International Electronic Journal of
Mathematics Education
Volume 2, Number 2, July 2007 www.iejme.com

EMPIRICISM, CONTINGENCY AND EVOLUTIONARY METAPHORS: GETTING BEYOND THE "MATH WARS"

Kurt Stemhagen

ABSTRACT. This article provides a philosophical conceptualization of mathematics *given the particular tasks of its teaching and learning*. A central claim is that mathematics is a discipline that has been largely untouched by the Darwinian revolution; it is a last bastion of certainty. Consequently, mathematics educators are forced to draw on overly absolutist or constructivist accounts of the discipline. The resulting "math wars" often impede genuine reform. I suggest adopting an evolutionary metaphor to help explain the epistemology/nature of mathematics. In order to use this evolutionary metaphor to its fullest effect in overcoming the polarization of the math wars, mathematical empiricism is presented as a means of constraint on the development of mathematics. This article sketches what an evolutionary philosophy of mathematics might look like and provides a detailed descriptive account of mathematical empiricism and its potential role in this novel way of thinking about mathematical enterprises.

KEYWORDS. Philosophy of Mathematics Education, Empiricism, Evolution, Math Wars, Philip Kitcher, John Dewey, Imre Lakatos, John Stuart Mill.

INTRODUCTION

As a middle school mathematics teacher, I was frequently frustrated by what went on in the classroom. Theorists and practitioners in other subject areas have worked to explicitly link the role of human agency to their respective disciplines and to find ways to apply school knowledge in reasonably realistic contexts. In language arts, science, and history, emphasis on traditional conceptions of subject matter as given and inert has shifted to an increased focus on the role people play in the creation and perpetuation of the disciplines. Furthermore, there has been at least a tacit change in the way knowledge within each discipline is conceived of, as the social dimensions of each are now recognized, if not embraced[1]. For a number of reasons, some of which will be the focus of this article, similar changes in the way mathematics is regarded has not taken place. I submit that the teaching and learning of mathematics has suffered as a result.

Copyright © 2007 by GOKKUSAGI

ISSN: 1306-3030

[1] In science, the work of philosophers of science, such as Kuhn and Popper, coupled with a post-Sputnik concern for relevant, applicable science education facilitated shifts in the way science is taught and learned in school. Increasingly, science class has become a place where students play the role of fledgling scientists. History class, with the advent of computers and other technology, has undergone a similar metamorphosis. Traditionally a place where students were expected to memorize names, dates and places; it has become a potentially dynamic forum for students to act as mini-historians, using the newfound ease of access to primary source data as a means to work as historians, discovering and interpreting historical material.

Part of the problem is that when educators turn to professional mathematicians for inspiration regarding conceptualization of the nature of mathematical enterprises and of the place of mathematics in the lives of humans, the results are often confusing. Many mathematicians hold the belief that mathematics bears no inherent ties to the physical world and possibly even to human life beyond the mathematics department (Hersh, 1997; Kline, 1980). Philosophical stances that tend to perpetuate this point of view often are forms of mathematical absolutism, that is, they describe mathematics as certain, objective, wholly logical, and based on objects or truths outside of human and earthly influence. Bertrand Russell provides a classic example of this way of thinking, referring to mathematics as a "beautiful world; it has nothing to do with life and death and human sordidness, but is eternal, cold and passionless…(it has) an immense dignity derived from the fact that its world is exempt from change and time…The only difficulty is that none but mathematicians can enter this enchanted region" (Griffin, 2002, p. 224)[2].

Recent reform efforts that have sought to include social facets of mathematical knowledge in mathematics education have worked against these particularly entrenched understandings of the mathematical enterprise. Reformers position themselves against these rigid conventions by offering subjective, relative, and fallible versions of mathematics. The resulting "math wars" position rigor–oriented, "back–to–basics" traditionalists against child–centered, application–focused constructivists. This "either/or" forced choice has made it so that many of the reform–oriented mathematics educators' work is rejected apriori by those who self–identify as traditionalists. Likewise, this polarization often makes it difficult for reformers to acknowledge that mathematics does seem to be an especially stable form of knowledge. My concern is primarily with the former situation, as resistance to the high quality work of many thoughtful reformers seems unnecessary and counterproductive to mathematics education[3].

There appear to be similar conflicts occurring in other areas of the curriculum. The phonics versus whole language debate seems, at first blush, to be analogous to the polarized confrontation at the heart of the math wars. However, there is at least one important difference. The whole language–phonics debate is primarily about teaching methods, whereas the math wars–while certainly concerned with teaching methods–also possess philosophical dimensions. In other words, while phonics–whole language debates tend to focus on ways to teach the subject, mathematical traditionalists and constructivists, while arguing over curriculum and methods on the surface, often do so from differing positions as to the very nature of mathematics. Lerman's three "levels" of mathematics education provides a useful model in this regard: "At the first level are the surrounding (sometimes called foundation) disciplines of psychology, sociology, philosophy, anthropology, (in our case) mathematics, and perhaps others. At the second level are mathematics education and other curriculum areas of educational research. At

---

2 Responses to a recent article on mathematics education and social justice (Stemhagen, 2006) suggest that contemporary attitudes share much with Russell's. One mathematics/computer science graduate student, presumably parroting talk within his discipline, claimed: "if there's one thing I'm sure of, it's that mathematics has nothing to do whatsoever with…any aspect of the physical world…math lives in its own separate Platonic world and we mere mortals can only hope for an occasional peek inside" (Kontorovich, 2007).

3 The competing websites *Mathematically Correct* (2007) and *Mathematically Sane* (2007) demonstrate this sharply oppositional position taking, each encouraging the cultivation of a perspective that fundamentally rules out the other.

the third level are curriculum and classroom practice" (Lerman, 2000, p. 20).

My claim is that underlying philosophical positions (Lerman's first level) can, at times, influence mathematics educational practices (his third level). In the context of the math wars, traditionalists tend to draw on absolutism–viewing mathematics as certain, permanent, given, and independent of human activity. Reformers, on the other hand, generally draw on philosophical constructivism and focus on the ways in which humans actively create mathematical knowledge, rendering mathematics contingent on human activity. While having much to offer mathematics education, both provide incomplete accounts of mathematical knowledge and mathematical activity. Absolutism suggests an understanding of mathematics that captures its unique stability and universality but that does not acknowledge its human dimensions. Conversely, constructivism tends to encourage understandings of mathematics that feature human involvement but, in doing so, seems to lose the ability to explain the remarkable stability and universality of mathematical knowledge[4].

Rorty's distinction (1999) between those who view phenomena as fundamentally "found" versus those who see it as "made" provides an interesting analogy, as mathematical absolutists can be thought of as seeing mathematical as knowledge as found, whereas constructivists often conceptualize it as made. A la Rorty (drawing on Dewey), I argue that a way beyond this either/or stalemate is needed if theoretical considerations are to have a role in improving mathematics education and I seek to provide a philosophical understanding of mathematics that is able to account for both its stability–universality and its contingent human–influenced qualities. I do this by conceiving of the discipline in evolutionary terms.

Evolutionary models run the risk of not being adequately sensitive to the strengths of the claims of mathematical absolutists (namely regarding the stability and universality of mathematics), and I argue that the constraints on mathematics posed by an empirical conceptualization of its development and even facets of contemporary practice are consistent with an evolutionary account and sufficiently restrictive to depict the stability of mathematics. Thus the bulk of the paper is devoted to providing an account of a particular form of mathematical empiricism and a defense of the merits of acknowledging empirical dimensions of the discipline as a means to develop ways of thinking about mathematics that retain the "best of both worlds"–the absolutist focus on mathematical stability and the constructivist contribution of illuminating the human and contingent dimensions of mathematics.

By way of disclaimer, my goal is not to settle any larger metaphysical or ontological scores as to the ultimate nature of mathematics. Instead, my work is situated in the philosophical tradition of American pragmatism and, accordingly, in this article I work to develop a functional conceptualization of mathematics *given the task of its teaching and learning*. It is possible, even likely, that adoption of other understandings of mathematics is more fruitful for those with other

[4] For more on the many varieties of constructivism, see Phillips (2000).

purposes. For example, the non–empirical even Platonic philosophical understandings often employed by pure mathematicians might serve very well as they engage in their disciplinary practices (Hersh, 1997). My pragmatic argument is that, because many working mathematicians and philosophers of mathematics find consideration of the empirical dimensions of mathematics distasteful and perhaps even fruitless, it does not follow that such consideration is not useful for those who teach and learn mathematics. In the words of Larry Hickman, for the pragmatist, "function trumps ontology" (2001, p. 32).

EVOLUTION AND MATHEMATICS

My claim is that mathematics is a discipline that has been largely untouched by the Darwinian revolution. In essence, it is a last bastion of certainty. Take, for example, the work of Daniel Dennett, respected philosopher and avowed evolutionist. In *Darwin's Dangerous Idea*, he develops an argument for adopting the process of evolution by natural selection as an overarching explanatory principle for almost all phenomena. He patiently explains how evolutionary biology works and then develops the thesis that our socio–cultural objects can also be thought of as an evolutionary development. He places cars, libraries, and political freedom on the "tree of life." In this way, Dennett recognizes that just as species could have turned out otherwise (he talks of the vast design space and how small a portion of it is taken up by the tree of life), so could have our socio–cultural artifacts.

Dennett places two things beyond the pale of natural selection: the laws of physics and logic/mathematics (Dennett, p. 129). Dennett forwards a fundamentally ahistorical and extra–human version of mathematics. To Dennett, while most things are changing over time, mathematics does not evolve – it just is. In what follows, I present the work of several thinkers in an effort to make philosophy of mathematics somewhat more amenable to evolutionary thinking, all the while being careful to consider the ways in which a focus on the empirical dimensions can provide a sufficient explanation of the constraints on evolutionary developments. Philip Kitcher's empiricism provides the map for this new direction. The mathematical thought of J. S. Mill, John Dewey, and Imre Lakatos are used to flesh out Kitcher's framework.

KITCHER'S MATHEMATICAL NATURALISM: A POINT OF DEPARTURE

Kitcher describes his project as offering a much–needed alternative to mathematical absolutism, as most work in philosophy of mathematics has featured arguments between factions within the absolutist school of thought (1988, p. 3). Kitcher explains that one reason why absolutism is so prevalent is that philosophers of mathematics typically have not had much choice, other than absolutism or some overly simple version of empiricism (p. 294). He attempts to develop a more careful version of empiricism as a critical component of his overall philosophy of mathematics. Kitcher claims that the physical manipulation of objects plays a role in actually creating mathematics (p. 5).

Kitcher's Debt to Mill's Empiricism

While it is beyond the scope of this paper to go into much detail on J. S. Mill and Gottlob Frege, a bit of background is in order, as Kitcher's empiricism draws on what Kitcher calls a formulation of: "Mill's optimal position" (1980, p. 215). Furthermore, their confrontation has much to do with the subsequent relative disrepute of mathematical empiricism (Kessler, 1980). Mill theorized that mathematics is not to be understood as the study of abstract objects, but instead that it consists of truths that are demonstrated by empirical observations, inductions from experience: "All numbers must be numbers of something; there are no such thing as numbers in the abstract. Ten must mean ten bodies, or ten sounds, or ten beatings of the pulse" (Mill, p. 167).

Mill was not content to relegate arithmetic only to relations between sets of specific instances. While he argued for the empirical origins of mathematics, he also recognized its stability and generalizability: "…though numbers must be numbers of something, they may be numbers of anything. Propositions, therefore, concerning numbers have the remarkable peculiarity that they are propositions concerning all things whatever…" (Mill, p. 167).

Frege's opposition to Mill's empiricism was intense and largely successful, as Frege's lasting influence and the relative obscurity of Millian mathematics attests. Frege understood Mill to say that number is a quality of groups of objects. Frege used color as an example of a quality that an object either possesses or not and he pointed out that number is a fundamentally different phenomenon. As for Mill's contention that number comes from arranging objects, Frege stingingly responded: "if Mill is right, we are very lucky that not all objects in the world are nailed down, for otherwise it would be false that 2 + 1 = 3" (1997, p. 94). Frege's critique of Mill effectively kept empiricism out of the mainstream in philosophy of mathematics discussions for the better part of the last century.

Links between the History and Epistemology of Mathematics

Kitcher states: "most philosophers of mathematics have regarded the history of mathematics as epistemologically irrelevant" (1983, p. 5). Kitcher's version of mathematics is very nearly the opposite of such mainstream accounts, as he argues that mathematics is a product of its past and that it can be better understood if this history is taken into account (1988, p. 298). Kitcher's ideas can be broken down into two main parts, the origins and development of mathematics (1983, p. 96).

Kitcher asks: "How do we come to know mathematics?" (1988, p. 297). Even though at this point he is working primarily within the context of the community of professional mathematicians and the portions of mathematics on which he is focused are the established axioms, rather than turn to proof as the sole means by which to "know" the veracity of mathematics, Kitcher turns to education. He proclaims: "In almost all cases…they were displayed on a blackboard or discovered in a book, endorsed by the appropriate authorities, and committed to the learner's memory" (Kitcher, 1988, p. 297). In short, according to Kitcher, our understanding of mathematics is based in large measure on the mathematics that we were taught in school. Kitcher points out that mathematics education makes it so that: "mathematical knowledge is not built up from the beginning in each generation" (p. 298). Implicit in this idea is that, without the common historically–funded version of mathematics that students take in during their school years, understandings of what mathematics is would be different.

Kitcher's second observation concerns the specific ways in which we come to understand mathematics: "some of this knowledge is acquired with the help of perceptions" (1983, p. 92). Kitcher points out that early in our mathematical development we use manipulatives, such as rods and beads and that later we use diagrams to understand more complex mathematics (geometry is one obvious example). Still more advanced mathematics has less in common with "everyday" physical objects but can be, Kitcher argues, linked to its empirical roots through a succession of transformations[5]. The idea that an individual can learn mathematics in an isolated and relatively synchronic manner is, to Kitcher, wrongheaded: "… the community supplements primary source (authorities) with local justifications, providing the student with ways of looking at mathematical principles which seem to make them obvious. So it comes to appear that the mathematician, seated in his study, has an independent, individual means of knowing the basic truths he accepts" (1983, p. 93).

Kitcher enriches the notion of a mathematical community by postulating that the mathematical practices in which the community engages can be conceived of as consisting of five components: a language unique to mathematicians, a set of accepted statements, a set of questions that are taken to be important and not currently settled, a set of reasonings used to justify accepted statements, and a set of views regarding how mathematics is to be done (1988, p. 299). After presenting his idea of mathematical practice, Kitcher is able to summarize his theory of mathematical development: "I claim that we can regard the history of mathematics as a sequence of changes in mathematical practices, that most of these changes are rational, and that contemporary mathematical practice can be connected with the primitive, empirically grounded practice through a chain of interpractice transitions, all of which are rational" (1988, p. 299). So Kitcher envisions mathematical development as taking place through a chain of knowers, each loosely bound by the mathematics they learned as well as the conventions particular to communities of mathematicians.

In terms of the origins of mathematics, Kitcher is a bit less clear. He certainly offers an account that is at least somewhat in line with Mill's empiricism, albeit a reconsidered version. At times, he sounds very Millian, such as when he explains how this "chain of knowers" began:

[5] Elsewhere, I use the work of C. S. Peirce to make the case that even the highly abstract endeavors of today's professional mathematicians have empirical dimensions. In fact, to Peirce, a primary role of the mathematician is to create manipulatable models of phenomena. For more on this, see Peirce (1898).

> Here I appeal to ordinary perception. Mathematical knowledge arises from rudimentary knowledge acquired by perception…our ancestors, probably somewhere in Mesopotamia, set the enterprise in motion by learning through practical experience some elementary truths of arithmetic and geometry. From these humble beginnings mathematics has flowered into the impressive body of knowledge which we have been fortunate to inherit. (1983, p. 5)

Kitcher intimates that the originators of mathematics made their mathematical observations in the midst of trying to solve practical problems. This seems a step in the direction toward the avoidance of the development of a version of the earliest mathematicians that has much in common with the solitary thinker sitting in his study that he dismissed as an unacceptable way to conceive of the contemporary mathematician.

This sketch of Kitcher's philosophy of mathematics depicts an unconventional theory that draws on a form of empiricism, history, and community in its effort to provide an alternative to absolutism. However, there are problems with Kitcher's work, including an overly passive account of how we use mathematics and that in spite of references to a chain of knowers, he does not render clear a connection between the empirical origins of mathematics and what he calls its current highly abstract state.

Kitcher's Work as the Basis for an Evolutionary Framework

Although I argue that Kitcher's theory is not truly evolutionary, it is fairly revolutionary. In bringing empiricism back into the conversation through his notion of perceptions as a critical component of the origins of mathematics, Kitcher has reintroduced psychological explanations of mathematics. Additionally, by linking contemporary practice to the origins of mathematics through a chain of groups of knowers, Kitcher has brought both the history of mathematics and the influence of community into play. Both accomplishments are important. In what follows, I use Dewey's pragmatic and psychological philosophy of mathematics to strengthen Kitcher's claims regarding the empirical origins of mathematics. Kitcher's reliance on Mill leads to an account of the empirical origins that does not sufficiently recognize the functional role of mathematics in human activity. The result is that Kitcher has trouble explaining how empirical origins have led to the highly abstract nature of contemporary mathematics, and Dewey's functional version of how human employ empirical objects is one way to connect today's mathematics to its origins, given Kitcher's general framework. Lakatos' historical explanation of the development of mathematics is next employed in order to flesh out Kitcher's notion of a "chain of knowers" and to help make the connection between the origins of mathematics and its contemporary state.

DEWEY'S PSYCHOLOGY OF NUMBER

In spite of John Dewey's massive influence within the philosophy of education, his work is rarely invoked in contemporary conversations within the philosophy of mathematics education. His non–presence is particularly vexing in that, Dewey's co–authored mathematics education book, *The Psychology of Number and its Applications to Methods of Teaching Arithmetic* (1900) featured a philosophical approach to both educational psychology and mathematics education. In what follows, I introduce Dewey's thought in order to argue that, in spite of an aversion to purely empirical or historical approaches, his philosophy of mathematics is not unfriendly to Mill's empiricism and Kitcher's historical conception. Additionally, Dewey's exploration of the psychological processes involved in an individual's coming to know mathematics provides a point of entry for human elements into a discipline (philosophy of mathematics) that has frequently worked to explain mathematics in non–human, anti–psychological terms.

Psychology has often been viewed largely as something to be overcome or ignored in philosophical work, as there is fear that mental processes can be a serious impediment to understanding how the world "really is." According to this traditional philosophic conception of psychology there is a sharp line between the mental and the physical. Dewey sought to mediate between those tendencies that focus disproportionately on either the mental or physical aspects of existence. Dewey saw this polarization as overly static and inaccurate, and one way he combated this way of thinking was to employ "psychology" in an unorthodox manner.

Dewey's inclusive and activity–sensitive psychology is at the core of his more general pragmatic beliefs. According to his pragmatic conception of how we know and what there is to know; knowledge, belief, and psychology are inextricably linked:

> I start and am flustered by a noise heard. Empirically, that noise is fearsome; it really is, not merely phenomenally or subjectively so. That is what it is experienced as being. But, when I experience the noise as a known thing, I find it to be innocent of harm. It is the tapping of a shade against the window, owing to movements of the wind. The experience has changed; that is, the thing experienced has changed not that an unreality has given place to a reality, nor that some transcendental (unexperienced) Reality has changed, not that truth has changed, but just and only the concrete reality experienced has changed… This is a change of experienced existence effected through the medium of cognition. (1910, p. 230)

In Deweyan terms, the world is as it is experienced. According to his immediate empiricism, or pragmatism, philosophers are obligated to employ psychology in understanding experience. This is radical, as to many philosophers, psychology is a seen as a barrier to logic, obscuring the contents of the logical, a priori realm. The Deweyan reconception of logic and psychology in light of human activity posits psychology and even logic merely as modes by which we undertake the act of figuring out how to live our lives. Psychology encompasses the mental processes by which we actually think (and live) and Dewey goes so far as to characterize logic as empirical in origin, coming about as fruitful methods of inquiry are recognized, emulated, and eventually formalized (1967, p. 138).

With regard to mathematics, Dewey conceived of number as transactional in nature – residing within the processes of mathematical activity. We come to use number only after a great deal of rational and abstract thought. In support of his claim that the sensory input with which we work, while rich in raw information as to the multiplicity of things in nature offers no insights regarding the notion of number, Dewey wryly noted: “There are hundreds of leaves on the tree in which the bird builds its nest, but it does not follow that the bird can count” (McLellan & Dewey, 1900, p. 23).

Dewey developed a technical descriptive account of how children come to know mathematical concepts. This account centered on the mental activities of children as they encountered various empirical situations. Without the psychological processes he detailed, there would only be the ideas of “much” and “many” but not the more refined notions of “how much?” and “how many?.” Dewey understood this simple sense of quantity coming about in light of the human need to measure in order to live more efficient and better lives (p. 42).

Dewey sought to blur the commonly understood distinction between counting and measuring. Traditionally conceived, counting is related to determining how many of something there are and measuring involves determination of how much of something there is. In other words, the question is whether some phenomenon is a series of parts of one whole, or a related group made up of individual units. Dewey’s pragmatic answer was that they may be either and that the context and the needs of the counter/measurer must be taken into account when answering the question.

So, to Dewey, mathematics can best be defined and understood by its use. The concept of a particular number (say three) does not reside within a group of three apples, beanbags or any other objects any more than it does in the symbol “3.” The concept of three emerges from the activities in which we engage that utilize quantification (measuring) as a means to an end. The accompanying pedagogy suggested by this philosophy of mathematics focused on measurement. This is a reasonable suggestion, as, to Dewey, all counting is measuring and all measuring is counting. Making measurement the vehicle for mathematical explorations ensured, according to Dewey, that number symbols will always be linked to concrete units and encouraged active, empirically–oriented, and contextualized conceptions of mathematical enterprises.

With his work in philosophy of mathematics and mathematics education, Dewey created an alternative to what he considered to be overly empiricist and rationalist options of his day. Thus, it would be easy to conclude that Mill’s empirical explanation of mathematical knowledge did not have much in common with Dewey’s. While Dewey’s version of mathematics emphasized the interplay between empirical objects and our actions, I argue that Mill’s philosophy of mathematics contained more than just simple empiricism; it showed a nascent acknowledgement of the role of human intent in the construction of mathematical knowledge[6]. Although Mill never fully articulated the pragmatic notion that mathematical knowledge is actually created by the interaction of our activities and the physical world, he did come fairly close. For example, Mill stated: “… Two pebbles and one pebble are equal to three pebbles… affirms that if we put one pebble to two pebbles, those very pebbles are three” (Mill, p. 168). Although not the major point that Mill is attempting to make, here he is clearly affording the

[6] While pragmatism and Millian empiricism might seem strange bedfellows, note that William James dedicated *Pragmatism*, to: “the memory of John Stuart Mill from whom I first learned the pragmatic openness of mind and whom my fancy likes to picture as our leader were he alive to-day” (James, 1978, p. 4).

human act of "putting" the pebbles a place in his understanding of the nature of the number 3. About Mill, Kitcher makes a similar point:

> We have seen that Mill suggests that apparent references to classes can be parsed away. His idea is to say that objects belong to a class is to assert that we regard those objects as associated… Thus the root notion in Mill's ontology is that of a collecting, an activity of ours, rather that that of a collection, an abstract object… At times, Mill seems to come very close to an explicit proposal of this kind. (Kitcher, 1980, p. 224)

The similarities between Kitcher's formulation of Mill's "optimal" position and Dewey's account of mathematics are striking. Dewey's general tenet of conceiving of mathematics as arising from human activity (measuring) certainly is friendly to Mill's account. There are also parallels between Dewey and Mill's ideas of how children come to know number. Mill's semi–recognition of the active role of the child in establishing the concept of number is somewhat sympathetic to Dewey's version. Kitcher says of Mill's version of learning arithmetic: "Children come to learn the meanings of 'set', 'number', 'addition' and so forth by engaging in activities of collecting and segregating" (1980, p. 224).

Dewey is helpful in fleshing out Kitcher's intriguing but under–developed idea that the first mathematical ideas were empirical. Whereas Mill's work aids the project of developing an evolutionary understanding of mathematics through its recognition of the role of the physical world in shaping the development of mathematics, Dewey–while similarly acknowledging the role of the physical world–goes farther. He makes clear the critical component of the role that human activity plays in this development.

While the physical world provides some constraint on the potential directions the development of mathematics may take, so too do the ways in which we choose to use mathematics. According to Dewey's pragmatic account, our mathematics is what it is – in addition to empirical limitations on how mathematics "work"– because of the ways in which we live our lives. If we lived our lives differently (due to chance, choice, accident, or other circumstance) it is conceivable that our mathematics would be different. For example, had our ancestors not needed to use geometry to organize their experiences as farmers, the ways in which we systematized spatial relations might be different than they are. Just as Kitcher warns about Mill, it would be a mistake to view Dewey as a simple constructivist, as Dewey's focus on function over structure makes clear. To Dewey, the development of mathematics is driven by the ways in which we use it (i.e., its functions). A common feature of most different brands of constructivism is their structuralism, as most constructivist accounts posit some underlying structure that can account for the development of mathematics. Dewey's functionalism suggests a means to consider whether the constructions are good ones. In other words, the functional approach tests constructions by acting on them. If the results are satisfactory, then the construction is more than mere belief. The beginnings of an evolutionary philosophy of mathematics is beginning to emerge, as the functionality of mathematical ideas leads to their acceptance and subsequent influence on future mathematical ideas.

LAKATOS AND MATHEMATICAL DEVELOPMENT

Imre Lakatos developed what he described as a "quasi–empiricist" philosophy of mathematics. He compared the discipline of mathematics to a Popperian version of science, putting forward a conception of mathematics that rejects absolute certainty as a goal and acknowledges the ways in which mathematicians develop mathematical knowledge through an unending series of hypotheses and critiques. Lakatos coined the term quasi–empiricism to show how mathematics and science are similar in method but differ as to content. Science uses the physical world as a basis for its experiments while mathematics uses mathematical ideas.

Although the comparison between mathematics and science is interesting and useful, I contend that Lakatos' work is pertinent to this project for another reason. Dewey's work offered a functionalist account of how lay people develop mathematical understandings. Lakatos, on the other hand, tended to focus on the ways in which mathematics is done within organized professional (or at least academic) communities of mathematicians where strictly absolutist versions of mathematics are the norm. Rather than thinking of proof as the way mathematicians uncover the immutable Truth that is mathematics, Lakatos conceived of proof more pragmatically. As Kitcher explains:

> His central thesis is that the role of proofs in mathematics is misunderstood, and he suggests that this crucial misunderstanding will affect the quality both of mathematical research and of mathematical education. The mistake is to regard proofs as instruments of justification. Instead we should see them as tools of discovery, to be employed in the development of mathematical concepts and the refinement of mathematical conjectures. (1977, p. 782)

Dewey thought of mathematics as a set of tools we have developed in order to solve problems in an effort to lead better lives. Lakatos shared an understanding of mathematics that employed the tool metaphor, but his focus was different than Dewey's. Lakatos' work was primarily concerned with the use of mathematics as a tool for inquiry within the discipline of mathematics, be it the professional or school varieties of mathematics. Lakatos describes the purpose of his work, *Proofs and Refutations*, as an attempt to cast doubt on the idea that formalist accounts of mathematics are sufficient[7]. He sets the stage for his own historically–oriented version: "Formalism disconnects the history of mathematics from the philosophy of mathematics, since, according to the formalist concept of mathematics, there is no history of mathematics proper" (1976, p. 2). He goes on to point out that formalism fails to accept most of what is typically thought of as mathematics as part of the discipline and that it offers a version of mathematics in which nothing meaningful can be said about its development: "None of the 'creative' periods and hardly any of the 'critical' periods of mathematical theories would be admitted into the formalist heaven, where mathematical theories dwell like the seraphim, purged of all the impurities of earthly uncertainty" (1976, p. 2).

Proofs and Refutations is the fictional story of a discussion set in a mathematics classroom. As Lakatos explains: "The class gets interested in a PROBLEM: is there a relation

[7] Formalism is a particular type of absolutism that posits mathematics as a set of formal rules.

between the number of vertices V, the number of edges E and the number of faces F of polyhedra – particularly of regular polyhedra – analogous to the trivial relations between the number of vertices and edges of polygons (they are equal)?" (p. 6)[8]. The class uses trial and error and comes up with a formula: $V - E + F = 2$[9]. Students surmise (conjecture) that it is likely that this formula will be true for all regular polyhedra. Attempts to falsify the conjecture fail, thus suggesting that a proof will demonstrate the truth of the statement. The bulk of the work is dedicated to what happens once a teacher offers the proof. What starts as a reasonably simple exploration of a particular mathematical problem becomes an increasingly complex discussion about the nature of mathematical inquiry and also of proof itself.

Proofs and Refutations would be an impressive achievement if it simply presented an account of a group of students struggling with the notion of proof, but there is another dimension to the work. A series of detailed footnotes explains how each development in the class discussion has an analog in the historical development of this particular piece of mathematics. One interpretation of his argument could be that the history of mathematics is relevant to its teaching and learning. While this is clearly a point of his, it seems likely that Lakatos had something more radical in mind. According to Kitcher: "I think that Lakatos has demonstrated that there are important issues about mathematical discovery that should not be neglected. The process of mathematical discovery cannot be dismissed (as it so often has been) as a series of 'happy guesses'" (1977, p. 196). In fact, I take a primary contribution of the work to be that however we choose to characterize mathematics, its development ought to be a consideration.

TOWARD AN EVOLUTIONARY PHILOSOPHY OF MATHEMATICS AND MATHEMATICS EDUCATION

Dewey's insistence of the import of individual psychology to the development of mathematics strikes a blow against the absolutist conceptions of mathematics as unsullied by the imperfect and psychology–ridden human mind. In grounding mathematical experiences in the context of our activities, Dewey's work also counters relativistic and idiosyncratic accounts of mathematics forwarded by radical constructivists, as mathematical knowledge is tied to how we relate to our physical world and can be judged by how well they help us live our lives. Dewey's detailed explanation of how humans come to develop the concepts of number also serves to enrich Kitcher's account of the empirical origins of mathematics. Likewise, Lakatos enriches the historical dimensions of Kitcher's account through an exploration of the ways in which formal mathematics develops, both within the discipline and the classroom. Additionally, Lakatos strengthens Kitcher's notion that the nature of mathematics and its teaching and learning are connected.

---

[8] A polyhedron is a three-dimensional object made up of plane faces, a regular solid.

[9] This formula means that for any regular solid, subtracting the number of edges from the number of vertices and adding the number of faces will come to 2. A cube, for example, has 8 vertices (corners). Subtract its 12 edges (where two faces (sides) meet) to get -4. Finally adding the 6 faces does yield 2. It is likely that trial and error refers to the class looking at cubes, prisms and other polyhedra in hopes of finding the pattern that they eventually found.

The understanding of mathematics presented in this paper grounds mathematical activity in the empirical world, while still recognizing the importance of mental activity in mathematical enterprises. Thus, this perspective is sensitive to the remarkable stability and universality of mathematics without describing it in absolute or otherworldly terms. At the same time, by grounding constructions in the empirical world, this perspective recognizes the role humans play in the creation of mathematical knowledge and the judgment of the worth of such constructions. This reconsideration of mathematical empiricism lays a promising foundation for the development of an evolutionary philosophy of mathematics.

Acknowledging the ways in which human interaction with the physical world affects the development of mathematics can help to get beyond the current philosophical stalemate between absolutist and constructivist philosophers of mathematics. Perhaps more significantly for this project, an evolutionary conception of mathematics – particularly one nourished by a rich and nuanced understanding of mathematical empiricism – offers a promising conceptual base camp from which mathematics educators might be able to find their way out of the morass of the math wars. The practical upshot of this work is not so much to seek to create a new pedagogy or curriculum based on the evolutionary account, although elsewhere I have presented classroom activities that draw on such a conception (Warnick & Stemhagen, 2007). Instead I hope that reintroducing empirical constraints on the development of mathematical knowledge will help make the constructivist/reformer versions of mathematics more palatable to some who currently see themselves allied with the traditionalist side of the math wars. That is, the reintroduction of empiricism helps to soften the sharp either/or forced choice of conceiving of mathematics as either fundamentally precast or as simply the product of however we choose to think about it.

REFERENCES

Dennett, D., (1995). *Darwin's dangerous idea: evolution and the meanings of life*. New York, Simon & Schuster.

Dewey, J., (1910). The postulate of immediate empiricism. *In The influence of Darwin on philosophy and other essays*. New York, Henry Holt and Company.

Dewey, J., (1967). *Reconstruction in philosophy*. Boston, Beacon Press.

Frege, G., (1974). Foundations of arithmetic. In M. Beaney (Ed.), *The Frege reader.* Malden, MA, Blackwell.

Hersh, R., (1997). *What is mathematics, really*? New York: Oxford University Press.

Hickman, L., (2001). Philosophical tools for a technical culture. In S. Rice (Ed.), *Philosophy of Education Society 2001*. Urbana, Ill: Philosophy of Education Society.

James, W., (1928). *Pragmatism: A new name for some old ways of thinking*. Cambridge, MA, Harvard University Press.

Kessler, G., (1980). Frege, Mill, and the foundations of arithmetic. *The Journal of Philosophy*, v LXXVII, no. 2, pp. 65–79.

Kitcher, P., (1977). On the uses of rigorous proof. *Science*, 196, pp. 782–783.

Kitcher, P., (1980). Arithmetic for the Millian. *Philosophical Studies*, 37, pp. 215–236.

Kitcher, P., (1983). *The nature of mathematical knowledge*. New York, Oxford University Press.

Kitcher, P., (1988). Mathematical naturalism. In W. Aspray and P. Kitcher (Eds.), *Minnesota studies in the philosophy of science*, *Volume XI*. Minneapolis, MN, University of Minnesota Press.

Kline, M., (1980). *Mathematics: The loss of certainty.* New York: Oxford University Press.

Kontorovich, L., (2007). *Aboslutely Regular: A math/computer science research blog*. Retrieved January 25, from http://absolutely–regular.blogspot.com/index.html.

Lakatos, I., (1976). *Proofs and refutations*. Cambridge, Cambridge University Press.

Lerman, S., (2000). The social turn in mathematics education research. *In Multiple Perspectives on Mathematics Teaching and Learning* (Ed. J. Boaler). Westport, CT: Ablex Publishing.

Mathematically Correct. (2007). Retrieved, April 16, 2007 from http://www.mathematicallycorrect.com.

Mathematically Sane. (2007). Retrieved, April 16, 2007 from http://www.mathematicallysane.com.

McLellan, J. & Dewey, J., (1900). *The psychology of number and its applications to methods o teaching arithmetic.* New York, D. Appleton & Company.

Mill, J., (1967). *A system of logic*, 8th ed. London, Longman's, Green and Co.

Peirce, C., (1898). Logic of mathematics in relation to education. *Educational Review*, pp. 209–216.

Phillips, D., (Ed.), *Constructivism in education: opinions and second opinions on controvesial issues.* Chicago, University of Chicago Press.

Rorty, R., (1999). Philosophy and social hope. New York: Penguin Books.

Stemhagen, K., (2006). Social justice and mathematics: Rethinking the nature and purposes of school mathematics. *Philosophy of Mathematics Education Journal*, 19. http://www.people.ex.ac.uk/PErnest/pome19/index.htm.

Toulmin, S., (1991). *Cosmopolis: the hidden agenda of modernity*. New York, Free Press.

Warnick, B. & Stemhagen, K., (2007). Mathematics teachers as moral educators: The implications of conceiving of mathematics as a technology. *Journal of Curriculum Studies*, Retrieved April 20, 2007 from http://www.informaworld.com/smpp/content~content=a769546823~db=all~order=pubdate.

Authors : **Kurt Stemhagen**

E-mail : krstemhagen@vcu.edu

Address : Virginia Commonwealth University

School of Education

P.O. Box 842020

1015 W. Main Street

Richmond, VA USA 23284-2020

Phone Number : +1 (804) 827 8415