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IDENTIFICATION OF HORIZON MATHEMATICAL KNOWLEDGE FOR TEACHING FRACTION DIVISION AT ELEMENTARY SCHOOLS

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ABSTRACT

Determining the kind of teachers' mathematical knowledge for teaching a topic of mathematics at elementary school is substantial most notaby for developing teacher training. The important purpose of this study was to propose a model by applying Delphi method for identifying the horizon mathematical knowledge for teaching fraction division at elementary schools in Indonesian context. The results were the procedure for identification horizon mathematical knowledge and eight statements of competences in mathematical knowledge for teaching fraction division at elementary schools. The recommendations based on the results were the model developed in this study which could be one of the many alternatives for identifying the horizon mathematical knowledge for teaching, and the results should be followed by an empirical validation process to analyze the ultimate effects on students' learning.

KEYWORDS
Horizon mathematical knowledge for teaching, delphi method, teaching fraction division, elementary schools

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Introduction

What defines effective student learning in mathematics as represented in the assessment of TIMMS or PISA is determined by a number of factors (Demir & Kilic, 2010; Gencturk, 2012). The mass media in Indonesia often attribute the student achievement in mathematics to the teachers as the main factor (Shadiq, 2013). However, despite the fact that teachers matter most, numerous factors contribute to the teachers' performances. One factor recently discussed in many researches in mathematics education is the mathematical knowledge for teaching (MKT). For instance, in 2014 such considerable dissertations as

Coddington (2014), Jackueline (2014), and Flake (2014), to name a few, are particular well-known works referred to the study of MKT. The abundant studies concerning MKT, most likely than not, could be related to the effect of MKT on student learning results. One of which is by Hill, Rowan & Ball (2005) who asserted that the teachers' MKT contributed positively to students. Similar conclusion was put forward by Metzler & Woessmann (2010) in Peru, which is growing into a developing country like Indonesia, who affirmed that "We find a significant effect of teacher subject knowledge on student achievement, drawing on data on math and reading achievement of 6th-grade students and their teachers in Peru. A one standard-deviation increase in teacher subject knowledge raises student achievement by about 10 percent of a standard deviation" (p. 20).

The MKT concept was first coined by Ball et al. (2008) who proposed that teachers require a great deal of knowledge and expertise in carrying out the work of teaching a particular subject matter, as shown in Figure 1. The teacher knowledge is distinguished into two domains, namely subject-matter knowledge and pedagogical content knowledge. The subject-matter knowledge grinds into more specific domains – common content knowledge, specialized content knowledge, and horizon content knowledge. In the following, this article outlines the horizon of mathematical knowledge for teaching (HMKT), which emerges as principally subtantial for mathematics-teaching, particularly one that represents the division of fraction, at elementary schools. In the following Figure 1, the grey color constitutes the area of this research.

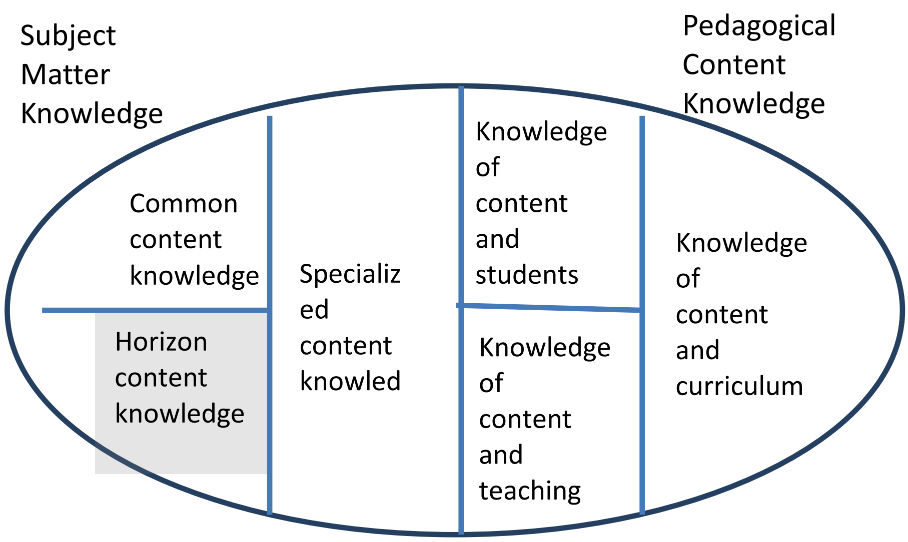

Figure 1. Locus of Horizon Mathematical Knowledge for Teaching (Ball et al., 2008, p. 403)

The amount of teacher's HMKT for teaching mathematics at elementary schools heavily depends on what institution of teachers' disciplinary training is and how it is provided to prospective teachers of elementary schools. The educators of teacher's disciplinary training institution in general believe that the teachers who teach mathematics certainly need some mathematical knowledge more than the mathematical knowledge being taught. Albeit there is no formal standard for deciding the content of HMKT in the curriculum of teacher education, the HMKT substance is included within the mathematics courses offered in the bachelor degree program of Elementary Teacher Education at some universities in Indonesia. Among others is the bachelor degree program of Teacher Education at Universitas Terbuka which offers four mathematics courses, only one of which is an advanced mathematics course. The advanced course consists of mathematical logic, linear algebra, and statistics, all presented very briefly. To date, there is no study yet to reveal whether the course content of HMKT is sufficient or not for teachers to teach all mathematics subject matters at elementary schools, except for fraction division. Sugilar (2015) concluded that the HMKT contents in a bachelor degree program of the elementary teacher accomodated 50% of the teacher need for teaching that subject. Meanwhile, the largest state university for prospective teacher education in Indonesia offers 11 mathematics courses with six of which being advanced mathematics. It seems that advanced mathematics or horizon mathematical knowledge has been typically based on intuition or what has been thought to be good for the prospective or practicing teachers with little empirical basis. To top it off, the need for mathematical knowledge for teaching at elementary schools relies on the available lecturers at the academic faculty. Accordingly, one might conjecture there is a substantial agreement about an instrument necessary for identifying what kind of HMKT a teacher needs to accomplish the task of teaching mathematics at elementary schools.

One of the approaches to what might stipulate MKT is through observation of teaching practices (Ball & Bass, 2003). Such observation focuses on various kinds of knowledge from which teachers apply to effective mathematics-teaching. Similarly, Charalambous (2016) explored the knowledge in the construct of teaching mathematics by drawing a distinction between studies that sought the teachers' knowledge and those that probed the actual teaching practices between two cohorts of elementary school teachers and university students with strong mathematical background. This approach largely discovered the advanced mathematical knowledge generated during the teaching practices. HMKT determined by this approach was related admittedly to the types of subject matter being taught and the level of mathematical knowledge of the students who might pop up a number of questions during the teaching process. A pivotal tenet for mathematics-teaching, the advanced mathematical knowledge might turn quite difficult to observe as teachers most likely responded to students' erroneous solution to a mathematics problem (Delaney, 2008).

Another approach is set up through teacher notions on what types of horizon of mathematical knowledge are most suitable for mathematics-teaching at elementary schools. This approach was implemented by Galant (2013) who interviewed 46 teachers. The research of this area was also spurred by Mosvold

& Fauskanger (2014) who pointed out that; (1) the teachers in this study tended to abandon the importance of HMKT in their teaching; (2) the teachers more concerned with such mathematics related to the topics they were teaching; and (3) the pre-service teachers did not concern with HMKT because they considered it as a difficult subject to learn, and therefore they thought it would degrade their professional identity as teachers.

Given that HMKT is of great importance for mathematics-teaching, including, but not limited to, that at elementary schools, it requires several other approaches, in addition to the aforementioned approaches, to identify HMKT most appropriate for numerous mathematical teaching conditions. According to Jakobsen, Thames, Ribeiro, & Delaney (2012), HMKT was necessary as teachers need advanced mathematical knowledge, even for teaching elementary mathematics. The teachers consequently would have; (1) comprehension that provided them with a sense of how the subject matter they taught was situated in and associated with broader disciplinary territories; (2) competence in developing intuition upon which the mathematical concepts being taught were reflected; and (3) necessary resources to recognize an extensive amount of mathematical knowledge for teaching.

However, despite its increasingly widespread interest and concern, what counts as HMKT and how it relates to student achievement have remained inadequately specified, as expressed by Mosvold & Fauskanger (2014) :"*Although horizon mathematical content knowledge is included in the framework of MKT, and researchers seem to agree about its importance, there is still a lack of empirical evidence both for the existence and importance of this particular aspect of teacher knowledge*" (p. 12). In addition to empirically demonstrating an approach and a model as an attempt to identify HMKT, this paper sought to fill in the scarce number of studies about HMKT from Indonesia's viewpoint as a developing country.

The study relied on Delphi method to explore the extent to which HMKT was identified for teaching practices of fraction division at elementary schools. The similar method has been employed by Winklbauer (2014) to identify technical competence of USA army. In addition to the identification of HMKT noted previously, the study sought to measure how HMKT was applied to teaching fraction division. Fraction division represents a wide range of elementary content and roughly maps onto a major focal area in elementary curriculum. Lo & Luo (2012) described that; (1) fraction was the subject which was challenging to learn for students and to teach for teachers considering the complexity involved, (2) high-degree competence of fractional numbers was a requisite to understanding algebra, and (3) fraction division involved all the concepts and skills needed in learning fractional numbers.

Research Methods

The study applied Delphi procedure to establish intuitive knowledge from experts on a particular problem (Heiko, 2012) with a deductive approach

(Hannafin, 2004). The study underwent several rounds, as described within the next section. The method allowed the study to seek consensus-building measurement among the representative experts that included a number of lecturers for graduate programs at mathematics-education universities to formulate competence statements of HKMT for mathematics teachers at elementary schools. The experts' ideas expressed in the competence statements would be rated and evaluated for appropriateness by the mathematics lecturers who taught the prospective elementary school teachers in the bachelor degree program. The competence statements were accompanied by a brief summary about the subject content so that the statements were well-defined to the raters. The following was the detail of the steps for identifying HMKT for teaching mathematics at elementary schools:

(1) The five experts of mathematics education from Universitas Bengkulu and Universitas Terbuka established some statements of competence related to horizon mathematical knowledge required for mathematics teachers at elementary schools based on the current curriculum. The consensus among experts was based on intuitive knowledge and such literatures as, among others, the works by Olanoff (2011) and Lo & Luo (2012) which examined the teachers' requirement of mathematical knowledge for teaching fraction division. At this point, 11 HMKT statements of competences were identified, as shown in Table 1.

(2) Based on the 11 HMKT statements of competence, a brief summary of the mathematics content related to each statement was developed to clarify the HMKT statements. For instance, the seventh statement of competence was briefly presented in Figure 2.

(3) The next step was to conduct a Delphi procedure to gain consensus among 18 tutors of mathematics courses in the bachelor degree program of Elementary Teacher Education at Universitas Terbuka. Those 18 tutors, all of whom came from several universities in Bengkulu Province in Indonesia, were participating as the expert panels. The number of whom was adequate since the number required in Delphi procedure is between 10 and 18 (Bourgeois, et al., 2006). The 18 expert panels received the list of HMKT statements in the form of a rating scale questionnaire from which the panels' rating was garnered. The rating scales were based on the relevance of HKMT statements that varied from 1 to 3, where 1 for "irrelevant", 2 for "in between", and 3 for "relevant" as to the teachers' competence for teaching fraction division at elementary schools. The instruction for the panels to grant ratings for each HKMT statement was available in the preface of the questionnaire. They were also urged to learn the brief summary of the content related to the HMKT statement to have a well-defined mathematics content listed in each of the HMKT statement. As previously stated, those HMKT statements were crucial for the elementary school teachers upon teaching fraction division. The teachers accordingly would employ the following; (1) opportunity that provided them with a sense of how the subject matter they taught was situated in and associated with larger mathematical

landscapes; (2) competence in developing intuitive knowledge upon which the mathematical concepts being taught were reflected; and (3) necessary resources to recognize an extensive amount of mathematical knowledge for teaching with respect to building their flexibility and self-confidence.

Tabel 1. The Initial HMKT Statements of Competences

Numb.	HMKT Statements
1	Definition of rational number
2	Set of rational number as infinite and countable set
3	Proof that all rational numbers are countable
4	Operation in set of rational number
5	Equality of two rational numbers and equivalence relation
6	Equivalence classes in rational numbers under equivalence relation
7	Set of rational number as a Group
8	Set of rational number as a Ring
9	Set of rational number as a Field
10	Set of rational number as an ordered field
11	Set of rational number as a dense field

The Delphi procedure employed in this study incorporated a number of rounds. After each round, the 18 panels were encouraged to correlate a judgement for each HMKT statement and also to come up with a new HMKT statement. The results of each round were aggragated and analyzed to compute the relevance of the HMKT statement with the teacher need and the level of consensus among diverse set of panels. The HMKT statement was provided with such measurement scales as low relevance yet high agreement, and vice versa. The analysis sought to solicit the HMKT statements and reach the correct responses through consensus-building among those panels. Since the sample size was no larger than 30, the statistical measure used was non-parametric analysis, which included the relevance and the agreement. The response stability was presented by median, inter quartile range (IQR), and Spearman's Rank Correlation. The IQR value was used to identify which HMKT statement is appropriate or not for the next round. The required range for an HMKT statement to be eligible for next round was IQR > 0.8 (Kalaian & Kasim, 2012). Once it hit the required range, the statement would be rephrased to clarify the original statement by putting it into words that were more easily comprehensible by the panels, while retaining the basic meaning. If the panels did not reach a consensus over the statement in the succeeding round, a response stability was measured with Spearman's Rank Correlation (Kalaian & Kasim, 2012). Once significant correlations emerged, stability of the panels' responses was accomplished, and thus further rounds were dispensable. At this point, the panels gained an agree-to-disagree consensus over the statement, which led the statement to be ruled out in HMKT list.

Understanding the Set of Fractional Numbers as a Group

Loosely, a group is a set of G under which the operation of * occurs, and which satisfies the following properties:

1) **Closure**
   Every element of a and b in G equals $a * b = c \ in \ G$.

2) **Associative**
   Every a, b, and c in G equals $a * (b * c) = (a * b) * c$

3) **Identity element**
   An element e in G equals $a * e = a$ for every a in G

4) **Inverse element**
   Every $a \neq 0$ in G there is an inverse element of G, $i.e. a^{-1}$, such that $a * a^{-1} = e$

A group with which commutative properties correspond, i.e. $a * b = b * a$, is called Abelian group. If the group operation is identified with multiplication, the group is defined as a multiplicative group. Likewise, a group whose operation is addition is referred to as an additive group.

To check your understanding to apply the group concept in fractional numbers, please do the following exercises.

1) Show that the set of rational numbers with addition forms a group.
2) To what identity element does a group of rational numbers with addition belong?
3) In a group of rational numbers with addition, what is the inverse of 2, 1/3, and 5/2?
4) Show that the set of rational number with multiplication forms a group.
5) To what identity element does a group of rational number with multiplication belong?
6) In a group of rational numbers with multiplication, what is the inverse of 2, 1/3. and 5/2?

Figure 2. The Set of Fractional Numbers as a Group

The formula for the Spearman's Rank Correlation, r_s, is as follows:

$$r_s = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$$

where, d_i is the difference between the ranks of the panels' response on the i^{th} item of the Delphi procedure, for example, from rounds 1 and 2, and n represents the number of experts in a panel (Kalaian & Kasim, 2012: p. 7). The following Table 2 is an example of computing Spearman's Rank Correlation for the item number 8 as appeared in the Table 5.

Table 2. An Example of Computing Spearman's Rank Correlation

Item	8			
	Round 1	Round 2	d	d^2
Expert 1	3	3	0	0
Expert 2	1	2	-1	1
Expert 3	3	2	-1	1
Expert 4	3	3	0	0
Expert 5	3	3	0	0
Expert 6	3	3	0	0
Expert 7	1	1	0	0
Expert 8	3	3	0	0
Expert 9	2	2	0	0
Expert 10	3	3	0	0
Expert 11	1	1	0	0
Expert 12	1	1	0	0
Expert 13	1	1	0	0
Expert 14	3	3	0	0
Expert 15	3	3	0	0
Expert 16	3	3	0	0
Expert 17	2	2	0	0
Expert 18	1	1	0	0
			Sum of d^2 =	2
$r_s = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$		= 0.998 (rounded to 1.00)		

Results and Discussions

The First Round

The 18 members of the panels engaged in the Delphi procedure included the mathematics tutors of the bachelor degree program of Elementary Teacher Education in Bengkulu regional service of Universitas Terbuka. A group of geographically assembled tutors was deliberately selected as the panels to allow easier communication during the data collection. They were distributed a questionnaire containing 12 HMKT competence statements for teaching the fraction division at elementary schools and a small printed book describing horizon of mathematical knowledge that overviewed the competence statements.

The competence in HMKT statements referred to the elementary school teacher, not the elementary students. This needed to be made clear as there might be a slight confusion as to whom the competence referred to. One of the panels wondered, "Do the competences belong to the elementary students?" (In 2013 National Curriculum, the student competences merely comprised problem-solving to fractional numbers.) To avoid such misleading information, the notion of the competence was available in the preface of the questionnaire to grant the whole panels thorough conception of at whom the competence aimed. This was essential prior to the distribution of the questionnaires.

The results of the first round were presented in Table 3. As the panels' rating was represented by median statistics, the average value of the competence statements was 3, implying that HMKT was substantially required by mathematics teachers who would teach fraction division at elementary schools. Aligned with that, the panels assessed that HMKT was reasonably attainable by those teachers who studied in the Elementary Teacher Education for Bachelor Degree. There was still an appropriate measure, however, to determine the degrees of the consensus among the panels over the average value. The panels' consensus was indicated by IQR, where > 0.8 IQR signifies a weak consensus in which subsequent round is necessary (Kalaian & Kasim, 2012). Table 3 revealed that there were five HMKT statements with IQR > 0.8, i.e. 7, 8, 9, 10, and 11. Those five statements were consequently engaged in the subsequent round of Delphi procedure.

Those five statements of competences were closely related to the following mathematical subjects; (1) the set of rational numbers as a group; (2) the set of rational numbers as a ring; (3) the set of rational numbers as a field; (4) the set of rational numbers as an ordered field; and (5) the set of rational numbers as a dense field. There was an overwhelming common ground among the panels that those five of HMKT competence statements failed to support teachers' competences for teaching fraction division at elementary schools. It turned out that those mathematical subjects were heavily associated with abstract algebra, which was not essentially required by those teachers in this sense.

Table 3
The First Round Results

HMK	The Panel Members																		M	IQR
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18		
1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3.00	0.00
2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3.00	0.00
3	3	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3.00	0.00
4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3.00	0.00
5	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	3	3.00	0.00
6	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3.00	0.00
7	3	3	3	3	3	3	1	3	2	3	3	1	1	3	3	3	2	1	3.00	1.00
8	3	1	3	3	3	3	1	3	2	3	1	1	1	3	3	3	2	1	3.00	1.75
9	3	3	3	3	3	3	1	3	1	3	1	1	1	3	3	3	2	1	3.00	2.00
10	3	3	3	3	1	3	1	3	1	3	1	1	1	3	3	3	2	1	3.00	2.00
11	3	3	3	3	3	3	1	3	2	3	1	1	1	1	3	3	2	1	3.00	2.00

In the first round, one of the panel members ruled on a new HMKT competence statement for the elementary school teachers to justify that dividing by a fractional number could be considered as multiplying by its inverse, a concept that was included in group theory. Accordingly, an HMKT competence statement was constructed along with a brief explanation about the pertinent mathematical concept, as can be seen in Figure 3.

Division by a Fractional Number as Multiplication of Its Inverse

Considering that set of rational numbers Q is a group within multiplication, division by a fractional number can be achieved by the inverse operation to multiplication. If a and b are rational numbers, then:

$$a : b = a \times b^{-1}$$

The proof is as follows:

$$a : b = \frac{a}{b} = \frac{a \times b^{-1}}{b \times b^{-1}} = a \times b^{-1}$$

By its very nature, the division by fractional number means multiplication by "the number that is turned upside down". The following example demonstrates such operation, since the set of rational numbers is a group within multiplication: $\frac{1}{3} : \frac{2}{3} = \frac{1}{3} \times \frac{3}{2} = \frac{3}{6} = \frac{1}{2}$

Exercise:

Show that in a group of rational numbers with addition, subtraction with a rational number is the addition with its inverse!

Figure 3. Division by a Fractional Number as Multiplication by Its Inverse

The Second Round

The five competence statements of HMKT, not yet to be approved by the panel members, were evaluated in the second round. In addition to the aforementioned five competence statements, a new statement was encouraged by the panel members pertaining to the operation of fraction division by multiplying the inverse. This led the second round to engage six statements. The result of the second round was presented in Table 4. The new additional statement was listed in number 12.

Table 3. *The Results of the Second Round*

HMK	The Panel Members																		M	IQR
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18		
7	3	3	3	3	3	3	3	3	2	3	3	2	2	3	3	3	2	2	3.00	0.75
8	3	2	2	3	3	3	1	3	2	3	1	1	1	3	3	3	2	1	2.50	1.75
9	3	1	2	3	3	3	1	3	1	3	1	1	1	3	3	3	2	1	2.50	2.00
10	3	1	2	3	2	3	1	3	1	3	1	1	1	3	3	3	2	1	2.50	2.00
11	3	2	2	3	3	3	1	1	2	1	1	1	1	1	3	3	2	1	2.00	2.00
12	3	3	3	3	3	3	3	3	2	3	3	2	2	3	3	3	2	2	3.00	0.75

In the second round, two out of six included statements gained consensus among the panel members with IQR < 0.8 and a median score of 3.00. The two statements revolved around the set of rational numbers as a group. The HMKT statement of number 7 delved into the group within operation of addition and multiplication. Statement number 8, as well as all those containing the concept of ring and field, was not yet to reach consensus in the preceding round. Statement number 12 was the application of group concept to reveal that the division by a fractional number equals to multiplication by its inverse. Recently propounded by one of the panel members in the previous round, the statement generated a median score of 3 and IQR of 0.75 which signified that the competence, pertaining to the statement, was of great importance for the teachers in demonstrating fraction division at elementary schools. The panels' altered consensus to statement number 7 was most likely due to the new statement of number 12, which shed light on the group concept in further comprehending the fraction division.

The other statements, number 8, 9, 10, and 11, did not achieve consensus yet in the second round. Those statements pertained to the concept of ring, ordered field, and dense field. All four were parallel to the concept of group yet had no marked correlation to the operation of fraction division. A few individual panels, however, modified their scores in this round. For instance, panel member number 2 raised his score from 1 to 2 on HMKT of statement's number 8, while panel member number 5 did similarly on HMKT of statement's number 10. When the panel members' different scores are significant enough to indicate unstable scoring, that means subsequent round is indispensable. Otherwise, the delphi procedure rounds off with the stable responses among the panel members. Those four statements consequently would be ruled on by the response stability test to delve deeper into the significance of the score alteration.

Response Stability

The response stability was measured by the correlation coefficient of Spearman's rank to represent the stability between two scores. As previously stated, significant correlation coefficient between scores of the first and the second round evoked stability of response which in turn led to the end of Delphi procedure, without further rounds. The critical value of the correlation coefficient for sample size (n) equal to 18 was 0.900. The calculation result of correlation coefficient was shown in Table 5.

Table 5. *Response Stability*

Stat. Number	Corr.Coeff. (r_s) *)	Conclusion	Decision
8	1.00	Stable	No need a next round
9	0.99	Stable	No need a next round
10	0.99	Stable	No need a next round
11	0.99	Stable	No need a next round

*) An example of calculating the Spearman correlation coefficient, r_s, was presented in the Table 2

In Table 5, the correlation coefficient of each item was greater than 0.900, suggesting that each of the four HMKT statements had secured a stable response among the panel members in the last two successive rounds and that the Delphi procedure therefore topped off with homogeneity of scale. The four statements disapproved by the panel members were ultimately withdrawn in the HMKT list.

Conclusion and Implication

In the frame of the Delphi procedure, the development of HMKT identification models for teachers who taught fraction division at elementary schools was fairly accomplished. In contrast to previous studies that identified HMKT based on teaching practice, the model relied on process of collecting intuitive knowledge possessed by experts working in the field of mathematics education and practitioners engaged in the education of elementary school teachers. The two-round Delphi method had solicited eight HMKT statements of competence as to fraction-division teaching in elementary school classes.

In Indonesian context, teacher's competence in HMKT for teaching fraction division is one of the competences imposed on teachers according to the standard of teacher competence in Law Number 14 of 2005 on Teachers and Lecturers in Article 8 which articulates that "The teacher must have academic qualifications, competencies, teaching certificate, healthy physically and mentally, as well as having the ability to achieve national education goals."

Furthermore, in Article 10 paragraph (1) states that "The competencies of teachers referred to in Article 8 includes pedagogical competence, personal competence, social competence, and professional competence acquired through education profession". Teacher's HMKT expertise to teach mathematics at primary schools is included into the demands of professional competence. According to the Regulation of the Minister of National Education of the Republic of Indonesia Number 6 of 2007 on Academic Qualification Standards and Teacher Competencies, the professional competence of teachers refers to a teacher mastery of the material, structure, concept, and the mindset of scientific support of teaching subjects. For subjects of mathematics at primary schools, education minister Law specifies that the professional competence of teachers include (1) expertise of conceptual and procedural knowledge, and the linkages between both in the context of matter of arithmetic, algebra, geometry, trigonometry, measurement, statistics, and mathematical logic, (2) capable of using mathematical horizontally and vertically to solve mathematical problems and problems in the real world, (3) capable of using conceptual and procedural knowledge, and the linkages between both in mathematical problem-solving, as well as their application in real-life mathematics, and (4) capable of using props, measuring tools, calculators and computer software. HMKT demonstrates an obvious connection to the professional competence set forth in clause (1), the conceptual and procedural knowledge and the linkages between both within the context of the mathematic course being taught at elementary schools.

The aforesaid teacher competence requirement defined by the Laws of the National Education System and Regulations of the Minister of National Education leads teacher's education and training in Indonesia to the following implication. The key issue the providers of teacher education and training must cope with is what should be taught to prospective teachers to fulfil the required competency standards? This study has sought to provide a model to identify one of the professional competencies of teachers to teach fraction division at elementary schools. This model is expected to come in handy to identify HMKT to other mathematics concepts at elementary schools or at higher education levels. In fact, it is highly encouraged to implement the model on other courses than mathematics. Equally importantly, further studies to re-evaluate, to verify and to expand the identification of the proposed HMKT and its effect on student learning outcomes would be greatly commendable.

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