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**FACTORS THAT IMPACT PRESERVICE TEACHERS' GROWTH IN CONCEPTUAL  
MATHEMATICAL KNOWLEDGE DURING A MATHEMATICS METHODS COURSE**

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**ABSTRACT.** Teachers' conceptual understanding of elementary mathematics is believed to be fundamental to effective classroom level mathematics reform. This study examined preservice teachers' change in conceptual mathematical knowledge after taking a reform-based mathematics methods course as part of a teacher certification program, and investigated the relationship between this change and factors such as preservice teachers' academic background, initial levels of conceptual and procedural mathematical knowledge and values, and the number of mathematics courses taken in high school and university. The results of this study suggest that the number of mathematics courses taken in high school may influence growth in conceptual mathematical knowledge, while preservice teachers' subject-area background and the number of university mathematics courses taken did not appear to influence growth in conceptual mathematical knowledge as needed to teach in a reform-based manner.

**KEYWORDS.** Preservice Teachers, Academic Background, Conceptual and Procedural Mathematical Knowledge, Conceptual and Procedural Mathematical Values, Mathematics Methods Course.

**INTRODUCTION**

Teachers' knowledge about teaching and learning has been cited as the most important predictor of students' success (Greenwald, Hedges & Laine, 1996). Furthermore, teacher's conceptual understanding of mathematics and their ideologies influence students' mathematical learning and values, which permit students to engage or not to engage in a mathematics course (Bishop, Clarke, Corrigan & Gunstone, 2006). It is important to consider how teachers' Copyright © 2009 by GOKKUSAGI

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mathematical knowledge (i.e., “knowledge of mathematical concepts and procedures”) and values (i.e., “mathematical conceptions and ideologies”) influence students’ mathematical knowledge and learning (Ambrose, 2004).

Improving preservice teachers’ conceptual mathematical knowledge before they begin their classroom practice enhances the mathematical knowledge and values that these teachers will initially bring to the classroom (Boyd, 1994; Kajander, 2005; Sowder, 2007). Previous work (Kajander, 2007) and empirical teaching observations suggest that some preservice teachers embrace and demonstrate conceptual change to a much greater extent than others. This study investigated factors such as preservice teachers’ initial capacity (initial levels of conceptual and procedural mathematical knowledge and values, academic background and number of mathematics courses taking at high school and university) that may affect their growth in conceptual mathematical knowledge during a teacher certification program, which included a 36-hour mathematics methods course. The course was designed to promote the concepts of mathematics reform as described by the National Council of Teachers of Mathematics (NCTM) (NCTM, 2000). The question that guided this study was:

1. To what extent do preservice teachers’ initial capacity (initial levels of conceptual and procedural mathematical knowledge and values, academic background and number of mathematics courses taking at high school and university) impact their growth in conceptual mathematical knowledge after taking a mathematics methods course during their teacher certification program?

## **FRAMEWORK**

### **Teachers’ Mathematical Knowledge**

Teachers’ knowledge of mathematics has become an area of concern in the last two decades. There has been an implicit disagreement over the knowledge of mathematics that teachers need to know in order to teach with deep conceptual understanding. Some researchers argue that teachers’ capabilities in higher level mathematics are the most important attributes (Hill & Ball, 2004). Others believe that higher level mathematics ability is not sufficient to teach, and believe that teachers must have knowledge about how to teach mathematics to students (Ma, 1999; Ambrose, 2004; Schommer-Aikins, Duell & Hutter, 2005). Hence, teaching mathematics to students should be treated as a system of interacting features to minimize the gap between teaching and students’ mathematical learning (Hiebert, Stigler, Givvin, Garnier, Smith,

Hollingsworth, Manaster, Wearne & Gallimore, 2005). This system of interacting features, such as the knowledge and values that teachers and students bring to the lesson, tasks presented in the classroom, teaching strategies, students' discourse and participation, the assessments and the physical materials available for teaching is what defines the learning conditions for the students (Ibid). Once the definitions of these learning conditions are established, then what matters is how these features together are enacted with students to help them achieve their goals (Ibid).

Teaching mathematics is a complex enterprise that entails making the content accessible, interpreting students' questions and ideas, and being able to explain concepts and procedures in different ways (Hill, Sleep, Lewis & Ball, 2007). Teachers need to have deep conceptual understanding of the mathematics they are teaching to their students and be able to illustrate to their students why mathematical algorithms work and how these algorithms may be used to solve problems in real life situations (Ibid). Hence, the skills required for teaching mathematics are multidimensional; this means that this capacity does not relate to one general factor such as mathematical ability or teaching ability but rather, it relates to a system of features that interact with one another to help teachers transfer mathematical knowledge to their students (Ibid).

### **Mathematical Values**

In this system of interacting features for teaching mathematics to students, teachers may opt to use a reform-based model for teaching mathematics, in which students may actively contribute to the construction of their mathematical knowledge rather than being passive recipients of information (Johnson & Munakata, 2005). Therefore, it is important to realize that teachers' beliefs about mathematics may influence students' perceptions of mathematical concepts and procedures. For our work, we define values as deeply held beliefs about what is important in mathematics learning. These values have a powerful impact on teaching (Ernest, 1989). In some cases, these values can encourage students to apply, or discourage them from applying, their mathematical knowledge to real life situations or other situations outside the classroom structure (Boaler, 1999). Classroom experiences together with teachers' mathematical values develop students' perceptions of mathematics (Ibid). Accordingly, some students develop the perception that mathematics is just made of numerous rules, formulas and equations that they must memorize; but in other cases, students may come to believe that mathematics is about interacting with the problem, being creative and finding a solution without following a fixed procedure (Ernest, 1989). Students who subsequently choose to become preservice teachers also

tend to arrive at teacher preparation programs with varying experiences and values (Kajander, 2007), and we were interested in the impact of these on subsequent growth.

### **Students' Reform-based Mathematical Learning**

The NCTM *Principles and Standards* (2000) have provided some of the fundamental characteristics of the mathematics curriculum in Ontario (Ross, Hogaboam-Gray & McDougall, 2002). Although the implementation of the reformed mathematics curriculum is not consistent across all the elementary and secondary schools in Ontario (Ibid), students taught using a reform-based approach have more opportunities to develop mathematical learning without merely memorizing formulas (NCTM, 2000). Such an approach minimizes students' fears and concerns about mathematical performance and encourages students to learn in a classroom climate in which risk-taking is encouraged and supported by the teacher and other students in the classroom (Hiebert, 1999). Furthermore, students taught using a reform-based approach are able to acquire greater skills in using mathematical tools to improve their prior knowledge and construct new knowledge than those taught with the traditional mathematics approach, in which the emphasis is more in mathematical procedures (Romberg, 1997). For example, Fennema, Franke and Carpenter (1993) tracked a teacher over four years as the teacher implemented a program that focused on helping students construct deep understanding of mathematical concepts and strategies for solving problems embedded in their everyday experiences. The researchers found that this teacher had a profound effect on her students. Her students solved more complex mathematical problems than other grade 1 pupils and adapted their mathematical procedures in response to problem requirements. Villasenor and Kepner (1993) found that children who were in a classroom that fully implemented mathematics reform were also more successful in traditional mathematics tasks. Hiebert (1999) argued that reform-based teaching programs promote students' deep understanding of mathematics. Cardelle-Elawar (1995) found that providing students with reform-based instruction and including mathematical tasks embedded in real-life experiences contributed to superior grades 3-8 student performance on mathematical problem-solving. Stein, Remillard and Smith (2007) found that the learning environment is a critical factor in students' mathematical learning and that the curriculum implemented in the classroom is more effective when the normative practices in the classroom promote a reform-based learning environment associated with students' mathematical understanding in problem-solving. The researchers also found that students' mathematical achievement was highest among students who experienced a standards-based curriculum in a reform-based learning environment over two consecutive years.

The implementation of reform mathematics, however, is a difficult process (Senger, 1998). Even teachers chosen as exemplars of reform mathematical practices regress from reform methods to traditional methods (Ibid). Indeed, some research studies show that the most challenging in the implementation of reform mathematics is the management of students' talk about mathematical reasoning, including finding the right balance between encouraging student construction of knowledge without leaving them floundering (Ball, 1993; Ross, Haimes, & Hogaboam-Gray, 1996; Smith, 2000). For example, Bosse (1998) studied the recommendations of the *Principles and Standards* (NCTM 1989; 2000) in light of a historical perspective in the United States, focusing on the educational high school reform movement in the mid-1990's. In this study, Bosse emphasizes that the NCTM *Standards* expect K-12 teachers to grasp and develop new curricula philosophically consistent with these *Standards* and related ideas of mathematical reform. Bosse's findings indicate that teachers and the public perceived the new curricular suggestions to be quite extensive and beyond the expertise of the K-12 teachers, whose preparation appeared insufficient to support the reform effort. Earl and Southerland (2003) conducted a similar study but with an emphasis on the perception of students on the impact of reform education in Ontario secondary schools. The researchers found that while some students were very accepting of the new curriculum, others found it to be very condensed and difficult. These varying responses may also suggest difficulties with teacher preparation and capacity.

Extensive evidence suggests that it is important to develop teachers' mathematical content knowledge and values so that teachers can more effectively support students' mathematical learning (Ball, 1990; Ma, 1999; Stipek, Givvin, Salmon & MacGyvers, 2001). The most powerful mechanism for overcoming the barriers to mathematics reform teaching may be appropriate teacher education complemented with professional development (Hill, Schilling & Ball, 2005). Since teachers' mathematical development contributes to students' mathematical success (Greenwald et al., 1996), such professional development is of crucial importance.

### **Teachers' Mathematical Development**

One way to facilitate teachers' mathematical development is by deepening their mathematical understanding and changing their epistemological beliefs via professional development experiences (Hill & Ball, 2004; Kajander, Keene, Siddo & Zerpa, 2006). Kajander et al. (2006) conducted a study of 40 in-service grade 7 teachers from urban and rural areas. She surveyed teachers before and after an eight-month intervention in order to examine mathematical understanding as well as beliefs about mathematics. She provided volunteer teachers with

professional development experiences that emphasized conceptual understanding of fundamental mathematics, appropriate use of manipulatives, use of representations and differentiated instruction. This included three days of professionally delivered in-service training on number and operation, as well as online courses for some of the participants. The researchers found that measureable changes in mathematical knowledge were possible even in such a short time. In addition, teachers' beliefs about the need to focus specifically on procedural learning decreased, which was indicative of a shift towards a more reformed based conception (Kajander, 2005). If conceptual aspects of learning also promote procedural learning with a less specific focus on procedural skills (NCTM, 2007), a diminished emphasis on procedural values which tends to be accompanied by an increase in conceptual values, may be an indicative of a shift to a more reform-based conception (Kajander, 2005). Other researchers (Ahmed, 1987; Ingvarson et al., 2005; Mundry, 2005), have however argued that longer time periods are needed for change.

Ball (1996) also found that the use of professional development experiences can change teachers' traditional ways of mathematical thinking. A deep conceptual re-examination of the mathematics itself can shape teachers' understanding of mathematical concepts and help them be more flexible when listening to students' new ideas and innovations. Teachers need experience linking concrete ideas and mathematical models to new generalizations and procedures, and such mathematics may be highly specific to teaching. Ball concludes that the lack of critical discussion and reflection during professional development experiences may cause teachers to formulate their own interpretation and implementations, making common standards difficult to establish.

Professional development experiences should include a vision that requires teachers to shift their mathematical thinking and values in order to deepen students' mathematical knowledge (Sowder, 2007). This shift in teachers' mathematical thinking and values should occur during their preservice training experiences (Ibid.). The current study has focused on examining factors that may impact preservice teachers' change in conceptual mathematical knowledge during a one year (36 hour) mathematics methods course.

## **THE STUDY**

### *Goals*

The goal of the study was to investigate, via a regression model, factors that may affect preservice teachers' change in conceptual mathematical knowledge after taking a mathematics methods course during their teacher certification program. Factors examined were preservice

teachers' subject area majors ("background"), preservice teachers' conceptual and procedural mathematical knowledge and values at the pretest, and the number of mathematics courses taken in each of high school and university.

### *Methodology*

The design used for this study was a One-Group Pretest-Posttest design. Since this design did not include a control group, a strong causal statement regarding the gains in conceptual mathematical knowledge of students was not possible. Uncontrolled variables such as history, maturation, instrument decay, regression to the means and attitude of subjects may influence the outcome of the study and therefore, were considered threats to the internal validity of the data (Linn, 1989). Nonetheless, the One-Group Pretest-Posttest design has been used in other educational research studies (Ibid) where the inclusion of a control group was not feasible or possible. In this study, the One-Group Pretest-Posttest design was implemented because the mathematics methods course is a compulsory course in the teacher certification program at our university, and thus it was not possible to create a control group.

The mathematics methods course included mathematical content related to patterning, numeracy, geometry and data management and the entire course was taught by one instructor. The National Council of Teachers of Mathematics *Principles and Standards* (National Council of Teachers of Mathematics, 2000) guided the teaching strategies used in the mathematics methods course (NCTM, 2000). Detailed field notes were kept during each class of the course to examine the learning opportunities offered to these preservice teachers' candidates. For instance, in the mathematics methods course, teaching was focused on enhancing preservice teachers' conceptual understanding of the fundamental mathematics needed for teaching at the junior intermediate level by encouraging the preservice teachers to make use of manipulatives and models to help them construct links between their understandings of mathematical concepts and procedures. In many instances, preservice teachers were required to investigate or co-construct models and justifications for standard procedures such as those for fraction or integer operations. For example, preservice teachers were asked to justify the standard fraction multiplication procedure using an area model of the factors and the product, by linking the areas to the numeric steps in the procedure. As well, they worked with various models and manipulatives to construct and justify a number of other operations and procedures. Examples are models of integer operations, and the construction of basic algebraic properties such as expanding binomial products or factoring quadratics.

In addition, the mathematical examples provided to the preservice teachers in the mathematics methods course were thoroughly discussed to allow all preservice teachers taking the mathematics methods course the opportunity to learn and build upon their existing knowledge regardless of their academic background. The curriculum delivered in the mathematics methods course was coherent in the sense that the mathematical problems and ideas were presented with the intention to better prepare preservice teachers to solve mathematical problems with more conceptual understanding. Moreover, preservice teachers were allowed the opportunity to share their ideas with other members of small classroom groups, and encouraged to find other ways to solve the problems and build upon their existing knowledge.

The mathematics methods course instruction aimed to implement as many as possible of the principles of reform in mathematics education by using the NCTM *Principles and Standards* (NCTM, 2000) as a guide. The interpretation of what mathematics reform really is may be a dilemma (Hiebert, 1999), as there is no consistent image of what reform should look like in the classroom, and even less consensus about how it should be measured (Ross, Hogaboam-Gray & McDougall, 2002). Based on the examination of field notes collected during the study year, which documented the activities in all of the mathematics methods course classes, the course exemplified many of the characteristics of mathematics education reform as described in previous research (Ibid). For instance, many examples given in class to the preservice teachers were open-ended problems embedded in real-life contexts, and many of these problems had more than one possible solution method. Furthermore, the instruction was focused on the construction of mathematical ideas through preservice teachers' talk rather than the transmission through lectures and presentations.

The instructor's role in the course was more of a co-learner and creator of a mathematical community rather than sole knowledge expert. The mathematical problems presented to the class were done with the aid of manipulatives and with access to other mathematical tools (calculators and computers) and the assessment of the class was integrated with every-day events. Hence, we believe that the mathematics methods course in this study used a number of key characteristics of reform mathematics. The course was treated as an intervention which potentially might enhance preservice teachers' mathematical knowledge and values.

### *Participants*

Data collected from 111 grades four to ten preservice teachers were used to examine preservice teachers' change in conceptual mathematical knowledge during a mathematics



methods course. All participants were preservice teachers from the one-year professional program in 2005-2006 and, while participation was voluntary, complete data were collected from over 90% of the teacher candidates in the four course sections surveyed. The participants were recruited from all four sections of the Curriculum Instruction in Mathematics course in a faculty of education in a medium sized urban university in Ontario, Canada.

### *Instrument and Measurements*

The instrument used to collect the pretest and post-test data was the Perceptions of Mathematics survey (POM). The validity and reliability of the POM instrument was established in previous studies (Kajander, Knee, Siddo & Zerpa, 2006; Kajander, 2007; Zerpa, 2008). The POM questionnaire was administered at the beginning of the mathematics methods course and again after completion of the six month (36 hour) course.

The POM was used to measure preservice teachers' conceptual and procedural mathematical values and conceptual and procedural mathematical knowledge. The strand measurements for conceptual and procedural mathematical knowledge included number and operations, algebra and measurement. Hence, four variables were measured in this study – conceptual and procedural knowledge, and conceptual and procedural values. These four variables were scaled out of 10 and provided information on preservice teachers' levels of mathematical knowledge and values at the pretest and post-test. In particular we were interested in preservice teachers' change in conceptual mathematical knowledge ( $\Delta CK$ ) from the pretest to the post-test.

Demographic variables such as mathematics courses taken in high school, mathematics courses taken at university, and academic background were also collected via the POM questionnaire. The conceptual and procedural mathematical knowledge and values responses measured by the POM questionnaire at pretest plus the demographic variables measured by the POM were used as independent variables and change in conceptual mathematical knowledge ( $\Delta CK$ ) was used as the dependent variable. Change in conceptual knowledge ( $\Delta CK$ ) was obtained by computing the difference between pretest and post-test conceptual knowledge data collected via the POM instrument.

A t-test for repeated measures was used to analyze the intervention effect between the pre and post-test for the dependent variable ( $\Delta CK$ ). Cohen's effect size (Cohen, 1998) for a repeated measures t-test was computed for the dependent variable ( $\Delta CK$ ) used in this analysis.

Pearson's Product Moment Correlations were explored between change in conceptual knowledge ( $\Delta CK$ ) and each independent variable: conceptual knowledge (CK), procedural knowledge (PK), procedural values (PV), conceptual values (CV), academic background in their undergraduate major (whether arts or more scientifically oriented), number of mathematics courses taken in university and highest year and level of mathematics taken in high school, using the pretest data. The strengths of the correlations were examined to identify which factors at the pretest (conceptual knowledge, procedural knowledge, procedural values and conceptual values) plus demographic variables (academic background, university mathematics courses and high school mathematics courses) significantly related to change in conceptual knowledge and potentially could be used as predictors of change in conceptual knowledge ( $\Delta CK$ ).

A regression analysis was performed to create a linear mathematical model as shown in Table 1. The beta standardized coefficients from the regression model were used to identify the variables or factors that had the highest impact on change in conceptual mathematical knowledge.

**Table 1.** Summary of Variables used for the Regression Model to Predict Preservice Teachers' Change in Conceptual Knowledge

Dependent Variable	Independent Variables
y: change in conceptual knowledge	X <sub>1</sub> : procedural mathematical knowledge X <sub>2</sub> : mathematics courses taken in high school X <sub>3</sub> : mathematics courses taken in university X <sub>4</sub> : procedural mathematical values X <sub>5</sub> : conceptual mathematical values X <sub>6</sub> : academic background
Predicting Equation	
$y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + C$ where: ( $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6$ ) are the unknown weights of the independent variables (C) constant value	

## RESULTS

The repeated measures t-test suggests that there was a significant improvement in preservice teachers' conceptual knowledge from the pretest to the post-test,  $t(110) = -15.04$ ,  $p < 0.025$ ,  $d = 1.43$  (large effect). The descriptive statistics presented in Table 2 as well as Pearson Product Moment Correlations presented in Table 3 summarize the pretest data and change in participants' conceptual knowledge from the pretest to the post-test. The results suggested that change in conceptual mathematical knowledge was significantly correlated to procedural

knowledge,  $r=-0.27$ ,  $n=111$ ; conceptual knowledge,  $r=-0.36$ ,  $n=111$ ; and high school mathematics level,  $r=0.24$ ,  $n=111$ .

**Table 2.** Descriptive Statistics Pretest Data for Change in Conceptual Knowledge and Factors

	Mean	Std. Deviation	N
$\Delta$ CK	3.81	2.66	111
High School Mathematics	1.45	0.50	111
University Mathematics	2.06	3.07	111
Background	1.26	0.44	111
PV	7.89	1.22	111
CV	7.83	1.22	111
PK	6.97	2.09	111
CK	0.97	1.41	111

*Note.*  $\Delta$ CK = change in conceptual knowledge from the pretest to the post-test; High School Mathematics = mathematical level gained from high school; University Mathematics = level of mathematics taken at university; Background = mathematics or non-mathematics major; PV = procedural values at the pretest; CV = conceptual values at the pretest; PK = procedural knowledge at the pretest; CK = conceptual knowledge at the pretest.

**Table 3.** Pretest Data Correlations between Change in Conceptual Knowledge and other Factors

	$\Delta$ CK	HIGHM	UNIVM	BACKM	PV	CV	PK	
HIGHM	Correl	0.24						
	Sig	0.01						
	N	111						
UNIVM	Correl	0.16	0.28					
	Sig	0.10	0.00					
	N	111	111					
BACKM	Correl	0.05	0.20	0.45				
	Sig	0.60	0.03	0.00				
	N	111	111	111				
PV	Correl	-0.01	0.10	-0.07	-0.04			
	Sig	0.31	0.28	0.46	0.67			
	N	111	111	111	111			
CV	Correl	-0.01	0.06	0.09	0.12	0.29		
	Sig	.923	0.54	0.33	0.11	0.00		
	N	111	111	111	111	111		
PK	Correl	0.27	0.31	0.36	0.11	0.04	0.14	
	Sig	0.00	.001	0.00	0.04	0.65	0.14	
	N	111	111	111	111	111	111	
CK	Correl	-0.36	0.25	0.18	0.27	0.07	0.17	0.25
	Sig	0.00	0.00	0.06	0.00	0.44	0.07	0.00
	N	111	111	111	111	111	111	111

*Note.*  $\Delta$ CK = change in conceptual knowledge from the pre to the post-test; HGHM = level of high school mathematics; UNIVM = level of mathematics taken at university; BACKM = mathematics or non-mathematics majors; PV = procedural values at the pretest; CV = conceptual values at the pretest; PK = procedural knowledge at the pretest; CK = conceptual knowledge at the pretest. Correlation is significant at the 0.05 level (2-tailed).

Based on the magnitude of the correlations found, a regression analysis as shown in Table 4 was performed to identify the weights of factors that affected change in conceptual mathematical knowledge the most. For this regression analysis, collinearity statistics were implemented by computing the variance inflation factor (VIF) as shown in Table 4. The variance inflation factor was found to be less than 10, which indicates that the independent variables are not linearly related.

The results from the regression analysis model suggest that change in conceptual knowledge is affected by preservice teachers' high school mathematics level ( $\beta=0.26$ ,  $p<0.05$ ), procedural knowledge ( $\beta=0.30$ ,  $p<0.05$ ) and conceptual knowledge ( $\beta=-0.52$ ,  $p<0.05$ ) pretest data. Nonetheless, the low value for  $R^2$  (0.35) as shown in Table 5, indicates that this model, although significant, leaves 65 percent of the variance in change in conceptual mathematical knowledge scores unexplained.

**Table 4.** Results of the Regression Analysis Beta Coefficients with Change in Conceptual Knowledge as the Dependent Variable Using the Pretest Data

Model		Unstandar Coeff		Standar Coeff	t	Sig.	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	IF
1	(Const)	.70	1.92		0.36	.71		
	HIGHM	.37	0.46	0.26	2.93	.00	0.83	1.206
	UNIVM	.03	0.08	0.03	0.33	.73	0.70	1.429
	BACKM	.35	0.55	0.06	0.62	.53	0.75	1.329
	PV	-0.23	0.18	-0.10	-1.26	.21	0.88	1.125
	CV	.09	0.18	0.04	0.50	.61	0.87	1.144
	PK	.38	0.11	0.30	3.35	.00	0.79	1.263
	CK	-0.98	0.16	-0.52	-5.99	.00	0.84	1.182

*Note.* Dependent Variable –  $\Delta CK$  = change in conceptual knowledge from the pre to the post-test. Independent Variables – HGHM = level of high school mathematics; UNIVM = level of mathematics taken at university; BACKM = mathematics or non-mathematics majors; PV = procedural values at the pretest; CV = conceptual values at the pretest; PK = procedural knowledge at the pretest; CK = conceptual knowledge at the pretest; VIF = variance inflation factor less than 10.

**Table 5.** Regression Analysis Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	0.58	0.35	0.301	2.23

Based on the independent variables that were significant (high school mathematics, and pretest procedural knowledge and conceptual knowledge), a trimmed model was created as shown in Table 6. The trimmed model as shown on the equation from the regression model below indicates that change in conceptual knowledge is affected by preservice teachers' high school

mathematics level ( $\beta=0.26$ ,  $p<0.05$ ), procedural knowledge ( $\beta=0.32$ ,  $p<0.05$ ) and conceptual knowledge ( $\beta=-0.50$ ,  $p<0.05$ ) pretest data. The variance inflation factor was less than 10, which indicates that the variables are linearly independent. The standardized coefficients for the equation below were obtained from Table 6.

#### The Equation from the Regression Model

$$\Delta CK = .26(HM) + .32(PK) - .5(CK)$$

where

$\Delta CK$  change in conceptual mathematical knowledge

HM level of high school mathematics

PK procedural mathematical knowledge at the pretest

CK conceptual mathematical knowledge at the pretest

This trimmed model, although significant, has a low value for  $R^2$  (0.35), which leaves 65 percent of the variance unaccounted for on change in conceptual mathematical knowledge scores as shown in Table 7. Hence this model may provide a useful starting point, but there may be other factors not addressed by this study which may account for the unexplained variance in the model.

**Table 6.** Trimmed Model Regression Analysis Beta Coefficients with Change in Conceptual Knowledge as the Dependent Variable Using the Pretest Data

Model		Unstandard Coeff		Standar Coeff	t	Sig	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	IF
1	(Const)	-0.11	0.85		-0.13	.89		
	HM	1.38	0.45	0.26	3.05	.00	.87	1.14
	PK	.40	0.10	0.32	3.76	.00	.86	1.15
	CK	-.95	0.15	-0.50	-6.03	.00	.90	1.10

*Note.* Dependent Variable –  $\Delta CK$  = change in conceptual knowledge from the pre to the post-test. Independent Variables – HM = level of high school mathematics; PK = procedural mathematical knowledge at the pretest. CK = conceptual mathematical knowledge at the pretest.

**Table 7.** Trimmed Regression Analysis Model Summary Using High School Mathematical, Procedural and Conceptual Knowledge as Independent Variables

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	0.57	0.33	0.309	2.21

## DISCUSSION

High levels of conceptual understanding of fundamental mathematics are important to teach mathematics to others with profound understanding (Ball, 1996; Hill & Ball, 2004; Ma, 1999). This study was conducted to examine factors that may affect preservice teachers' growth in conceptual mathematical knowledge during a mathematics methods course.

The literature indicated that the number of university mathematics courses taken by preservice teachers during their undergraduate majors does not increase their conceptual understanding of fundamental mathematics (Ball, 2004; Foss, 2000), and this finding was supported by our study. In fact, preservice teachers may need specialized mathematics courses in order to deepen their conceptual understanding of elementary content (Ball, 2004; Kajander et al., 2006; Ma, 1999; Sowder, 2007).

Similarly, academic background at university did not correlate to change in conceptual mathematical knowledge. These results suggest that academic background and mathematics courses taken at university do not play a significant role in supporting growth in preservice teachers' conceptual mathematical knowledge. On the other hand, the results suggest that preservice teachers' knowledge of fundamental mathematics as gained from high school as well as preservice teachers' levels of conceptual and procedural mathematical knowledge at the start of their teacher preparation program do appear to support growth. In particular, those participants with stronger procedural skills (which seem reasonably related to the likelihood that they had also taken more years of high school mathematics courses) whose conceptual understanding was weaker tended to develop the most during the program. In our personal experience with preservice teachers, those who appear committed but who describe themselves as having survived mathematics by working hard and memorizing and who typically describe themselves as "weak" or "afraid" of mathematics, often blossom when they are exposed to the reasons behind why the methods they learned previously make sense. The results of the model and the predictive factors it indicates align well with our experiences.

We needed to find the weight of each independent variable in the mathematical model in order to examine their impact on change in conceptual mathematical knowledge. Since the correlations only indicated the strength of the relationship between the dependent variable and each independent variable, we conducted a regression analysis to explore the impact of the related independent variables (preservice teachers' levels of conceptual and procedural mathematical knowledge and values at the pretest, high school mathematics courses, university mathematics courses and academic background) on change in conceptual mathematical knowledge.

Based on the results of the regression analysis, the level of high school mathematics attained and the levels of conceptual and procedural mathematical knowledge at the pretest were the best predictors of change in conceptual knowledge. Furthermore, the beta standardized coefficients (values obtained by standardizing all variables to unit variance before the regression was run) within the model indicated that the preservice teachers' conceptual mathematical knowledge at the pretest had the highest weight. This means that each value of the coefficient of preservice teachers' level of conceptual mathematical knowledge at the pretest is the expected increase on change in conceptual knowledge with a 1-unit increase in preservice teachers' level of conceptual mathematical knowledge at the pretest when other regressors are held constant. For instance, with preservice teachers' levels of procedural knowledge at the pretest and the level of high school mathematics variables held constant, each increase from preservice teachers' level of conceptual mathematical knowledge at the pretest is associated with a decrease of -0.50 unit on change in conceptual knowledge. In other words, preservice teachers with high levels of conceptual mathematical knowledge at the pretest may tend to change less in conceptual mathematical knowledge according to this regression model. Conversely, the conceptually weaker student seemed to have grown the most in conceptual mathematical understanding over the intervention.

In addition, the regression model in this study shows that with initial levels of conceptual knowledge and the level of high school mathematics variables held constant, each increase from preservice teachers' levels of procedural mathematical knowledge at the pretest is associated with an increase of 0.32 unit on change in conceptual knowledge, which means that preservice teachers with high levels of procedural mathematical knowledge at the pretest may tend to change more in conceptual mathematical knowledge.

Finally, the results of the regression analysis show that with preservice teachers' pretest levels of procedural and conceptual mathematical knowledge variables held constant, each increase from the level of high school mathematics is associated with an increase of 0.26 units on change in conceptual knowledge. Hence, preservice teachers with more high school mathematics courses may change more in terms of conceptual mathematical knowledge.

This combination of attributes paints a possible picture of students who, knowingly weak in conceptual understanding, nevertheless persevere and take more high school mathematics courses, which they survive by memorizing and using procedural methods rather than by ever managing to develop much conceptual understanding. Such a combination of factors appears to be typical for students who grow most in conceptual knowledge over the methods course. It must

be remembered that the regression model shows that although high school mathematics and preservice teachers' levels of conceptual and procedural mathematical knowledge at the pretest were the best predictors of change in conceptual knowledge, the low value for  $R^2$  indicated that 65 percent of the variance was unaccounted for in terms of predicting change in conceptual knowledge. Thus in order to account for a higher percentage of the variance, other factors should be taken in consideration in future models. A larger sample may be needed to create a stronger linear model.

### **SUMMARY AND CONCLUSIONS**

Teachers' conceptual mathematical understanding is considered an important element in mathematics reform (Hiebert, 1999); therefore, teachers need to have a profound understanding of the mathematical concepts that they will be teaching to their students in the classroom (Ma, 1999; Sowder, 2007). Hence, in order to better improve teacher's conceptual understanding of mathematical concepts as an important element of classroom mathematics reform, it might be helpful to determine which factors most impact preservice teachers' growth in conceptual mathematical knowledge during a mathematics methods course (Boyd, 1994; Ross et al., 2002).

The results of this study indicate that the number and level of high school mathematics courses taken and the levels of conceptual and procedural mathematical knowledge at the pretest seemed to have impacted preservice teachers' conceptual mathematical growth the most. In particular, participants who initially demonstrated higher procedural skills but weaker conceptual understanding seemed to benefit the most in terms of conceptual growth from the methods course.

Our work might suggest that assessing preservice teachers' initial levels of conceptual and procedural mathematical knowledge as well as their levels of high school mathematics may help determine how much preparation, via a mathematics methods course or other courses, is needed to improve preservice teachers' conceptual mathematical knowledge. This way, it may be possible to help preservice teachers better develop appropriate mathematical understandings to support the development of teaching strategies that reflect reform-based mathematics curricula. In addition, these findings seem to underscore the importance of continued professional development opportunities to deepen teachers' conceptual mathematical knowledge. Via enhanced professional development opportunities, teachers may be better able to teach mathematics to their students in environments in which students can improve their mathematical knowledge by making use of concepts; an environment in which students will be able to integrate



concepts and procedures to develop better mathematical strategies when solving problems (Rittle-Johnson et al., 2001).

These findings also may have implications for mathematics educators of preservice teachers because the findings highlight the importance of creating and studying the effects of specialized mathematics methods courses to better prepare preservice teachers in teacher certification programs. Universities need to provide opportunities to prepare preservice teachers for reform-based teaching, so that preservice teachers can gain more competence in supporting students' reform-based learning during their classroom practices.

We suggest that methods courses should include reform-based content related to multiple mathematics strands, and be guided by The National Council of Teachers of Mathematics *Principles and Standards* (National Council of Teachers of Mathematics, 2000). The focus of these courses should be to improve preservice teachers' conceptual understanding of fundamental mathematics by encouraging preservice teachers to make use of manipulatives and models to help them construct necessary connections between the understanding of mathematical concepts and the development of procedures.

Preservice teachers' mathematical development in these specialized courses should be centred around solving mathematical problems that allow them to share ideas with others to build understanding, and be based on a strong emphasis on conceptual mathematical understanding. The content of these courses should be coherent in the sense that the problems and ideas should be offered with the intention of better preparing preservice teachers to solve mathematical problems with more conceptual understanding. This study contributes to the growing body of knowledge that argues that more university courses in mathematics are not the answer to develop teachers' understanding. We argue that specialized methods courses, or other specialized mathematics-related courses, are needed to help teachers learn how to teach mathematics in a reform-based manner, in order to support sustained student growth.

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