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## CRITICAL MATHEMATICS EDUCATION: RECOGNIZING THE ETHICAL DIMENSION OF PROBLEM SOLVING

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**ABSTRACT.** In this paper, I examine the notion of ‘real life’ mathematical applications as possible sites for ethical reflection in school mathematics. I discuss problems with the ‘real’ in mathematics education, and show how these problems are often based on faulty cognitive theories of knowledge transfer. I then consider alternative visions of mathematical application and suggest that attention to classroom discourse and the craft of mathematics offer ways of introducing the ethical into school mathematics.

**KEYWORDS.** Application, Ethics, Social Justice, Critical Mathematics Education.

### THE CONCRETE

Freire (1970/1998) advocated for a critical pedagogy that was grounded in the “present, existential, concrete situation” ( 76) whereby teaching might begin with the lived experiences of students, accessing their emotional and ethical ties to the situations in which they struggled for voice and equity. The aim of critical pedagogy, according to Freire, is to access the complex ethical dilemmas and power relations inscribed within a given context in order to trigger “moral outrage” (Iyer et al, 2004, p. 356) and increase student participation and social action. The emotion that fuels outrage, unlike that which underpins guilt, can become a source of political agency in the service of the disadvantaged (Leach et al, 2002). When outrage is buttressed with strong problem solving skills, students are able to envision how they are implicated in the experiences of others, and how they might go about redressing the situation for the benefit of all.

Mathematics education often seems bereft of ethical principles that might fuel moral outrage, but it does, in theory, furnish students with strong problem solving skills. These skills are both quantitative and qualitative insofar as students master procedural and conceptual knowledge as well as particular problem solving strategies (or habits of attention) so as to determine when

and how to apply mathematics in various contexts. These contexts of application can be considered the “concrete situation” to which Freire refers, as they are often ‘real life’ applications meant to connect to the lived experience of students. Although ‘Real life’ applications have the potential to tap student emotion and trigger ethical reflection, most examples of ‘real life’ applications are less concerned with citizenship and social justice and more concerned with enhancing mastery of mathematical skills while addressing the student in terms of profit and consumerism. In this paper, I examine the notion of ‘real life’ applications as possible sites for ethical reflection. I discuss problems with the ‘real’ in mathematics education, and show how these problems are often based on faulty cognitive theories of knowledge transfer. I then consider alternative visions of mathematics application and suggest that the concept of craft offers a way of introducing the ethical into school mathematics.

### **CRITICAL MATHEMATICS EDUCATION**

“Critical mathematics education” (Skovsmose & Borba, 2004) is an attempt to re-conceive school mathematics as a site of political power, ethical contestation, and moral outrage. Critical mathematics education refers to a set of concerns or principles that function as catalysts for re-conceiving and redesigning the lived experience of school mathematics. These concerns or principles are meant to target issues of political agency in society through an examination of mathematics education. The approach addresses political issues in relation to teaching and learning mathematics, confronts the problems of access and opportunity according to skin colour, gender and class, and examines the cultural re-inscription of power through applications of mathematics in society (Skovsmose & Borba, 2004, p.207). Various proponents of critical mathematics education have pursued this agenda in different ways: designing new mathematics curricula that address social justice issues (Mukhopadhyay & Greer, 2001; Gutstein, 2006; Tate 2005), studying the role of mathematics teacher disposition towards social justice pedagogy (de Freitas, 2008a; & Rodriguez & Kitchen, 2005; Zevenbergen, 2003), deconstructing the instructional strategies unique to school mathematics that inhibit increased participation (Adler, 2001; de Freitas, 2008b; Morgan, 2006; Walshaw, 2005), generating a socio-political ethics of mathematics education (Skovsmose, 2005; Skovsmose & Valero, 2001; Valero, 2004), and offering visions of alternative teaching practices (Brown, 2001; Valero & Zevenbergen, 2004). Although many of these authors define their unique approach in different terms, one can trace a collective movement in the research community that takes form in relation to the concept of

*critical pedagogy*. As a paradigm for student-centered teaching practice, critical mathematics education resonates with Freire's critical agenda, and offers educators and researchers a wide range of methodologies for exploring the concerns listed above.

It is not, however, a panacea. Gutstein (2008) remarks that the challenges to critical mathematics education often seem insurmountable when one is faced with the stalwart institutional practices that currently structure school mathematics and policy. Mechanisms of cultural reproduction that continue to sustain systemic inequity in/through education cannot simply be named and then easily abolished. Despite the efforts of many in the last century, studies continue to reveal correlations between socio-economic status and mathematics achievement, and indicate little progress has been made regarding these patterns of inequity (Gates, 2006). Studies of mathematics teacher practice continue to show that the vast majority of mathematics teachers are still teaching skill and drill in ways that serve only a select set of students (Confrey & Kazak, 2006).

Because application problems rely on a shared understanding of the 'real', they tend to produce large differences in measured achievement between social classes (Cooper, 2001, p. 256). Applications and 'realistic' tasks have proven to be the most problematic, or rather the most revealing, in terms of differentiated cultural performances (Cooper & Dunne, 2004). Empirical work on assessment items has shown that student confusion about the border between the 'real' life or 'everyday' realm and the mathematical is correlated to socio-economic position, gender, and ethnicity (Cooper, 1998a, 1998b). In other words, when students are asked to complete an application, their actions will be conditioned and constituted by their socio-cultural position. These studies have shown that students designated working class are three times more likely to solve a 'realistic' problem with a 'realistic' answer (taking into account subjective and ethical issues) and fail to read the code of the problem as one demanding a mathematical solution (Cooper & Dunne, 2004, p. 71). Since these 'realistic' applications are a reflection of reform movements in mathematics that attempt to move away from the esoteric and towards more 'meaningful curriculum', it seems crucial that we unravel the ways that applications may *not* address our aim of increasing mathematics participation in schools. Instead, they may demand an even more esoteric performance by introducing another level of code based on one's own enculturation into the discursive practices of school mathematics.

Skovsmose and Borba (2004) are careful to suggest that the critical approach must always attend to the "what if not" of school mathematics, that it must investigate the possible - think the otherwise - and explore "what could be." (p. 211). They argue that researchers and educators

must imagine alternatives that trouble the current situation by actively and creatively generating visions or descriptions of a mathematics education that is more inclusive, more playful, more relevant. The approach is profoundly hopeful and imaginative and offers educators a positive (and critical) means for professional development. “It confronts what is the case with what is not the case but what could become the case.” (Skovsmose & Borba, 2004, p. 214).

### **MODIFYING THE MAINSTREAM TEXT**

One way of practicing critical mathematics is to revise what is offered through the mainstream curricula using the concerns mentioned above as a guide. Consider, for instance, the ‘real’ life applications generated by The Consortium of Mathematics and its Applications (COMAP) ([www.comap.com](http://www.comap.com)), which is a non-profit organization dedicated to enhancing mathematics instruction through emphasis on modeling. The term modeling is used to refer to the practice of ‘real life’ problem solving through the use of mathematical applications. In practice, modeling is used to “understand, predict and control events in the real world” (Dossey, J.A. et al, 2002, p. 13). Like any good application problem, modeling problems do not necessarily prescribe the kinds of tools or methods that might be appropriate in obtaining a solution. A good application problem refrains from supplying instructions or specifying the best tools for the job. After all, deciding whether a particular method is suitable or not should be part of the problem solving process. This decision process or reflective deliberation – this stage when one reflects on the suitability of the mathematical methods to the given problem – is the crucial moment when ethical reflection might enter the application of mathematics.

Textbooks too often specify the appropriate tool to be used for the given problem, and leave out the crucial ethical moment of reflecting on whether the means suit the ends. COMAP comes close to creating this moment of ethical reflection, since they create curriculum that aims to capture the messiness of ‘real life’ problems, and each year sponsor a series of contests for students at various levels. These contests involve one messy problem and no scripted solution technique. Teams of four high school students from all over the world have thirty-six hours to create a solution and submit it to COMAP via the internet.

Unfortunately, since its inception, COMAP's high school mathematical modeling contest problems are almost always without ethical or political context<sup>1</sup>. The messiness of these problems pertains to the complex and difficult physical aspects of the 'real' world, but avoids the ethical messiness of student agency and political action. I don't mean to detract from this other kind of messiness, but rather to point to the infrequency of problems that are couched in political or ethical issues. Successful students – like those who select to participate in the COMAP contests – are trained to read the 'real' or 'realistic' application task as one without ethical significance. For instance, the 2006 contest problems, although wonderfully challenging, were woefully disconnected to the complexity of the ethical world. Consider this problem about a "South Sea Island Resort":

A South Sea island chain has decided to transform one of their islands into a resort. This roughly circular island, about 5 kilometers across, contains a mountain that covers the entire island. The mountain is approximately conical, is about 1000 meters high at the center, appears to be sandy, and has little vegetation on it. It has been proposed to lease some fire-fighting ships and wash the mountain into the harbor. It is desired to accomplish this as quickly as possible.

Build a mathematical model for washing away the mountain. Use your model to respond to the questions below.

- How should the stream of water be directed at the mountain, as a function of time?
- How long will it take using a single fire-fighting ship?
- Could the use of 2 (or 3, 4, etc.) fire-fighting ships decrease the time by more than a factor of 2 (or 3, 4, etc.)?
- Make a recommendation to the resort committee about what to do.

(COMAP, <http://www.comap.com/highschool/contests/himcm/2006problems.html>)

The problem is a 'fake' problem and easily recognized as unrealistic from an environmental perspective. Moreover, the driving principle behind the application is profit, as is often the case in school mathematics. If the mountain of sand were actually washed into the surrounding water, one could well imagine that the cost to marine life would be unimaginable. The problem refuses to address any of the environmental or political issues that might be relevant to the context. Rather, the problem hails the student as business man or engineer and demands

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<sup>1</sup> There are the usual questions related to elections and fair representation, but aside from these, COMAP seems committed to keeping the high school problems apolitical. the COMAP "math serve" contest was introduced as a means of bridging mathematics applications and community service, but this contest was closed due to lack of interest.

that the best solution be one acceptable to a resort committee. It is conceivable that a team response might take into consideration the environmental damage of flattening the island, since the task does not prescribe the method of solution, and thereby allows for distinct answers, and also, one presumes, distinct kinds of assessment, but the guiding questions would dissuade one from pursuing this aspect of the problem. The code for determining the preferred kind of solution is found in the statement: "It is desired to accomplish this as quickly as possible."

In order to make this problem more 'critical', while still inviting mathematical application, one could ask students to consider the problem from multiple perspectives, each of which having different desired solutions. One could add a few statements regarding the environmental impact of the task, such as: "A ring of coral reef with 1 km diameter surrounds the island. Every cubic meter of sand deposited on the reef kills 1 square meter of coral". Students could then be asked to create two models, one favoring the resort committee interests, and the other favoring an island environment committee. The problem would begin to access the ethical dimensions of applying mathematics in this context. In designing the task as a debate about the contested value of each alternative action, and in attending to the ways that particular solutions will serve particular segments of society, the students are enacting critical mathematical practices. Another approach might involve rewriting the code statement: "It is desired to accomplish this as quickly as possible while minimizing the damage to the local environment." This minor change in the statement forces the students to recognize the ethical consequences of their desired efficiency. By introducing the conditional into the statement, the best solution is still the one that maximizes the speed of completing the task, but within the constraints of ethical action.

This sort of modification can be achieved by teachers on a daily basis by revising typical textbook questions so that they address the student in terms other than profit and consumerism. For instance, pre-service teachers in my *Critical Literacy in Mathematics and Science Education* course are given a set of 'real' life applications and asked to re-write these in ways that address "citizenship, environmentalism or social responsibility". For many pre-service teachers, their first inclination is to simply attach humanitarian goals to the problem, without altering any of the information. In the following example, most pre-service teachers simply added a sentence that declared Kim's intention to raise money for a charity, instead of for profit.

Kim is having a sale at his store and wants to advertise. He has decided to promote his sale through radio and TV. He wants to have his ad broadcast at most ten times and does not want to spend more than \$2400. For a thirty-second spot, the TV station charges \$300 dollars. The radio station charges \$200 dollars for a thirty-second spot. The TV station has roughly 8000 viewers, while the radio station has 6,000. Kim wants to know how many

times he should run the advertisement in each of these media outlets to maximize the number of times the advertisement is heard.

After discussing the mathematics involved in the problem, I then offered another more radical version, pointing out that the mathematical skills were the same in both.

Kim is in charge of a homeless shelter in an impoverished community. The shelter is a big space – like a gym, with movable walls – and it totals 2400 square feet. She accepts applications for a 6 month period. She has space for two kinds of applicants : (1) individual applicants and (2) family applicants. A family requires 300 square feet whereas an individual only requires 200 square feet.

She can accept no more than 10 applicants in total, due to the time it takes to process the applications.

$$x \geq 0$$

$$y \geq 0$$

$$x + y \leq 10$$

$$300x + 200y \leq 2400$$

These inequalities (also supplied in the first version) summarize the constraints on how many individuals and families she can accept. She knows that when she houses people they are likely to put money back into the local community and improve services generally. These are people who are often on the edge of poverty and they need a step up before they can get back into regular housing. Based on research she has done over the first year of the Shelter being open, she has noticed that families tend to put back 8000\$ into the community whereas individuals put back 6000\$ (based on a 6 month period).

If Kim hopes to maximize the amount of money that is put back into the community, how many of each kind of applicant should she accept?

With continued exposure to the practice of re-writing problems, pre-service teachers became more adept at modifying textbook questions, and also became more critical of the ‘real’ as it was presented in applications. At the end of the semester, the following problem was given and pre-service teachers were asked to modify it:

Sampson’s dog, Cecil, is tied to a post by a chain 7 meters long. How much play area does Cecil have? Express your answer to the nearest square meter. (Serra, 2008. p. 451)

Although some of the new versions of the problem indicated that pre-service teachers were still struggling with the critical agenda of the course, and some indicated that they hadn’t yet grasped the proper grammar required to convey the appropriate mathematics (For instance, one

student wrote “The students are going to clean up the garbage on an area of the beach. If they start at the life guard chair and walk out 7 meters, how much area will they clean?”), most pre-service teachers produced alternatives that pushed the problem into areas of social justice and environmentalism. Below are two such examples:

1. Mr. Smith from the Institute of Natural Disasters visited the epicenter (center) of an earthquake in California. The furthest reports of activity were from 16 miles away. Mr. Smith was told to report the area of damage to the Disaster Relief Board of California. With this information, they could estimate the amount of volunteers and money needed to help local families rebuild houses; provide food, shelter and other necessities. What is the area in square miles for the effects of the earthquake? Express your answer to the nearest square mile.

2. Crude oil leaking from the hull of an oil tanker is spreading in a circular plume on the surface of the water. Within 5 minutes of the hull being punctured, the plume has reached a buoy located 7 meters from the ship. How large is the area of contamination after just 5 minutes of leakage? After 10 minutes, the plume is 14 meters from the ship. How large is the area of contamination now? After 15 minutes, the plume is 21 meters from the ship. If you extrapolate using this information, how large an area will be contaminated with crude oil after 30 minutes? What does this say about how quickly clean-up needs to get underway in order to minimize damage to the environment?

Despite this new critical focus, applications of this sort require further critique because they continue to honor the mathematics as politically neutral (Valero, 2004, p. 14). Moreover, the modified versions tend to incorporate more “everyday” language and will doubtless prove even more challenging for students who are ill-equipped for the code switching required in completing ‘real’ life applications within school mathematics. In the next section, I discuss the ramifications of honoring mathematics as politically neutral, and I look more carefully at problems inherent to the discourse of mathematics.

### **ETHICAL FILTRATION**

According to Ernest (2001), “Critical mathematics education aims to empower learners as individuals and citizens in society, by developing mathematical power both to overcome barriers to higher education and employment and thus increasing economic self-determination, and by fostering critical awareness and democratic citizenship via mathematics.” (288). Note that the mathematics itself is seen as a neutral tool that might be used for good or evil. Ernest’s vision assumes that there is nothing inherent to the mathematics itself (that being the various semiotic



practices that comprise the doing of mathematics) that might compromise such democratic applications. He lists possible applications that are grounded in democratic and social justice ideals: surveys and analysis of homeless statistics, property values, patterns in petrol use, corporate profits, and gender and racial bias. These excellent ideas for curricula, however, are often reduced, through the application of a mathematical model, to extremely inadequate and often unethical representations of the ‘real’ experiences of those under study.

*Rethinking Mathematics* (Gutstein & Peterson, 2005) is a collection of practical examples and reflective essays that attempts to offer teachers concrete instances of social justice mathematics curricula, including activities that explore the cost of war and poverty and the connections between race, privilege and inequity. Again, these activities are compelling and powerful classroom applications, and I have used them in my own classrooms. But the act of simplifying the contextual factors to allow for a mathematical application often creates a problem that is less ‘realistic’ and dangerously naïve. Recognizing the risks in application – even when guided by inclusive democratic goals - is crucial if we are to develop a more far reaching critical mathematics.

Application is a complex concept that involves content mastery and emphasis on the five NCTM<sup>2</sup> process standards (problem solving, reasoning and proof, communication, connections, and representation). Applying mathematics to the ‘real’ world requires recognizing both the messiness of life contexts and the limitations of the mathematical tools to adequately represent or model such messiness. Reflecting on whether a given application is suitable or not is a crucial aspect of all application. Unfortunately, mathematics “in action” (Skovsmose, 2008) often entails an “ethical filtration” whereby the agent (be it a student or engineer or other) reduces the complex ethical and political situation to a set of abstract parameters which are then combined into a simplifying model primarily used for prediction. According to Skovsmose (2008), ethical filtration is built into the practice of mathematics in action, that is to say, the act of stripping away the contingent and the subjective are inherent to mathematical problem solving. For instance, Skovsmose recounts how a group of students were given the assignment “Family support in a micro society”, in which they were shown portraits of 24 families and asked to formulate ethical principles for distributing child benefits. They were then asked to determine a set of parameters and an algorithm for child benefit distribution.

In the process of turning the verbally-formulated principles for distribution into functional algorithms, the students experienced how the original principles needed to become

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<sup>2</sup> National Council of Teachers of Mathematics

simplified. At times the principles were almost ignored when mathematics was brought into operation to do the distribution. The students experienced the general phenomenon that when mathematics is brought into action, a new discourse takes over. The ethical principle, which might have guided the initial considerations, becomes substituted by the technical administration of the system. This is an ethical filtration, and it is a common consequence of bringing mathematics into action. (Skovsmose, 2008, p. 167)

It's crucial to consider this ethical filtration process as something that derails attempts at critical mathematics. Moreover, one can see that mathematics education *actually* teaches students to enact ethical erasure so as to successfully generate solutions that seem unambiguous. Aside from the preponderance of applications in textbooks that are blatantly unreflective ("Your task is to find the best location to build the new gas station so as to maximize your profit."), one can see in the Skovsmose example, that even those tasks with a political or ethical principle underpinning their design involve a re-contextualizing practice of abstraction that sacrifices the ethical in order to achieve one solution. It is hard to imagine how moral outrage might be sustained in a mathematics classroom where ethical filtration is ultimately operative.

Since ethical issues are often the source of our sense of agency and action, one can draw connections between the process of ethical filtration in mathematics education and the process of disengagement in school mathematics. Textbooks fail to offer students controversial problems to which they might apply mathematical skills, and teachers often don't have the alternative resources to introduce current issues into the curriculum. Students perceive mathematics as disconnected from the world of language and politics precisely because these textbooks offer such a limited sense of contextual relevance (Dowling, 1998). This disconnection serves the status quo because it creates a public perception that mathematics belongs to an abstract realm of dispassionate ability. Recent research on student identity in mathematics suggests that successful high school mathematics students perceive their success in terms of conformance, compliance and lack of agency (Boaler & Greeno, 2000; Povey et al, 2004; Walshaw, 2004). Many students perceive school mathematics as a site of complete erasure where identity and agency are sacrificed. This perception is not simply a reflection of frustrated students who have given up on mathematics; Boaler and Greeno (2000) interviewed successful students in high school mathematics and found that a significant majority believed that success in mathematics was simply about following rules and denying personal agency (196). In the eyes of many educators, this research seems to suggest that "traditional math is bad for society" (Gutstein & Peterson, 2005, p. 14).

## KNOWLEDGE TRANSFER

Applying mathematics to ‘real life’ situations has traditionally been theorized as a form of transferring knowledge from one context to another (de Abreu, 2002, p. 329). The concept of knowledge transfer comes from cognitive theories of learning that posit developmental models of skill acquisition (de Abreu, 2002, p. 330). According to this approach, application and problem solving are theorized as higher level skills which involve transferring more basic skills –skills that have successfully been un-situated and internalized - to new and unfamiliar situations.

Challenges to the concept of knowledge transfer have come from ethnographers who have studied the context-dependent nature of cognition (Greenfield & Lave, 1982), arguing that cognition and skill acquisition cannot be abstracted from the socio-cultural context, that doing mathematics is a deeply embodied cultural practice, and that applications of knowledge are complex social enactments of identity. Applications of school mathematics are not at all straight forward, nor, it seems, are they at all frequent in the lived experiences of students. In their famous study of “street mathematics” and tailoring, Greenfield and Lave concluded: “It appears that neither schooling nor tailoring skills generalize very far beyond the circumstances in which they are ordinarily applied.”(p.199).

Cobb and Yackel (1996) theorized student learning in classrooms according to layers of collective discursive norms: social norms, socio-mathematical norms, and classroom mathematical practices. They defined socio-mathematical norms as “normative understandings of what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant.” (461). Other socio-cultural researchers have built on these situated theories of learning, and have developed new ways of making sense of mathematics education as a cultural practice (Lerman & Zevenbergen, 2003). These researchers read the mathematics classroom through a discursive lens, studying the ways that identities are constituted in the classroom through discursive practices and enactments of power relations.

Reading “application” as a cultural practice – instead of a transfer of knowledge- allows researchers to ask new questions about the ethics of application. In application problems, there is always a presumed distinction between the new and the familiar content, and one can hear different modalities (of affect, certainty, and authority) as teachers and students speak through their process of problem solving in the unfamiliar situation. Attention to the language of uncertainty in problem solving, such as hedging or equivocation (such words as “maybe”, “probably”, “almost”, “wonder”, “sort of”), is an important “think aloud” teaching method that

models the process of conjecture and speculation. Conjecturing and speculating require a language of uncertainty – a form of communication that is deliberative, in process, tangential, inconclusive. “Hedges” are frequently used (Rowland, 2008). In contrast, many associate the domain of mathematics with precision, exactness, and definitive answers. There is no humility in the language of certainty, no moment to introduce the “what if not” conjectures that help students reflect on the consequences of their actions. I believe that “think aloud” methods of modeling humility can help students embrace/celebrate a language of uncertainty as they enter into an application problem. Sustaining that humility as one considers possible solution strategies can be achieved through continual attention to the language of epistemic authority. Presenting application problems in terms of hesitation, hedging, and ambiguity, and asking students to model their responses on this same language, will teach students to contextualize their solutions in terms of their own authority and social position. Reflecting on the language of uncertainty allows us to reflect on the ethical dimension of our problem solving, to reflect on the implication of our proposed solutions. Although think aloud strategies point to the process of deliberation and thereby locate or situate the problem solver in a particular context – with a corresponding particular agenda – such discursive practices do not represent the tightly packed noun phrases and concise phrasing found in mathematics textbooks. The task of bridging these two radically different registers remains a daunting challenge for both educators and researchers.

### **CONCLUSION: MATHEMATICS AS CRAFT**

The word application comes from Latin where it means “a joining to, an attaching of oneself to” (<http://www.etymonline.com/index.php>), and similarly the word apply means “to attach to, to devote oneself to” to “lay, fold or twist” or “to plait, to braid, to intertwine” (<http://www.etymonline.com/index.php>). These definitions suggest that application is a process of merging oneself with that which is outside of oneself. This subjection of oneself to the task at hand, this folding of self into other, is what makes application a form of ethical action. “Joining” is an ethical action that brings the world together with shared concerns. Joining is awkward and problematic and never innocent. Accordingly, applications are always ethical and material – we are implicated in our applications in ethical and material ways. We enter the situation when we apply our knowledge to it, we interfere when trying to understand the other, we trace our own interests onto the contexts to which we apply our tools.

de Abreu (2002) suggests that we abandon the language of knowledge transfer and speak of a “tool kit” – a collection of community skills – that would be in circulation to a lesser or greater extent in mathematics classrooms. The tool kit would be comprised of various numeracy strategies, methods of spatial reasoning, notation and vocabulary proficiency, and, of course, instruments to be used when decoding applications and ‘realistic’ problems. The tool kit metaphor promotes a “craft” vision of doing mathematics education, suggesting that instruction be seen as apprenticeship in communities of practice, and leading to mastery of these tools (“to plait, to braid, to intertwine”) and ultimately to full membership in the community. de Abreu (2002) quotes Resnick, Pontecorvo, and Saljo (1997) in arguing that the concept of tool is appropriate in the mathematics context:

The concept of tool is expanded ... beyond the conventional view of a tool as a physical artifact. Not only physical artifacts but also concepts, structures of reasoning, and the forms of discourse that constrain and enable interactions within communities qualify as tools. Vygotsky ... originally distinguished tools from signs, or language. However, subsequent influential developers of theories of socially situated cognition ... have suggested that many kinds of thinking, as well as physical actions, are carried out by means of tools. (p.3)

As a metaphor, the tool kit effectively positions mathematical skills in the public domain, and demands that we envision learning as a material and cultural process of enacting particular kinds of performances. It shifts our attention away from the issue of inner ability, and focuses on identity and cultural performance. The tool kit metaphor, however, fails to address two crucial aspects of critical pedagogy. First, novices often misuse tools in important ways, and this is often how the new is introduced into a community of practice. If the process of apprenticing is merely enculturation without critique, then how will the new be introduced? How will inequity be named and redressed? If “The master’s tools will never dismantle the master’s house.” (Lorde, 1984), then which tools are we in search of? Second, and related to the first point, is that the tool kit approach doesn’t include any instruments for ethical reflection on the use of the tools. The ethics of craft are often about doing ‘good work’ with passion and attention to detail. But I would like to suggest that in some instances, when a craft is sufficiently central to the power structure of a culture (for instance, in the case of medicine, or for that matter, education), there must be a code of ethics about how (and whether) one should apply the tools. In medicine, various ethical oaths have historically guided the application of powerful medical knowledge and tools. The medical oath is meant to remind doctors (and those who submit themselves to their care) that medical power/knowledge must be in the service of those in need of assistance. We characterize medicine

as a “caring profession”, but isn’t the issue of care implicated in all uses of power? Why wouldn’t such an oath accompany the application of any powerful tool to the world?

The NCTM introduced their “equity principle” as an ethical oath for teachers, to remind them that their role was to ensure that all students should have the opportunity to learn “powerful mathematics”. In this paper I have argued that we need another ethical principle to accompany the mastery and application of powerful mathematics. The American Association of Mathematics adopted a code of ethics in 1995, but it pertains almost entirely to issues regarding scholarship and citation (Hersh, 2008). Why not construct an ethics of mathematical application, as we have for medicine? The application oath might simply demand that the mathematical agent (be it a student or a teacher or other) must reflect on the ethical consequences of her/his mathematical actions in the ‘real’ world, and seek to serve ‘real’ others in need of assistance through the use of these powerful tools. If applied mathematics is conceived as a “social contract” (Davis, 2008), and if mathematics “both sustains and binds us in its steady and unconscious operation” (2), then time spent in our classrooms on ethical reflection will serve the social justice goals of critical pedagogy.

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