel Selection in Greece and by the regional governments in Spain.

## THEORETICAL FRAMEWORK

Regarding the choice of a theoretical framework, the model called Mathematics Teacher's Specialised Knowledge, abbreviated as MTSK (Carrillo-Yañez et al., 2018), establishes suitable categories for our study. This model is a revision of the *Mathematical Knowledge for Teaching* model, abbreviated MKT (Ball et al., 2008), which develops in the case of mathematics teaching the main ideas presented in Shulman (1986). The designers of the MTSK model themselves define it as "based on the idea that the specialization of the mathematics teacher's knowledge derives from his profession, that is, the knowledge he possesses will be specialized as long as it is necessary for him to develop his work as a mathematics teacher" (Montes et al., 2013, p. 404).

In fact, for our analysis, we need the two major domains already presented in Shulman (1986) which are the Mathematical Knowledge (MK), and Pedagogical Content Knowledge (PCK). We describe these domains as in (Carrillo-Yañez et al., 2018): MK contains the mathematics content itself, the interlinking systems which bind the subject and how one proceeds in mathematics; PCK contains the mathematics teaching theoretical knowledge, characteristics inherent to learning mathematics, and instruments designed to measure students' level of ability in understanding, constructing and using mathematics. These domains are used to organize the courses of the GRS education degrees by content type as described in Method section.

The second pillar in the theoretical framework is the report NCTQ (2008) from the *National Council of Teacher Quality*. This report is based on the mathematical contents and processes collected in the *Principles and Standards for School Mathematics* NCTM (2000) and sets five standards for teacher education in mathematics at the elementary education level to achieve a high-quality education program. These standards refer to the depth of conceptual knowledge of mathematics that pre-service teachers must begin to acquire and demonstrate in their studies, the requirements for admission to the programs, the coordination and opportunities for practice-teaching, and the selection of teacher trainers. Precisely, the statements of the standards are the following (NCTQ, 2008, p. 2):

**Standard 1:** Aspiring elementary teachers must begin to acquire a deep conceptual knowledge of the mathematics that they will one day need to teach, moving well beyond mere procedural understanding. Required mathematics coursework should be tailored to the unique needs of the elementary teacher both in design and delivery, focusing on four critical areas: (1) Numbers and operations; (2) Algebra; (3) Geometry and measurement and -to a lesser degree; (4) Data analysis and probability

**Standard 2:** Education schools should insist upon higher entry standards for admittance into their programs. As a condition for admission, aspiring elementary teachers should demonstrate that their knowledge of mathematics is at the high school level (geometry and coursework equivalent to second-year algebra). Appropriate tests include standardized achievement tests, college placement tests, and sufficiently rigorous high school exit tests

**Standard 3:** As conditions for completing their teacher preparation and earning a license, elementary teacher candidates should demonstrate a deeper understanding of mathematics content than is expected of children. Unfortunately, no current assessment is up to this task

<sup>3</sup> <https://www.kodiko.gr/nomothesia/document/288704/nomos-1268-1982>

<sup>4</sup> [https://www.et.gr/api/DownloadFeksApi/?fek\\_pdf=20070201466](https://www.et.gr/api/DownloadFeksApi/?fek_pdf=20070201466)

<sup>5</sup> <https://eurydice.eacea.ec.europa.eu/national-education-systems>

<sup>6</sup> [https://www.edu.ro/sites/default/files/\\_fi%C8%99iere/Legislatie/2022/LEN\\_2011\\_actualizata\\_2022.pdf](https://www.edu.ro/sites/default/files/_fi%C8%99iere/Legislatie/2022/LEN_2011_actualizata_2022.pdf) and <https://www.aracis.ro/wp-content/uploads/2022/10/5.-Standarde-C5-28.09.2022.pdf>

<sup>7</sup> [https://www.boe.es/diario\\_boe/txt.php?id=BOE-A-2007-22449](https://www.boe.es/diario_boe/txt.php?id=BOE-A-2007-22449)

<sup>8</sup> <https://www.boe.es/eli/es/o/2007/12/27/eci3854>

**Standard 4:** Elementary content courses should be taught in close coordination with an elementary mathematics methods course that emphasizes numbers and operations. This course should provide numerous opportunities for students to practice-teach before elementary students, with emphasis placed on the delivery of mathematics content

**Standard 5:** The job of teaching aspiring elementary teachers' mathematics content should be within the purview of mathematics departments. Careful attention must be paid to the selection of instructors with adequate professional qualifications in mathematics who appreciate the tremendous responsibility inherent in training the next generation of teachers and who understand the need to connect the mathematics topics to elementary classroom instruction.

The NCTQ report evaluated (and created a ranking of) the teacher education programs of a large sample of U.S. universities and, as in its more recent studies (NCTQ, 2018), it points out mismatches between the desired standards and the concrete reality of the study programs of those universities.

As commented in the previous paragraphs, the MTSK and MKT models stress the fact that the mathematical knowledge of an elementary teacher is intimately linked to his or her teaching practice. This is indeed at the origin of the introduction by Shulman (1986) of the PCK domain: There exists a mathematical subject knowledge *for teaching* (some reflections on the PCK can be found in Star, 2023). The NCTQ standards are in the same line:

Mathematical courses for elementary teacher preparation should be tailored to the unique needs of the elementary teacher both in design and delivery.

Any analysis or discussion of this paper is done under this important assumption: The existence of special needs, as concentered in the description of the MTSK domains, for the mathematical preparation of an elementary teacher.

## METHOD

This work is a descriptive and comparative study of the courses devoted to mathematics and mathematics education in the GRS education degrees. The number of universities which offer a primary and/or early childhood education degree in GRS is 9 in Greece (100% public), 25 in Romania<sup>9</sup> (84% public), and 63 in Spain (63,3% public). Spanish data come from the 2020/21 academic year (see Nolla et al., 2021), Greek from 2021/22 and Romanian from 2022/23. The study examines the implementation of part of the specific policy of initial teacher training in mathematics, so it falls into the Type II of international comparative studies proposed in Chabbot and Elliot (2003).

Data have been collected from an exhaustive inspection of the documents obtained from the official website of each University in two steps: First, via the title, an initial selection of mathematics and/or mathematics education subjects has been made; Second, checking the contents and the bibliography indicated in the syllabi (whenever possible), the initial selection has been classified according to their:

1. *Nature*: Compulsory courses (C), which all students must pass, or optional courses (O), under choice of the student
2. *Content type*
  - Mathematics education courses (ME), which essentially contains Pedagogical Content Knowledge
  - Mathematics courses (M), which essentially contains Mathematical Knowledge
  - A combination of mathematics and mathematics education courses (M&ME) and
  - Mathematics and other sciences courses (MS).

This last case is exceptional and appears when mathematics is presented in the context of a sciences course.

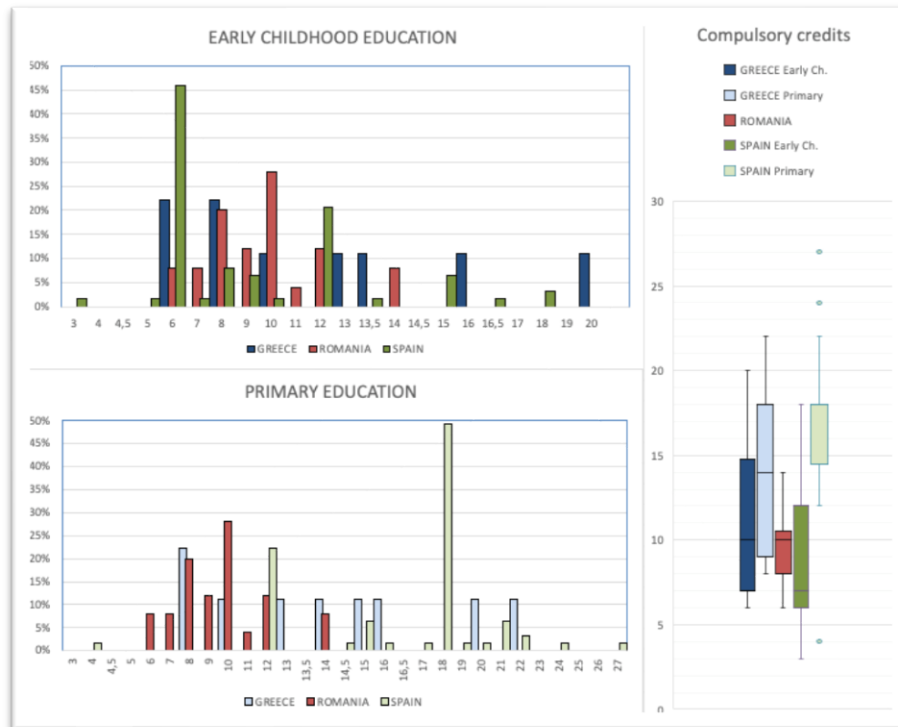
The content type allows us to assign a model to each University, according to the predominance of M, ME and M&ME and MS subjects. These categories are defined as follows:

- Mathematics Education model (**ME-model**): Degrees which contains only ME courses
- Mixed model (**M&ME-model**): Only M&ME courses
- Combined models: Subjects of type M&ME and other types
  - Mixed-Mathematical model (**M&ME+M-model**): Contains M&ME, M subjects but not ME subjects
  - Mixed-Education model (**M&ME+ME-model**): Contains M&ME, ME subjects but not M subjects
  - Mixed-combined model (**Combined-model**): Contains subjects of the three types
- Separated model (**M+ME-model**), there are courses on mathematics and courses on mathematics education, but not of mixed nature
- Mathematics model (**M-model**), only courses with mathematics contents
- Transversal model (**MS-model**), mathematics subjects are combined with other sciences.

<sup>9</sup> In the case of University Babeş-Bolyai Cluj-Napoca and University Lucian Blaga din Sibiu, we are considering each language program as if they were different universities, because the curricula are different.

**Table 1.** Number of ECTS compulsory credits in mathematics or mathematics education

	Mean	Min	Q1	Median	Q3	Max	Mode
GREECE Early Childhood	11,2	6	8	10	14	20	8
GREECE Primary	14	8	10	14	16	22	8
ROMANIA	9,5	6	8	10	10	14	10
SPAIN Early Childhood	8,84	3	6	7	12	18	6
SPAIN Primary	16,75	4	14,75	18	18	27	18
GRS Early Childhood	9,23	3	6	8	12	20	6
GRS Primary	14,63	4	12	15	18	27	18

**Figure 2.** Distribution in % of total universities for each GRS country of compulsory credits in mathematics/ mathematics education (Source: Authors' own elaboration)

In view of our classification, the variables considered are:

1. Number C of credits of compulsory courses in mathematics and mathematics education. This variable is related to S1, S2 and S3.
2. Number O of optional credits in mathematics and mathematics education, related to S1 and S4.
3. Number of credits in mathematics and mathematics education for each content type defined above, related to S1 and S3.
4. Start-end mathematics or mathematics education courses: first and last year of compulsory courses in such disciplines, related to S3.
5. Existence of a specialization program in mathematics, related to S4.

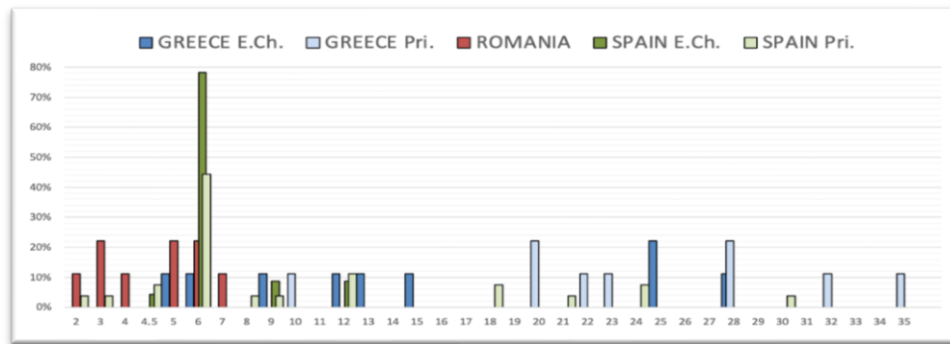
## RESULTS

In this section we present the tables and graphics of the variables considered in this study. The complete data of subjects and credits of GRS universities is presented in **Appendix** at **Table A.1** (Greece), **Table A.2** (Romania), and **Table A.3** (Spain).

The interplay between the collected data and the specific goals becomes apparent here. In **Table 1** we present basic statistics on the variable C of compulsory credits, and **Figure 2** contains the GRS distribution of compulsory credits. The information on the variable O of optional credits is collected in **Table 2**. The distribution of optional credits among the universities which offer such credits is presented in **Figure 3**.

**Table 2.** Percentage of Universities with optional ECTS credits in mathematics and/or mathematics education

	Early Childhood Education	Primary Education
GREECE	100%	100%
ROMANIA		36%
SPAIN	37%	41%



**Figure 3.** Distribution of the number of optional credits in mathematics and/or mathematics education (Source: Authors' own elaboration)

Start	Early Ch SPAIN				Start	Primary SPAIN			
1°	3	0	2	1	1°	0	5	17	4
2°		13	9	3	2°		7	16	10
3°			25	4	3°			1	3
4°				3	4°				0
End	1°	2°	3°	4°	End	1°	2°	3°	4°

Start	Early Ch. GREECE				Start	Primary GREECE			
1°	0	2	2	1	1°	0	2	3	2
2°		1	3	0	2°		0	2	0
3°			0	0	3°			0	0
4°				0	4°				0
End	1°	2°	3°	4°	End	1°	2°	3°	4°

Start	ROMANIA		
1°	3	13	3
2°		5	0
3°			1
End	1°	2°	3°

**Figure 4.** Starting and end mathematics and/or mathematics education compulsory courses (Source: Authors' own elaboration)

**Table 3.** Universities with specialization programs containing mathematics and/or mathematics education contents in GRS

Universities	SPAIN		GREECE		ROMANIA
	Early Ch.	Primary	Early Ch.	Primary	
	1	4	0	0	0

**Table 4.** University models for GRS countries in terms of the type of subjects

	Early Childhood Education						Primary Education							
	ME	M+ME	M&ME	M&ME+M	M&ME+ME	Comb	MS	ME	M+ME	M&ME	M&ME+M	M&ME+ME	Comb	MS
<b>Greece</b>	2	2	1	3		1		1	7	1				
<b>Romania</b>		25							25					
<b>Spain</b>	51	5	2				5	2	18	29	2	8	4	

In **Figure 4** we collect the starting and end courses of compulsory subjects of mathematics/mathematics education: The starting (respectively end) course is the first (respectively last) course of a compulsory subject.

In **Table 3** we present the number of GRS universities with a specialization program containing mathematics in the title, and finally, **Table 4** presents the classification of the universities within the models described in the Method section.

## DISCUSSION

In view of S4, we confront our data with the standards of NCTQ (2008), stated in the Theoretical Framework.

### Standard 1

Let us first remark that the area of data analysis is becoming more important in our society and, consequently, in education. In fact, the so-called computational thinking is, in some sense, starting to define a new paradigm in education, where science and mathematics are sometimes defined as computational endeavors (Weintrop et al., 2016). In particular, data practices, as proposed in Weintrop et al. (2016)-collecting, creating, manipulating, analyzing, and visualizing data-, constitute a relevant part of this ability. The comment “-to a lesser degree-”, in reference to the data analysis and probability area, might be reconsidered in an updated formulation of Standard 1.

Along this paper we have been showing important elements which have to appear in the mathematics and/or mathematics education courses of the education degrees, which in fact are quite independent of the country. To accomplish Standard 1 as described above, it would seem natural to reach a consensus on a minimal number of necessary credits of mathematics/mathematics education. In fact, this number is independent of the length of the degree, which allows us to consider Romanian data in absolute terms, although the degree is shorter. As a first approach to this minimum, we can contemplate the recommendations of the NCTQ report (2008, p. 8): 115 hours, 12 ECTS, as the time necessary to cover the mathematical contents (without counting mathematics education contents). Let us discuss what GRS data say on this minimum according to **Table 1**.

First thing to observe is that in Spain the number of compulsory credits in mathematics/mathematics education courses is significantly smaller in the early childhood education degree than in the primary school education degree. The median is 7 versus 18. However, in Greece the gap is smaller, 10 versus 14. Certainly, Standard 1 considers the whole period of elementary education and the 12 credits recommendation takes account of mathematical contents which does not appear in early childhood education. However, the skills and attitudes described above are equally desirable for these teachers, and, moreover, they face the extra difficulty of transferring them to their students. In this sense, Greece is closer to this standard since most prospective teachers take at least two courses in mathematics and/or mathematics education. However, in Spain most graduates in early childhood education only take one course, 5 to 7 credits.

Second thing is to compare the medians of GRS, considering the primary school in GS, and the Romanian data. The median in Greece is 14 credits, 10 in Romania, and 18 in Spain. The Romanian data are very homogeneous, following most universities (72%) the minimal law requirement of one mathematics and one mathematics education course. Nevertheless (see **Tables A.1, A.2, and A.3 in Appendix**), even adding the optional courses, half of the universities in Romania does not reach this median. This is different in Greece where, adding the optional credits, all universities reach the median of 14 compulsory credits. In Spain, the number of universities not attaining the median when adding compulsory and optional credits is 15 (remarkably, all of them are private institutions).

Third thing to remark is the range of variation on the number of compulsory credits. Let us study, as before, the primary school in GS, and the Romanian data. This range goes from 8 to 22 in Greece, 6 to 14 in Romania, and 4 to 27 in Spain. The minimums: 8, 6 and 4, (and even the first quartiles Q1 of GR: 10 in Greece, 8 in Romania) are far from Standard 1 proposal. Furthermore, the ranges reveal an absence of consensus on a minimal number of credits. We come back on this in the Conclusions.

Finally, let us confront the number of compulsory credits of mathematics of the GS primary and R degrees with the NCTQ suggestion (12 credits). We can set the hypothesis that for the mixed credits M&ME the percentage of mathematical content could vary between 50% and 75%. Just for this comment, assume the maximal value of 75%. Adding to the compulsory credits of mathematics the 75% of the M&ME compulsory credits, only 1 out of 15 universities of Greece and 29 out of 64 in Spain reach the minimal suggested value. In Romania, even doing the same addition, only 2 out of 25 institutions reach 12 credits. With this data, it seems that GRS countries are not in line with the suggestion of a minimum of 12 compulsory credits in mathematics.

### Standard 2

This standard focuses on the mathematical knowledge requirements for the candidates to enter an education degree. In principle, this standard sounds redundant. The main way to access a GRS education degree follows from an upper secondary education period of type ISCED 3 (European Education and Culture Executive Agency, Eurydice, 2016, p. 12), so that the knowledge of mathematics of applicants is guaranteed. However, there are some elements which raise some doubts on this fact: The heterogeneity of the students, which are coming from different baccalaureates, some of them without mathematics in their last period (Asensio Muñoz et al., 2022; Potari, 2001); the GRS Pisa results, below the average; the level of proficiency in mathematics of a graduate student in an education degree, which seems to be susceptible of improvement (see Standard 3 for further details). In Romania there exists an access test to education degrees, but we do not have evidence that it evaluates mathematics.

In the absence of an access test, it seems natural to place a course of mathematics as soon as possible in the degree to evaluate if the high entry standards are met and, if not, to have room to reach this level along the studies. This is the case, see **Figure 4**, in Greece, where 55.5% of universities in early childhood education and 77.8% of primary education start mathematics courses in the first year. It also occurs in Romania with a 79.2%. However, in Spain, only 9.5% of the early childhood education degrees and 41.3% of the primary education degrees place mathematics subjects in the first year. It is relevant here to remember that there are some law limitations in Spain: mathematics and mathematics education are placed in the Didactic and Disciplinary Module and not in the Basic Training Module, whose subjects are preferably placed in the first courses.

### Standard 3

We have no evidence that the final level of performance in mathematics is measured in any way in the GS education degrees. It is understood that the training received throughout the degree is enough for such performance. Similarly, in Romania, although there is a practice period after the degree, we are not aware of any final test on mathematical contents. The GS system to become a public employee includes an examination with mathematical contents. We agree, in principle, with this philosophy: graduation means proficiency and only when competition is required to hire someone for a position, additional tests must be passed.

Nevertheless, some studies and a common perception (Alsina, 2020; Făt, 2016; Frangopol, 2011; Nortes & Nortes, 2013, 2018; Potari, 2001; VVAA, 2020) seem to show that the level of proficiency in mathematics of a graduate in education is susceptible of improvement. The existence of this final test could help to make an accurate diagnosis on the situation in order to design improvements, if necessary.

While Standard 2 is related to the first course on mathematics, Standard 3 is related to the end course (see **Figure 4**). A compulsory course in the last year of the degree could provide a double opportunity: to check this desired final proficiency in mathematics, and to better link this last course to the Practice Module. The design of the GRS education degrees does not take care of these opportunities, as the data of **Figure 4** show.

### Standard 4

As said above, prospective teachers must learn meaningful mathematics and also how to activate the so-called mathematical sense in their future students. Such mathematical sense is usually defined as the set of skills related to the mastery of mathematics contents which leads to use them in a functional way (see Ramírez Uclés et al., 2019 and references therein). This includes, generalizing Standard 4, the number, spatial, measurement and stochastic senses (and possibly others). The understanding of the mathematical concepts and their manipulation must be intimately linked. This enlightens how to present mathematics to prospective teachers and what activities to propose: contents and methods must be clearly connected, and also related to the mental processes of the students. This concludes the relevance of a close coordination between mathematics and mathematics education courses. In the following we analyze how the GRS systems face such coordination in view of the data of **Table 4**.

For the early childhood degrees, 81% universities in Spain have opted for a mathematics education model ME, according to what is described in the Spanish law: A globalizing and integrated curriculum. The law completely determines the model in Romania, where all universities have the M+ME model. The Greek educational system does not have such a predominant model, and a variety of models appears. It is worth to mention that every Romanian university and 6 out of 9 universities in Greece offers a compulsory course in mathematics. This is not the case in Spain, where just 5 out of 63 (8%) dedicates a compulsory course to mathematics.

In relation to primary education, the Spanish system mostly chooses an integration of both knowledges, in line with this standard: 46% of the degrees follow a mixed model M&ME and this proportion reaches 62% if M&ME+M and M&ME+ME are included. The second choice in the Spanish system is the separate model M+ME with 29% universities. On the other hand, the predominant option in GR is the separated model M+ME: 78% Greek universities and the whole Romanian ones. This division between mathematical contents and mathematics education contents does not fit with Standard 4. It would be of interest to know if, in the M+ME models, there exists a good coordination between subjects of each type. Moreover, it would be also relevant to know how mathematics and mathematics education contents are integrated in subjects of mixed type, that is, to what extent such integration depends on the background of the teacher or on the department in charge of the course. This needs a careful analysis of the current courses beyond the official documents.

### Standard 5

This standard addresses the fact that the mathematics teacher educators for pre-service teachers must be carefully chosen and be clearly aware of their responsibility. To train mathematics teachers needs professionals with deep knowledge of mathematics but who do not forget that this mathematics is going to be presented to elementary students. Standard 5 invites to overcome dualities of type: “What is needed to know only is how to teach mathematics” versus “what prospective teachers need to learn is mathematics” (Potari, p. 82, 2001).

In **Table 2** we present the percentage of universities which offer optional courses related to mathematics. All Greek universities have mathematics-related optional courses in their curricula, being by far the GRS country with the largest offer. The range of courses varies between 5 and 28 credits in early childhood degrees, and, respectively, between 10 and 35 in primary degrees. However, in Romania, since the degree is shorter, the number of optional courses is very limited: in fact, only 36% universities offer them, in a range from 3 to 7 credits. In Spain, only 37% universities in early childhood education (range 4.5-12) and 41% in primary education (range 6-30) offer such optional courses.

To take some optional courses related to mathematics could partially fill the gap (see Standard 1) between the recommended compulsory credits and the actual ones. Additionally, it could help to form teachers with a special background on mathematics. This could be important for several reasons:

- Teachers specially prepared in mathematics can act as dynamic agents of mathematics education in their schools,
- They can lead research on mathematics education tightly linked to daily classroom situations, easily transferable to the community,
- They can contribute as university teachers in education degrees,
- They could be responsible for the mathematics aspects of the professional development and continuous education of teachers.



Data from the previous paragraph are complemented with the number of GRS universities with a specialization related to mathematics (see **Table 3**): No one in GR and only 5 in S. This is further developed in the Conclusions section, but it shows that specialization in mathematics is not considered as important in the GRS education degrees.

## CONCLUSIONS

Prospective elementary GRS teachers are prepared in bachelor's degrees, whose subjects are measured in ECTS credits. As proposed in S1, we have collected the number of credits of mathematics or mathematics education courses of compulsory or optional type and presented them in the results. **Tables A.1, A.2, and A.3** in the **Appendix** contain the list of universities and their subjects as can be found in their official web pages and classified via the theoretical framework described in the Introduction.

In **Table 1** one can check the heterogeneity of the number of compulsory credits of mathematics/mathematics education. As commented in the discussion on Standard 1, this number is smaller in the early childhood studies with respect to the primary ones, especially in Spain. Moreover, medians suggest that Romanian students spend less time (10 ECTS in mathematics/mathematics education courses) than Greek primary education students (14 ECTS), who in turn spend less time than the Spanish ones (18ECTS) However, in early childhood, Spain has the lowest median with 7 ECTS, while GR median is 10.

The ranges of credits (in each country and among them) appear to show the absence of a consensus on the number of credits necessary for an undergraduate student to set the basis for a competent elementary mathematics teacher, see S2. The mode in Spain is 6 credits in early childhood studies and 18 in primary education, with a much larger frequency than the other values. The forementioned 6 and 18 credits were suggested in Nolla et al. (2021) as a first indicator for a minimal number of credits, respectively for early childhood and for primary education. The GR data do not confirm this suggestion for primary education: In Greece, only two universities offer a number of compulsory credits over 18, and the maximum number of compulsory credits in Romania is 14 (two universities). The GRS data in primary school give the impression that the 15 credits of the GRS median (see **Table 1**) could be an indicator between the 10 Romanian and the 18 Spanish credits. If we assume 15 credits as a first approach to this searched minimum, we observe that 4 of 9 Greek universities; all Romanian universities; and 15 of 63 Spanish universities do not reach this number. In contrast, data in early childhood education in Greece agrees with the proposed minimum of 6 ECTS. However, 2 out of 9 of Greek universities do not reach this minimum while in Spain half of the universities offer a number of credits less than or equal to 6. This suggests that the Spanish education system assumes that one course in mathematics/mathematics education is enough to accomplish the expectations on an early childhood mathematics pre-service teacher.

We have organized (see **Table 4**) the GRS degrees according to their model of mathematics education. In Spain, at least formally, mathematics and mathematics education contents seem to be generally integrated. In GR the situation is different: These two knowledges are mainly separated. Some more details can be found in the discussion of Standard 4.

The document NCTQ (2008) fixes five standards on the preparation of prospective teachers in mathematics. We have confronted our data, see S3, with these standards in the Discussion Section, and, in general lines, the GRS education degrees are not in tune with them. Our findings, based on the collected data, show a panorama of the situation but also the need for further studies: For Standard 1, an analysis that moves away from the syllabus to the real practices of the courses (contents, methodologies, and evaluation); for Standards 2 and 3, a comparative study of existent admittance and final proficiency tests; for Standard 4, an analysis of the coordination among the courses, and between the courses and the practicum; for Standard 5, a study of the background in mathematics and mathematics education of the teacher educators, and of the departments which they belong to.

With respect to the S4 objective, let us study the GRS offer of optional courses in mathematics (see **Table 2**). All Greek universities provide such optional credits, while only 36% Romanian universities, 37% Spanish universities in early childhood degrees, and 41% Spanish universities in primary education degrees do it. It is relevant to say that we do not know if the optional courses are taught every year. From the RS scarce offer, one can infer that specialization in mathematics is not considered relevant in these two countries. The Romanian case is connected with the length of the degree. The Spanish case is further developed in Nolla et al. (2021). However, the number of optional credits in Greek universities is quite significant, bigger than 20 credits in 7 of the 9 primary school degrees. With the preventions suggested above, it seems that the Greek system is most aware of the importance of taking extra courses on mathematics or mathematics education.

Education is a crucial element of a society, a UNESCO Sustainable Development Goal, concretely SDG4<sup>10</sup> states:

Ensure inclusive and equitable quality education and promote lifelong learning opportunities for all.

Elementary teachers are, among others, responsible of the education of the new generations. It is consequently very important to take care of the preparation of such teachers, and concretely, as focused on this paper, in mathematics and mathematics education as a relevant part of the background of their students. The analysis presented here can be of service to improve such training: the experiences of some countries could be an inspiration for other countries. We have dealt with the cases of Greece, Romania, and Spain but this study can be extended to other education systems. We encourage researchers of other countries to collect the data of their education degrees and compare with the ones presented here.

<sup>10</sup> <https://www.sdg4education2030.org/the-goal>

**Author contributions:** All authors contributed to the conceptualization, methodology, writing and editing. **LA:** collected data from universities in Greece, **AI:** collected data from universities in Romania; **AN & RM:** collected data from universities in Spain. All authors have sufficiently contributed to the study, and agreed with the results and conclusions.

Álvaro Nolla, Roberto Muñoz, Alexandru Iosif, Lamprini Ananiadi

**Funding:** The authors declared that no funding source is reported for this study.

**Ethical statement:** The authors stated that the study does not require ethical committee approval since the information used comes from freely available data in the public domain (university websites), and the analysis of datasets, are either open source, from the data collection or obtained from other researchers.

**Declaration of interest:** No conflict of interest is declared by the authors.

**Data sharing statement:** Data supporting the findings and conclusions are available upon request from the corresponding author.

## REFERENCES

- Alsina, Á. (2020). La Matemática y su didáctica en la formación de maestros de Educación Infantil en España: Crónica de una ausencia anunciada [Mathematics and its didactics in the training of early childhood education teachers in Spain: Chronicle of an announced absence]. *La Gaceta de la RSME*, 23(2), 373-387.
- Asensio Muñoz, I. I., Arroyo Resino, D., Ruiz Lázaro, J., Sánchez Munilla, M., Ruiz de Miguel, C., Constante Amores, I. A., & Navarro Asencio, E. (2022). Perfil de acceso a la universidad de los maestros en España [University access profile for teachers in Spain]. *Educación XX1*, 25(2), 39-63. <https://doi.org/10.5944/educxx1.31924>
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407. <https://doi.org/10.1016/j.jmathb.2019.100718>
- Baroody, A. J., Clements, D., & Sarama, J. (2019). Teaching and learning mathematics in early childhood programs. In C. Brown, M. McMullen, & N. K. File (Eds.), *Handbook of early childhood care and education* (pp. 329-353). Wiley Blackwell Publishing. <https://doi.org/10.1002/9781119148104.ch15>
- Carrillo-Yañez, J., Climent, N., Montes, M., Contreras, L. C., Flores-Medrano, E., Escudero-Ávila, D., Vasco, D., Rojas, N., Flores, P., Aguilar-González, A., Ribeiro, M., & Muñoz-Catalán, M. C. (2018). The mathematics teacher's specialised knowledge (MTSK) model. *Research in Mathematics Education*, 20(3), 236-253. <https://doi.org/10.1080/14794802.2018.1479981>
- Chabbot, C., & Elliott, E. J. (Eds.) (2003). *Understanding others, educating ourselves: Getting more from international comparative studies in education*. Committee on a Framework and Long-term Research Agenda for International Comparative Education Studies. National Research Council. The National Academic Press.
- European Education and Culture Executive Agency, Eurydice. (2016). *The structure of the European education systems 2016/17: schematic diagrams*. European Commission. <https://data.europa.eu/doi/10.2797/464130>
- European Education and Culture Executive Agency, Eurydice, Delhaxhe, A., Birch, P., & Piedrafita Tremosa, S. (2018). *Teaching careers in Europe: access, progression and support* (A. Delhaxhe, Ed.). Publications Office. <https://data.europa.eu/doi/10.2797/309510>
- Făt, S. (Coord.) (2016). *Educația matematică din România* [Mathematics education in Romania]. Societatea de Științe Matematice din România [The Society of Mathematical Sciences from Romania]. **Error! Hyperlink reference not valid.**
- Frangopol, P. T. (2011). *Mediocritate și excelență. O radiografie a științei și învățământului din România* [Mediocrity and excellence. An x-ray of science and education in Romania] (Vol. 4). Casa Cărții de Știință.
- Gasteiger, H., & Benz, C. (2018). Enhancing and analyzing kindergarten teacher's professional knowledge for early mathematics education. *The Journal of Mathematical Behavior* 51, 109-117. <https://doi.org/10.1016/j.jmathb.2018.01.002>
- Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Lawrence Erlbaum Associates Publishers. <https://doi.org/10.4324/9781410602589>
- Méndez Coca, M., Belmonte Gómez, J. M., Pizarro Contreras, N., & Ramírez García, M. (2021). Formación matemática en el grado de maestro de educación infantil: Análisis de las guías docentes de las universidades públicas españolas [Mathematics training in the early childhood education teacher's degree: Analysis of the teaching guides of Spanish public universities]. In A. Vico-Bosch, L. Vega Caro, & O. Buzón-García (Eds.), *Entornos virtuales para la educación en tiempos de pandemia: Perspectivas metodológicas* (p. 756-780). Ed. Dykinson.
- Montes, M., Contreras, L. C., & Carrillo, J. (2013). Conocimiento del profesor de matemáticas: Enfoques del MKT y MTSK [Mathematics teacher knowledge: MKT and MTSK approaches]. In A. Berciano, G. Gutiérrez, A. Estepa, & N. Climent (Eds.), *Investigación en Educación Matemática XVII* (pp. 403-410). Servicio Editorial de la UPV/EHU, Bilbao.
- NCTM (2000). *Principles and standards for school mathematics*. NCTM.
- NCTQ. (2008). *No common denominator: The preparation of Elementary Teachers in Mathematics by America's Education Schools*. National Council on Teacher Quality. [https://www.nctq.org/dmsView/No\\_Common\\_Denominator\\_NCTQ\\_Report](https://www.nctq.org/dmsView/No_Common_Denominator_NCTQ_Report)
- Nolla, Á., Muñoz, R., Cerisola, A., & Fernández, B. (2021). La formación inicial de los maestros en matemáticas y su didáctica [Initial training of teachers in mathematics and its didactics]. *Revista Interuniversitaria de Formación del Profesorado*, 96(35.1), 185-208. <https://doi.org/10.47553/rifop.v96i35.1.85882>

- Nortes, A., & Nortes, R. (2013). Formación inicial de maestros: Un estudio en el dominio de las matemáticas [Initial teacher training: A study in the field of mathematics]. *Profesorado: Revista de Currículum y Formación del Profesorado*, 17(3), 185-200.
- Nortes, R., & Nortes, A. (2018). ¿Tienen los futuros maestros los conocimientos matemáticos elementales? [Do future teachers have basic mathematical knowledge?]. In L. J. Rodríguez-Muñiz, L. Muñiz-Rodríguez, A. Aguilar-González, P. Alonso, F. J. García-García, & A. Bruno (Eds.), *Investigación en educación matemática XXII* (pp. 397-406). SEIEM.
- Potari, D. (2001). Primary mathematics teacher education in Greece: Reality and vision. *Journal of Mathematics Teacher Education*, 4, 81-89. <https://doi.org/10.1023/A:1009983015697>
- Ramírez Uclés, R., Ruiz Hidalgo, J. F., Flores Martínez, P., & Fernández Plaza, J. A. (2019). Tareas que desarrollan el sentido matemático en la formación inicial de profesores [Tasks that develop mathematical sense in initial teacher training]. *Educación Matemática*, 31(1), 131-143. <https://doi.org/10.24844/EM3101.05>
- Senk, S. L., Tatto, M. T., Reckase, M., Rowley, G., Peck, R., & Bankov, K. (2012). Knowledge of future primary teachers for teaching mathematics: An international comparative study. *ZDM Mathematics Education*, 44, 307-324. <https://doi.org/10.1007/s11858-012-0400-7>
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14. <https://doi.org/10.3102%2F0013189X015002004>.
- Star, J. R. (2023). Revisiting the origin of, and reflections on the future of, pedagogical content knowledge. *Asian Journal for Mathematics Education*, 2(2), 147-160. <https://doi.org/10.1177/27527263231175885>
- Tatto, M. T., & Menter, I. (2023). The importance of comparative framing in the study of teaching and teacher education. In C. J. Craig, J. Mena, & R. G. Kane (Eds.), *Studying teaching and teacher education (Advances in research on teaching, Vol. 44)* (pp. 281-297). Emerald Publishing Limited, Leeds. <https://doi.org/10.1108/S1479-368720230000044028>
- VVAA. (2020). *La educación matemática en las enseñanzas obligatorias y el bachillerato [Mathematics education in compulsory education and high school]*. El Libro Blanco de las Matemáticas. Real Sociedad Matemática Española y Fundación Ramón Areces. <https://www.fundacionareces.es/recursos/doc/portal/2020/10/14/libro-blanco-de-las-matematicas.pdf>
- Weintrop, D., Beheshti, E., Horn, M., Orton, K., Jona, K., Trouille, L., & Wilensky, U. (2016). Defining computational thinking for mathematics and science classrooms. *Journal of Science Education and Technology*, 25, 127-147. <https://doi.org/10.1007/s10956-015-9581-5>

## APPENDIX

Table A.1. Data of Greek universities

Region	University	Type	Early Childhood Education						Primary Education								
			ECTS		Subject type (C)			Courses		ECTS		Subject type (C)			Courses		
			C	O	ME	M	M&ME	Start	End	Total	C	ME	M	M&ME	Start	End	
Aegean Islands (Rhodes)	U. of the Aegean	Public	20	25	20				1	3	42	22	16	6		1	4
Central Greece	National and Kapodistrian U. of Athens	Public	13	25	8	5			1	4	52	20		20		1	4
Crete (Rethymnon)	U. of Crete	Public	8	12		4	4		1	2	36	8	4	4		1	2
Epirus	U. of Ioannina	Public	16	28	6	6	4		2	3	35	13	5	8		1	3
Macedonia	Aristotle U. of Thessaloniki	Public	6	6			6		1	2	31	8	8			2	3
Patras	U. of Patras	Public	10	15	5	5			2	3	20	10	5	5		2	3
Thessaly (Volos)	U. of Thessaly	Public	6	5	6				2	2	36	16	5	12		1	2
Thrace (Alexandroupolis)	Democritus U. of Thrace	Public	13,5	9		9	4,5		1	3	50	15	5	10		1	3
Western Macedonia (Florina)	U. of Western Macedonia	Public	8	13		4	4		2	3	42	14	10	4		1	3

Table A.2. Data of Romanian universities

Region	University	Type	ECTS		Subject Type (C)			Course	
			C	O	ME	M	M&ME	Start	End
Alba	U. 1 Decembrie 1918 din Alba Iulia	Public	8		4	4		1	2
Arad	U. Aurel Vlaicu din Arad	Public	7		3	4		1	2
Bacău	U. Vasile Alecsandri din Bacău	Public	10	7	5	5		2	2
Braşov	U. Transilvania din Braşov	Public	10	6	5	5		2	2
Bucureşti	U. din Bucureşti	Public	12		4	8		1	3
Cluj	U. Creştină Dimitrie Cantemir	Priv	10	5	5	5		1	2
Constanţa	U. Spiru Haret din Bucureşti	Priv	8		4	4		2	2
Dolj	U. Babeş-Bolyai Cluj-Napoca (Romanian)	Public	6		3	3		3	3
Galati	U. Babeş-Bolyai Cluj-Napoca (German)	Public	9		5	4		1	2
Iaşi	U. Babeş-Bolyai Cluj-Napoca (Hungarian)	Public	10	5	5	5		2	2
Oradea	U. Tehnică din Cluj-Napoca	Public	8		5	3		2	2
Argeş	U. Ovidius din Constanţa	Public	12		6	6		1	2
Sibiu	U. din Craiova (más sedes)	Public	10		3	7		1	2
Suceava	U. Ştefan cel Mare din Suceava	Public	8		5	3		1	1
Dâmboviţa	U. Valahia din Târgovişte	Public	11	3	8	3		1	2
Gorj	U. Constantin Brâncuşi din Târgu Jiu	Priv	10	2	7	3		1	2
Mureş	U. George Emil Palade din Târgu Mureş	Public	14	4	8	6		1	2
Timiş	U. de Vest din Timisoara	Public	7		4	3		1	2
Ilfov	U. Adventus din Cernica	Public	6	3	3	3		1	1

