

The Impact of Teaching Approaches on Students' Mathematical Proficiency in Sweden

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The present study examines the effect of two differently structured methods, traditional and problem-solving, of teaching children mathematics the first five years in school as well as differences between boys' and girls' achievement depending on teaching approaches. The progress made by these students is presented by the five component measures of their mathematical proficiency; productive disposition, conceptual understanding, procedural fluency, strategic competence and adaptive reasoning. The tests (test in pre-school and national test in school year five) employed in this study were developed by an expert group contracted by the National Council of Education in Sweden. Differences between School A and School B, and boys and girls, on mathematical skills at 11 years of age were examined using t-tests for independent samples. The t-test was performed on raw scores across the entire sample. The results show that there are no significant differences between teaching methods when assessing procedural fluency. Students' progress in conceptual understanding, strategic competence and adaptive reasoning is significantly better when teachers teach with a problem-based curriculum. In order to develop aspects of self-efficacy, the results show that pupils would better benefit from a traditional curriculum. Boys and girls who have been taught with similar methods perform equivalent in both the traditional and the problem solving group.

Keywords: mathematics proficiency, teaching methods, learning outcomes, gender

The influence of the learning environment upon knowledge development has received relatively little attention in the field of mathematics teaching and learning (Boaler, 1999; Samuelsson, 2008). Even so, teachers often expect researchers to provide that kind of knowledge in mathematics didactics.

What happens in the classroom has an impact on students' opportunity to learn. The activities in the classroom, the repeated actions in which students and teachers engage as they learn are important because they constitute the knowledge that is produced (Cobb, 1998). There is some evidence that different teaching styles can have different impacts on student achievement (Aitkin & Zukovsky, 1994) and that the choice of teaching approaches can make an important difference in a student's learning (Wentzel, 2002). The synthesis of meta-analysis and reviews of Teddlie and Reynolds (2000) gives evidence for positive relationships between achievement and varied classroom settings. Case (1996) argues that a variation of teaching methods is important because different teaching methods draw attention to different competencies in mathematics (e.g. Boaler, 2002; Samuelsson, 2008). Thus, the mode of teaching method in mathematics seems to be important for students' development of mathematical proficiency.

In the present study, the effectiveness of two teaching approaches, traditional and problem solving, in mathematics is examined.

Mathematical Proficiency

The mathematics curriculum during elementary school in Sweden has many components, but there is a strong emphasis on concepts of numbers and operations with numbers. From an international perspective, mathematics knowledge is defined as something more complex than concept of numbers and operations with numbers. Kilpatrick et al. (2001) argue for five strands which together build students' mathematical proficiency. The five strands provide a framework for discussing the knowledge, skills, abilities, and beliefs that constitute mathematical proficiency. In their report they discuss,

1. *Conceptual understanding* is about comprehension of mathematical concepts, operations, and relationships. Students with conceptual understanding know more than isolated facts and methods. Items measuring conceptual understanding are for instance: "Your number is 123.45. Change the hundreds and the tenths. What is your new number?"
2. *Procedural fluency* refers to skills in carrying out procedures flexibly, accurately, efficiently, and appropriately. Students need to be efficient in performing basic computations with whole numbers (e.g., $6+7$, $17-9$, 8×4) without always having to refer to tables or other aids.
3. *Strategic competence* is the ability to formulate, represent, and solve mathematical problems. Kilpatrick et al. (2001, p.126) give the following example of item testing strategic competence: "A cycle shop has a total of 36 bicycles and tricycles in stock. Collectively there are 80 wheels. How many bikes and how many tricycles are there?"
4. *Adaptive reasoning* refers to the capacity for logical thought, reflection, explanation, and justification. Kilpatrick et al. (2001) gives the following example where students can use their adaptive reasoning. "Through a carefully constructed sequence of activities about adding and removing marbles from a bag containing many marbles, second graders can reason that $5+(-6)=-1$. In the context of cutting short bows from a 12-meter package of ribbon and using physical models to calculate that 12 divided by $\frac{1}{3}$ is 36, fifth graders can reason that 12 divided by $\frac{2}{3}$ cannot be 72 because that would mean getting *more* bows from a package when the individual bow is *larger*, which does not make sense" (p.130).
5. "*Productive disposition* is the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (Kilpatrick et al., 2001, p.5). Items measuring productive disposition are for instance: "How confident are you in the following situations? When you count $8-1= ___+3$ (completely confident, confident, fairly confident, not at all confident)."

The present study focuses on how different teaching methods affect aspects of students' mathematical proficiency.

Teaching Approaches Affecting Student Learning

There are very few studies focusing on how different teaching methods affect students' calculation and conceptual understanding as well as self-regulated learning skills, but there are several studies that focus on closely related areas.

For learning in general, Granström (2006) shows that different teaching approaches in classrooms influence the outcomes for students in different ways. Settings where students are allowed and encouraged to cooperate with classmates and teachers give the students more opportunities to understand and succeed. Similarly, Opendekker and Van Damme (2006) stress that good teaching involves communication and building relationships with students. Boaler (1999, 2002) reports that practices such as working through textbook exercises or discussing and using mathematical ideas were important vehicles for the development of flexible mathematical knowledge. One outcome of Boaler's research was that students who had worked in textbooks performed well in similar textbook situations. However, these students found it difficult to use mathematics in open, applied or discussion-based situations. The students who had learned mathematics through group-based projects were more able to apply their knowledge in a range of situations. Boaler's research gives evidence for the theory that context constructs the knowledge that is produced.

In a review of successful teaching of mathematics, Reynolds and Muijs (1999) discuss American as well as British research. A result of their review is that effective teaching is signified by a high number of opportunities to learn. Opportunity to learn is related to factors such as length of school day and year, and the amount of hours of mathematics classes. It is also related to the quality of classroom management, especially time-on-task. According to research in the area, achievement is improved when teachers create classrooms that include (a) substantial emphasis on academic instruction and students' engagement in academic tasks (Brophy & Good, 1986; Griffin & Barnes, 1986; Lampert, 1988; Cooney, 1994), (b) whole-class instruction (Reynolds & Muijs, 1999), (c) effective question-answer and individual practices (Brophy, 1986; Brophy & Good, 1986; Borich, 1996), (d) minimal disruptive behaviour (Evertsson et al., 1980; Brophy & Good, 1986; Lampert, 1988; Secada, 1992), (e) high teacher expectations (Borich, 1996; Clarke, 1997), and (f) substantial feedback to students (Brophy, 1986; Brophy & Good, 1986; Borich, 1996). Aspects of successful teaching are found in a traditional classroom (lecturing and drill) with one big exception- in successful teaching, teachers are actively asking a lot of questions and students are involved in a class discussion. With the addition of active discussion, students are kept involved in the lesson and the teacher has a chance to continually monitor students' understanding of the concept being taught.

On the other hand, negative relationships have also been found between teachers who spend a high proportion of time communicating with pupils individually and students' achievement (Mortimer et al., 1988; OfSTED, 1996). Students' mathematics performances were low when they practiced too much repetitive number work individually (OfSTED, 1996). A traditional direct-instruction/active teaching model seems to be more effective than a teaching model that focuses on independent work.

Another teaching model discussed in the literature is the one dependent on cooperative, small-group work. The advantage of problem-solving in small groups lies in the scaffolding

process whereby students help each other advance in the Zone of Proximal Development (Vygotsky, 1934/1986). Giving and receiving help and explanations may widen students' thinking skills, and verbalising can help students structure their thoughts (Leiken & Zaslavsky, 1997). The exchange of ideas may encourage students to engage in higher-order thinking (Becker & Selter, 1996). Students who work in small groups are developing an understanding of themselves and learning that others have both strengths and weaknesses. Programmes that have attempted problem-solving in small groups as a teaching method report good results, such as improved conceptual understanding and higher scores on problem-solving tasks (Goods & Gailbraith, 1996; Leiken & Zaslavsky, 1997).

Samuelsson (2008) used a split-plot factorial design with group (i.e., traditional, independent work, and problem-solving) as a between-subject factor and time (i.e., before and after a 10 week intervention) as a within-subject factor. In that design, *traditional approach* means that teacher explained methods and procedures from the chalk board at the start of the lessons, and the students then practice with textbook questions. *Independent work* means that students work individually on problems from a textbook without a teacher's introduction to the lesson; teachers just helped students who asked for it. *Problem solving* means that students were introduced to different ideas and problems that could be investigated and solved using a range of mathematical methods. Students worked in groups of four, and they discussed and negotiated arithmetic issues with each other and with the teacher, both in groups and in whole-class discussions. There were a total of seven dependent variables in the study. There were three measures of mathematics abilities; that is, a total score of mathematics ability, calculation, and conceptual understanding. Measures related to self-regulated learning skills such as internal and instrumental motivation, self-concept, and anxiety were also used as dependent variables. The results showed that there are no significant interaction effects between group and time according to total arithmetic ability and calculation. However, differences in students' progress in conceptual understanding may be explained by the teaching method. Traditional work as well as problem-solving seems to have more positive effects on students' development of conceptual understanding than independent work does.

To develop aspects of self-regulated learning skills, teachers, according to Samuelsson (2008), would be advised to use traditional work or problem-solving. Problem solving appears to be more effective in developing students' interest and enjoyment of mathematics than does traditional work or independent work. Also traditional work and problem-solving are more effective than independent work for students' self-concept.

Thus, different teaching methods also seem to influence students' self-regulated learning skills (interest, view of the subject's importance, self-perception, and attribution) (Boaler, 2002). Students who were expected to cram for examinations describe their attitudes in passive and negative terms. Those who were invited to contribute with ideas and methods describe their attitudes in active and positive terms that were inconsistent with the identities they had previously developed in mathematics (Boaler, 2002). A negative attitude towards mathematics can be influenced, for instance, by too much individual practice (Tobias, 1987) as well as by teachers who reveal students' inabilities. Students who do well in school (Chapman & Tunmer, 1997) demonstrate appropriate task-focused behaviour (Onatsu-Arvillomi & Nurmi, 2002), and they have positive learning strategies. If the students are

reluctant in learning situations and avoid challenges, they normally show low achievement (Midgley & Urdan, 1995; Zuckerman, Kieffer, & Knee, 1998).

As a result, the choice of teaching method not only affects mathematics achievement but also students' self-regulated learning skills.

Impact of Gender on Mathematical Achievement

There are studies discussing mathematics as a boy's subject in terms of girls' lack of confidence in their ability (Hyde, Fenneman, & Lamon, 1990; Terwilliger & Titus, 1995) and in terms of attitudes to mathematics as a school subject (Francis, 2000) and girls' achievement (Ernest, 1998). There are other studies arguing that teaching approaches where students practice rote learning (Ridley & Novak, 1983) and rote following (Scott-Hodgetts, 1986) are positive to girls. Boys are more interested in classroom settings where it is possible to take risks and where it is possible to find out different ways to solve problems - a more creative environment (Rodd & Bartholomew, 2006).

Researchers are attributing the problem to gender rather than focusing on wider context (Ernest, 1991). Rogers (1995) and Burton (1995) argue that the instruction of mathematics as a complete body of knowledge affects girls in a negative way more than boys. Boaler (1997) asserts that we must stop blaming the girls who do not participate in discussions in classrooms and who underachieve. Classroom norms frequently serve to exclude girls.

All of these studies above discuss boys' and girls' achievement in mathematics on a secondary or high school level. In this study, focus is on differences in mathematical proficiency between boys and girls taught the same way the first five years in compulsory school.

Predictions and Specific Research Questions

The following predictions and specific research questions were examined in relation to different areas of mathematical proficiency in this study.

1. It was predicted that students in the non-textbook problem solving group would show strength in all areas of mathematical proficiency except the procedural area (cf. Boaler, 1999, Samuelsson, 2008). That is, they would perform significantly higher than the traditional group. In the procedural area the author predicted an equal result in the distribution (Samuelsson, 2008). The research question was formulated as: Which teaching approach, traditional or problem solving is most effective for developing students' mathematical proficiency?
2. It was predicted that girls would have the advantages over boys in the traditional group (cf. Ridley & Novak, 1983, Scott-Hodgetts, 1986) and that boys would have the advantages over girls in the problem solving group (cf. Ross & Bartholomew, 2006). The research questions were formulated as: a) Do girls have the advantage over boys in a traditional teaching context? b) Do boys have the advantage over girls in problem solving context?

Method

This research was part of an ongoing longitudinal study of the impact of different learning contexts on students' mathematical proficiency.

Participants

A total of 105 students in four different classes were included in the study. They attended to two different schools (46 in school A and 59 in school B) in the same neighbourhood. These schools recruit students from a part of Sweden with low socioeconomic status. According to the National Council of Education, there were no significant differences in socioeconomic background between the two schools- when they started in grade 1, there were 115 students. After five years, some students had moved to other schools and some new students had arrived to the participating schools. The new students are not included in this study because they had been involved in other teaching approaches in mathematics than the experimental schools.

Age in months, gender, and performance on a standardized test in language and mathematics in pre-school were similar across classes for each school. The students were all 7 years old when they started grade 1; there were 57 female students and 48 male students. Thus, there were four (equal) groups of mathematics students attending school at grade 1 (group 1 (27 students), group 2 (26 students), group 3 (28 students), and group 4 (24 students), and these classes were assigned to two different teaching approaches in mathematics. Five years later, all students were given a National Test in mathematics.

During these five years a lot of observations were made on the schools. These observations are an important source describing the teaching approaches later in this text. These observations also led us believe that the two schools had predominantly the same teaching methods (School A traditional work with textbook, and School B problem solving) throughout the five years.

Measures of Mathematics Skills at Preschool Level

The tests (test in pre-school and National Test in school year five) employed in this study were developed by an expert group contracted by the National Council of Education. The pre-school test was used to see if there were any differences according to students' mathematical skills when they began school. The pre-school test is an oral test and measures simple calculation, conceptual understanding, number sense and problem solving. The test was given by one of the researcher in the research group. An independent t-test showed that there were no significant differences between the two groups assigned to different teaching methods on the test in total as well as their abilities in calculation and conceptual understanding, number sense and problem solving.

The National Test in school year five involved all the competencies that are important in order to develop a mathematical proficiency (Kilpatrick et al., 2001). The national test contains five subtest focusing different aspects of mathematical skills. To control our classification, all items were classified to one mathematical skill by six teachers and three teacher educators. The teachers and researchers agreed on 93% of the classification. All disagreement were discussed and resolved in the research group. (A sample of items related to

different mathematical skills is presented in the Appendix.). Cronbach's α estimates of reliability based on internal consistency for each measure of mathematical skills are based on item-level analyses in both groups.

Table 1

Means, standard deviation and significance level for differences between School A and School B for mathematics skills in pre-school

Mathematics Skill Measure	School A	School B	<i>p</i>
Calculation ^a	4.87 (2.43)	5.04 (2.33)	.73
Number sense ^a	5.43 (2.33)	4.80 (2.04)	.14
Conceptual understanding ^b	5.70 (2.26)	5.07 (2.12)	.15
Problem solving ^c	2.95 (1.01)	2.97 (1.16)	.96

Note: ^a maximum score = 10; ^b maximum score = 8; ^c maximum score = 5

Conceptual understanding was measured by 46 item and involved measurement, number sense and geometry ($\alpha=.89$). Procedural fluency was measured by 9 items ($\alpha=.81$). The procedural items require the student to perform addition, subtraction, multiplication and division. The procedural fluency test items involve decimals, fractions and whole numbers. Because the calculations are presented in a traditional format, the student is not required to make any decisions about what operations to use or what data to include. Strategic competence was measured by 16 items and required the student to formulate, represent, and solve the problems ($\alpha=.74$). Items related to productive disposition measure a student's belief in diligence and one's own efficacy. Twelve items measured this aspect of mathematical skill ($\alpha=.80$). Adaptive reasoning was measured by ten items ($\alpha=.71$). The items tested the student's capacity for logical thought, reflection, explanation, and justification. A sample of items testing different mathematics skills are presented in Appendix.

Differences in Teaching Approaches for Each School

In this study, two different teaching approaches (traditional work with a textbook, and experimental problem solving) were compared. Both schools followed the national curriculum. Two classes in School A were taught in the traditional way. This means that the teacher explained methods and procedures from the chalkboard at the start of the lessons, and the students then practiced with textbook questions. The students never made any choice about mathematical procedures to be used. They just followed the procedures they had just been taught and completed a lot of work in lessons.

The other two classes in School B were introduced to different ideas and problems that could be investigated and solved using a range of mathematical methods. Students worked in pairs or groups of four and discussed and negotiated mathematical issues with each other and with the teacher. This negotiation took place both in groups and in whole-class discussions. No mathematics textbooks were used in the school during these five years.

The two teaching approaches are very similar to Boalers (1999) description of Amber Hill and Phoenix Park School. The main difference seems to be on the age of students. Boaler's

students are 16-18 years old whereas the students in this study are 7-11 years old. Table 2 summarizes the different teaching approaches.

Table 2

Differences in teaching approaches for School A and School B

School A	School B
Teacher explanations from the chalkboard in every mathematics lesson	Teacher explanations from the chalkboard are rare
Closed questions	Open problems
Textbooks	Projects
Individual work	Work in pair or groups
Disciplined	Relaxed
High work speed	Low work speed

Data Analysis Methodology

Differences between School A and School B, and boys and girls, on mathematical skills at 11 years of age were examined using *t*-tests for independent sample. The *t*-test was performed on raw scores across the entire sample. Bonferroni's correction for setting the alpha level of $p < .05$ was used. The outcome of Bonferroni's test suggested an alpha level $p < .009$. With reference to Bonferroni the level was set to $p < .009$ (Abdi, 2007). The magnitude of mean differences was calculated using Cohen's *d* (Cohen, 1988). Cohen's *d* can be interpreted in terms of the percentage of nonoverlap in two distributions. An effect size lower than 0.3 is considered being small and indicates an overlap of 78.7%. An effect size of 0.50 is considered to be moderate with 67% overlap in the distribution of two samples. In this study Cohen's *d* is presented in Table 4.

Results

Descriptive statistics for the overall accuracy scores for the five mathematical tasks for each achievement group, and correlation among the tasks, are displayed in Table 3 and Table 4. In order to see if the five competency areas indeed represent distinct skill areas, I calculated the correlations among them. These correlations are reported in Table 3 below.

Table 3

*Correlations(*r*) among the measures used in the study*

Measure	1	2	3	4	5
1. Productive disposition	-	.304**	.249*	.272**	.034
2. Procedural fluency		-	.381**	.233*	.529**
3. Conceptual understanding			-	.786**	.434**
4. Strategic competence				-	.427**
5. Adaptive reasoning					-

Note: * $p < .05$; ** $p < .01$

Low correlations are desirable as they indicate that the competencies indeed represent distinct skills. The correlations between different competencies important to the mathematical proficiency are relatively low except the correlation among conceptual understanding and strategic competence. The strength of the relationship is indicated by the correlation coefficient (r) and the significance of the relationship is expressed in probability levels (p). A correlation lower than 0.5 is generally described as *weak* (Magnusson, 2007). The questions in the National Test measure different aspects of mathematical proficiency (National Agency of Education, 2009).

Teaching Approaches Impacting Mathematical Proficiency

The first hypothesis predicted higher scores in School B in all areas except procedural fluency. Table 4 presents the result on mathematical skills for all students in School A compared with all students in School B. School A demonstrates a significantly lower scores than School B on conceptual understanding, $t(103)=5.34$, $p<.009$, strategic competence $t(103)=2.69$, $p<.009$, and adaptive reasoning $t(103)=4.17$, $p<.009$.

The findings presented in Table 4 also demonstrate that students' productive disposition $t(103)=2.75$, $p<.009$ are significantly higher in School A - a result that was not predicted. Finally, Table 4 shows that there is no significant difference in procedural fluency between the groups.

Table 4

Means, standard deviation for differences between School A and School B for mathematics skills in school year 5 (11 years of age) and effect size (Cohen's d)

Measure	School A	School B	d
Productive disposition	29.77 (5.20)	27.14 (4.56)	0.26*
Procedural fluency	7.70 (1.90)	7.68 (1.80)	no sig.
Conceptual understanding	27.37 (8.64)	34.69 (5.33)	0.45*
Strategic competence	8.67 (3.45)	10.22 (2.44)	0.25*
Adaptive reasoning	4.67 (2.48)	6.69 (2.01)	0.38*

Note: * $p<.009$

Cohen's d estimation on these skills varied between .25 and .45. It means that the percentage of non-overlap in the two distributions was between 17% and 30%. The effect size represents the extent to which the groups differ from one another or the degree to which the variables are related. If the d value is approximately .20, the effect size is regarded as small. That is, researchers would conclude that any difference between groups is not especially pronounced. If the d value is over .30, the effect size is regarded as medium and researchers would conclude that the difference between groups is not modest. In this study the overlap, the similarity between groups on different aspects of mathematical competencies, is between 73% (medium) to 80% (small).

Teaching Approaches Impacting Mathematical Skills on Boys and Girls

In the second hypothesis, it was predicted that girls would take advantages over boys in the traditional group and that boys would take advantages over girls in the problem solving group. Table 5 shows that there are no significant differences between boys' and girls' results in the traditional teaching approach group.

The problem solving approaches have the same impact on boys as on girls. The results in Table 6 demonstrate no significant differences in any mathematical skill important to students' mathematical proficiency.

Table 5

Means, standard deviation for differences between boys and girls in School A for mathematics skills in school year 5 (11 years of age)

Measures	School A		<i>p</i>
	Boys	Girls	
Productive disposition	30.40 (4.27)	29.28 (5.85)	.47
Procedural fluency	7.75 (2.29)	7.65 (1.57)	.10
Conceptual understanding	28.15 (9.17)	26.77 (8.34)	.60
Strategic competence	9.25 (3.18)	8.23 (3.63)	.33
Adaptive reasoning	5.05 (2.54)	4.38 (2.43)	.37

Table 6

Means, standard deviation for differences between boys and girls in School B for mathematics skills in school year 5 (11 years of age)

Measures	School B		<i>p</i>
	Boys	Girls	
Productive disposition	27.24 (4.92)	27.13 (4.29)	.99
Procedural fluency	7.89 (1.81)	7.48 (1.81)	.39
Conceptual understanding	34.82 (4.60)	34.58 (5.99)	.86
Strategic competence	10.50 (2.30)	9.97 (2.57)	.41
Adaptive reasoning	6.46 (2.00)	6.90 (2.03)	.41

Discussion

This study was designed to investigate the effects of two different teaching approaches on students' mathematical proficiency, productive disposition, conceptual understanding, procedural fluency, strategic competence and adaptive reasoning. The study resulted in a number of findings of practical significance. It is important to emphasize that the intervention conducted did not involve total control over the classroom setting. It is virtually impossible to control everything that could happen in an everyday classroom.

Thus, the research group has tried to minimize the effect of certain variables (teaching activity, student activity, content taught, etc.). In order to have control on the teaching approaches, researchers and teachers met once a month to discuss the mathematics teaching in

the groups. The researchers wanted to make sure the quality of teaching was decent and acceptable, and was perceived to be so by the students. These discussion groups helped teachers focus on their actions in the classroom.

The most notable result is that the research group has been able to demonstrate differential effects on productive disposition, conceptual understanding, strategic competence and adaptive reasoning. When these kinds of descriptions are presented, the interpretation of the results is at least as important as the description. The result will be discussed with respect to earlier research in related areas. The following interpretations seem to be plausible and are possible starting points for further studies.

Teaching Approaches and Mathematical Proficiency

Many researchers argue that different teaching methods draw attention to different learning outcomes (Cobb, 1998; Case, 1996; Suchman, 1997; Vygotsky, 1934/1986; Boaler, 2002, Samuelsson, 2008). The current study provides support for this view. In this study, it is obvious that different teaching approaches have different impacts on different aspects of students' mathematical proficiency. Problem-based learning is significantly better for improving students' performances in conceptual understanding, strategic competence and adaptive reasoning, competences that strongly correlate with each other (see Table 3). These results are consistent with other studies investigating program that focused on problem-solving in small groups (Goods & Gailbraith, 1996; Leiken & Zaslavsky, 1997). One possible explanation is that students who worked in problem-solving classes were exposed to a higher level of reasoning, and that they accept this reasoning as valid. In traditional work, students generally interact with teacher, while students working on problem-solving interact with both their peers and their teachers (Oppendekker & Van Damme, 2006). In addition, active participation and the communication of thought processes with people of higher ability seem to be critical underlying factors when students are developing their conceptual understanding, strategic competence and adaptive reasoning. The language is a medium for discussing how to proceed and for restructuring ideas of peers' divergent and sophisticated range of strategies. From this perspective, discussion provides students with the opportunity to explore variations between their own and their partners' knowledge and thinking, correct misconceptions and fill gaps in understanding (Granström, 2006). Most importantly, in a problem-solving activity, students need to convince themselves and their partners of the correctness of a particular method. It appears that learning conditions characterised by communication are positive for students' understanding of conceptual understanding, strategic competence and adaptive reasoning.

Samuelsson (2008) reported that there were no significant differences between a problem-solving approach and traditional approach according to conceptual understanding. One possible explanation is that teachers in Samuelsson's (2008) study provided students with explanations and relevant concepts more often than teachers did in School A. A lot of the mathematical learning that takes place within the first five years of schooling is related to everyday practice. Teachers therefore, do not always provide students with mathematical concepts to the extent that they do when students are older.

The communication processes do not have the same impact on procedural fluency in this study. Students who worked in the traditional classroom achieved higher scores, but not

significantly higher, than students who worked in a problem-solving environment, regarding procedural fluency. This is not surprising, since they have practiced their procedural skills more than the problem-solving groups.

Students' productive disposition, beliefs in diligence and one's own efficacy, is affected significantly more if students work traditionally. Student beliefs about their efficacy can be developed by four main sources of influence (Bandura, 2000). The most effective way of creating a strong sense of efficacy is through mastery experiences. In this case, the mathematical questions were very similar to questions in a normal textbook in school year 1-5 in Sweden. This fact could explain why students in School A reported their ability higher than students in School B. Another explanation for students judging their ability higher in School A than School B is that students in School B are more aware of their own abilities. Earlier research has shown that students in small groups are developing a better understanding of themselves as learners, both strengths and weaknesses (Becker & Selter, 1996).

In an overview of research of effective mathematics teaching, Reynolds and Muijs (1999) argue for one specific teaching method, similar to what is called a traditional method in this study, as the most effective method for learning mathematics. In their review, they do not focus on how different teaching methods affect different mathematical skills. The results show how different teaching methods affect student's mathematics achievement in general. The specific contribution of this study is to clarify how different teaching methods affect different mathematical skills. From a teacher's perspective, when mathematics work is complex (Kilpatrick et. al., 2001), it is essential to know how different methods affect students' different learning outcomes.

Teaching Approaches, Learning Outcomes and Gender

Earlier research in mathematics teaching and learning argues that teaching approaches where students are taught in a traditional environment is positive for girls (Ridley & Novak, 1983; Scott-Hodgetts, 1986) and that boys have advantages in environments where they are able to take risks and solve problems (Rodd & Bartholomew, 2006). This study provides us with different results. The results of this study show that there is no variation between boys and girls according to different mathematical competences. One explanation for this result could be the magnitude of time of the intervention. Both boys and girls were taught in the same way for five years, they came from similar socioeconomic backgrounds and they grew up in the same neighbourhood. Under these conditions it was not possible to see any differences between boys and girls according to mathematical proficiency.

Another explanation could be the age of the students. The first five years in school mathematics is relatively concrete. In the beginning of year six, the mathematics curricula become more abstract. The widely cited meta-analysis by Hyde et al. (1990) found that gender differences in mathematical ability at high school, favour boys and are small in magnitude. Maybe these differences become clearer later in school when mathematics becomes more abstract. Research that links gender with levels of concrete instruction would be informative for future discussions about gender differences in mathematics. In this study, teaching approaches rather than gender seem to explain differences in achievement.

Didactical Implications

Teaching methods where students are able to use their language in order to discuss mathematical problems seem to have a positive effect on students' conceptual understanding, strategic competence and adaptive reasoning. Traditional method and problem solving approaches have equivalent impact on student's procedural knowledge. Boys and girls who have been taught with similar methods perform equally in both traditional and the problem solving groups. This study gives evidence that no single method affects all areas of mathematical proficiency with the same impact. An eclectic approach to instruction may best work to develop all dimensions of learning outcomes.

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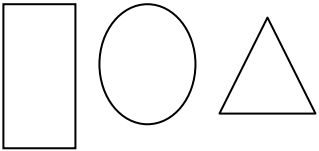
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Appendix - Examples of Items Testing Different Mathematics Skills

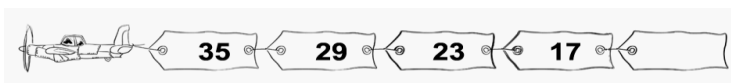
Productive Disposition

How confident are you in the following situations?	Completely confident	Confident	Fairly confident	Not confident at all
Tell names of these figures 				
Measure with a ruler				
When you count $8-1= ___ +3$				
When you count $403-125=$				
When you count $96/3=$				
When you count $36 \times 100 =$				

Conceptual Knowledge

- a) Draw a line under the largest number.
 1.49 1,499 1.5 1,099

- b) Study the pattern and state the number that is missing.



- c) Your number is 123.45. Change the hundreds and the tenths. What is your new number?

Procedural Fluency

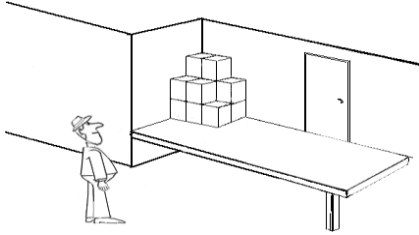
- a) $5 + 4 = 7 + ______$
 b) $9 - 4 = ______ + 3$
 c) $______ - 5 = 8$
 d) $18 + 6 = ______ \cdot 4$

Strategic Competence

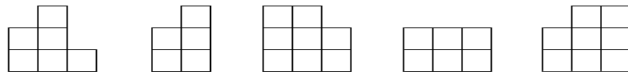
- Peter's car holds 15 gallons of gas. Anna's car holds 10 gallons of gas, and John's car holds 20 gallons of gas. How many more gallons does Peter's car hold than Anna's?
- Each of the four people has five dollars. How much money do they have together?

Adaptive reasoning

- There are several milk boxes outside the school restaurant. How many milk boxes are there? (Please count all boxes, even the ones you cannot see.)



- How does the man see the milk boxes from his angle? (Mark the correct picture)



- Complete the pattern.

