

undergraduate nursing students' abilities to calculate drug dosages, only 7% correctly answered the questions on ratio and proportion, although the average score on the test was 55% (Jukes & Gilchrist, 2006).

Research Method

The present study involved entry-level Diploma students from a comprehensive university in the South Africa. In this university, the students are streamed as mainstream or extended stream, depending on their prior academic performance. If applicants do not qualify academically for the entrance requirements of their chosen diploma, they are placed in the extended stream, and allowed extra time to complete the qualification. Extended stream courses qualify for additional government subsidies. The sample consisted of university entry level students who were enrolled for mathematics, a service course for national diploma studies in engineering and science. Based on a request from the researchers, out of a population of 120 students in three cohorts (Electrical Engineering, Civil Engineering and Analytical Chemistry), 94 students (54 mainstream and 40 extended stream) voluntarily took part in the study.

Adopting a positivist paradigm and a quantitative research approach, the study applied a survey design. The instrument collected general data such as age, Grade 12 mathematics scores, gender, course and stream. The fractions part in the instrument consisted of 20 items, of which three were multiple choice questions (MCQs) and the rest open-ended calculations. The lead researcher compiled the instrument after a study of the pertinent literature, including various Trends in International Mathematics and Science studies (TIMSS). Questions were selected to cover the following topics: notation, magnitude and magnitude on a number line (B1); operations on fractions (B2); operations combined with SI unit conversions (B3); ratio and proportion (B4) and percentage and percentage increase and decrease (B5). The skills tested are required in all engineering and science courses. An attempt was made to cover four levels of skills in the test, namely knowing; performing routine procedures and/or measurements; using complex procedures and lastly, solving problems as envisaged by (DoBE, 2011a). The allocation of a question to one of the four categories mentioned above was not always made apparent. Long et al. (2014, p.8) assert that such allocation "depends on the level of knowledge acquired by the learner". The first category was 'knowing' and the allocation of questions to this category seemed simple, but in retrospect it was found to be more difficult than expected as Long et al. (2014) posited:

Even a seemingly simple category such as "knowing", can be problematic.... Whilst there is an element of memory involved, in that recalling facts, terms, basic concepts and answers forms part of this component, this component also embraces knowledge of specifics (terminology and specific facts), knowledge of ways and means of dealing with specifics and knowledge of the universal abstractions in a field (principles and generalisations, theories and structures) (Long et al., 2014, p. 4).

Hence, experienced mathematics lecturers from the same faculty were

figures show nexus to the test scores-the average test score was similar (47.8%). Considering their choice of career, it stands to reason that most of the students from this sample (86 or 91.5%) regarded mathematics as very important. This is significant, since Thomson and Hillman (2010) assert that students who value mathematics are more likely to be successful in their tertiary study endeavours.

The students' average scores per question have been summarised in Table 2, sorted in the descending order. B1-B5 in Table 2 refer to the topics covered, B1 for notation, magnitude and magnitude on a number line; B2 for operations on fractions; B3 for operations combined with SI unit conversions; B4 for ratio and proportion and B5 for percentage and percentage increase and decrease. The skills level in Table 2 refers to one of four levels of skills, namely knowing (K), performing routine procedures and/or measurements (R), using complex procedures (C) and lastly solving problems (S). There were five questions in each of these categories.

Table 2¹. Frequency Distributions: Correct answers to Test Questions (n = 94)

Question Number	Skills level	Topics covered	Analytical Chemistry		Civil Engineering		Electrical Engineering		Total	
			Count	Percentage	Count	Percentage	Count	Percentage	Count	Percentage
3e	K	B2	40	87%	17	68%	15	75%	72	77%
5	R	B3	32	70%	19	76%	16	80%	67	72%
3b	R	B2	31	67%	19	76%	17	85%	67	71%
3d	R	B2	29	63%	24	96%	14	70%	67	71%
1	K	B1	29	63%	18	72%	10	95%	66	70%
7	C	B4	28	61%	17	68%	8	40%	53	60%
4	R	B3	27	59%	15	60%	13	65%	55	59%
11	K	B5	27	59%	15	60%	11	55%	53	58%
8	C	B3	28	61%	12	48%	11	55%	51	54%
2a	C	B1	22	48%	15	60%	12	65%	50	54%
2b	C	B1	23	50%	14	56%	12	65%	50	54%
10	C	B4	21	46%	16	64%	7	35%	44	47%
3a	R	B2	20	43%	10	40%	8	40%	38	41%
13	S	B5	13	28%	12	48%	9	45%	34	37%

¹Percentages in this table were calculated as a proportion of the number of students who answered each question. In the rest of the paper, percentages were calculated as a fraction of 94, the sample size. It was assumed that students who did not offer an answer,

Table 5. Central tendency & dispersion: test scores in percentages (n = 94)

Scores	Mean	S.D.	Min	Quartile 1	Median	Quartile 3	Max
B1	58.8	38.2	0.0	33.0	67.0	100.0	100.0
B2	46.9	20.2	0.0	29.0	43.0	57.0	86.0
B3	61.4	33.4	0.0	33.0	67.0	100.0	100.0
B4	39.6	29.4	0.0	25.0	50.0	50.0	100.0
B5	32.2	24.7	0.0	0.0	33.0	33.0	100.0
B	47.8	19.6	7.0	31.0	50.5	63.0	86.0

The data revealed a significant relationship between the self-reported Grade 12 Mathematics score and the B1 Score and a statistically significant relationship between the self-reported Grade 12 Mathematics score and both the B3 Score and the average score for the test (the B score) (Figure 1 and Tables 6 and 7). According to Gravetter and Wallnau (2009), correlations are statistically significant at the 0.05 level for $n = 94$ if $|r| \geq 0.203$ and practically significant if $|r| \geq 0.300$.

Table 6. Pearson Product Moment Correlations–B1 (Notation, magnitude and magnitude on a number line) score to B score and Mathematics Grade 12 (n = 94) score

	<i>Mathematics Grade 12</i>
B1 score	0.347
B2 score	0.110
B3 score	0.228
B4 score	0.169
B5 score	0.030
B score	0.291

Table 7. Contingency Table - Mathematics Grade 12 and B Scores

B Score	Mathematics Grade 12							
	30 - 49%		50 - 59%		60 - 100%		Total	
0 - 39	19	58%	9	33%	6	18%	34	36%
40 - 100	14	42%	18	67%	28	82%	60	64%
Total	33	100%	27	100%	34	100%	94	100%
Chi ² (d.f. = 2, n = 94) = 11.70; p = .003; V = 0.35 Medium								

The meta-cognition of the students was probed in the last question (Q15) by asking them to rate their scores in the test:

Please indicate what, in your opinion, you most likely scored on this test by writing down the letter of the score category:

A: 0 – 19%	B: 20 – 39%	C: 40 – 59%	D: 60 – 79%	E: 80 – 100%
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A statistically significant correlation was measured between the answers (B15 scores, Figure 2) and the B4 and B scores (Table 9).

Table 9. Contingency Table – Cohort and B15

B15	Cohort						Total	
	Analytical Chemistry		Civil Engineering		Electrical Engineering.			
0 - 19%	1	2.1%	2	8.0%	0	0.0%	3	3.2%
20 - 39%	9	18.8%	2	8.0%	1	5.0%	12	12.9%
40 - 59%	18	37.5%	10	40.0%	6	30.0%	34	36.6%
60 - 79%	11	22.9%	7	28.0%	7	35.0%	25	26.9%
80 - 100%	9	18.8%	4	16.0%	6	30.0%	19	20.4%
Total	48	100%	25	100%	20	100%	93	100%

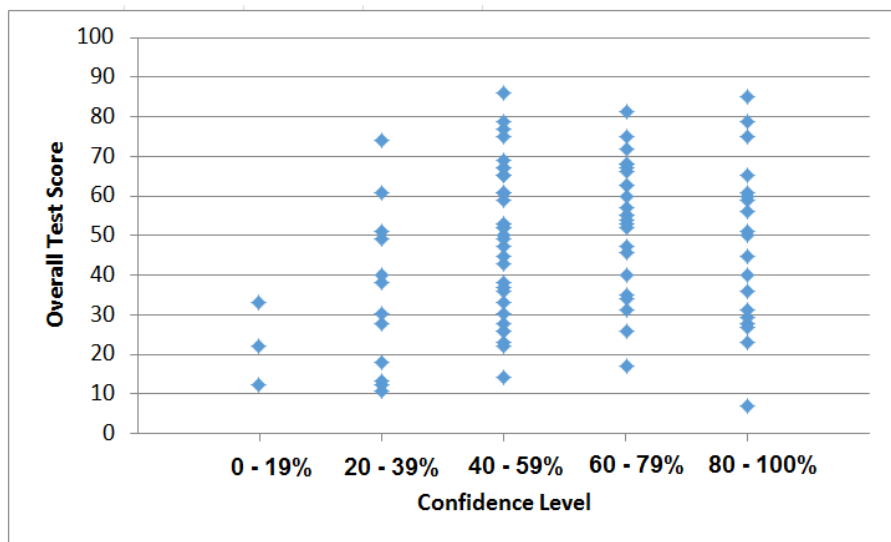


Figure 2. Relationship between Meta-cognition or Confidence level (B15) and Overall Test Score (B Score)

NOTATION, MAGNITUDE & MAGNITUDE ON A NUMBER LINE (B1, 58.8% AVERAGE) (Q1 & 2A, 2B)

This division seemed to work out perfectly, but for the fact that the distance between zero and the first subdivision was $\frac{22}{100}$ units and the distance between each successive subdivision, was substantially smaller at $\frac{2}{100}$ units.

Table 10. Scores for Question 2

Question 2	Mainstream			Extended Stream	
	Electrical Engineering (20)	Civil Engineering (17)	Analytical Chemistry (17)	Civil Engineering (8)	Analytical Chemistry (32)
correct answers for Q2a	13 (65.0%)	11 (64.7%)	10 (58.8%)	4 (50%)	12 (37.5%)
correct answers per stream	34 (63%)			16 (40%)	
sample Q2a	50 (53.2%)				
correct answers for Q2b	13 (65%)	9 (52.9%)	10 (58.8%)	5 (62.5%)	13 (40.6%)
correct answers per stream	32 (59.3%)			18 (45%)	
sample Q2b	50 (53.2%)				

The summary of the scores for Cohort for B1 (Notation, magnitude and magnitude on a number line) is given in Table 11.

Table 11. Contingency Table – Cohort and B1 score

B1 Score	Cohort						Total	
	Analytical Chemistry (49)		Civil Engineering (25)		Electrical Engineering (20)			
0 to 39	26	53%	9	36%	7	35%	42	45%
40 to 100	23	47%	16	64%	13	65%	52	55%
Total	49	100%	25	100%	20	100%	94	100%
Chi ² (d.f. = 2, n = 94) = 2.91; p = .233								

OPERATIONS ON FRACTIONS (B2, 46.8% AVERAGE) (Q 3A-F & Q9)

In Question 3b, students had to calculate $\frac{1}{5}$ of a decimal fraction - a Grade 6 skill, which is also revised in Grades 7 and 8 (DoBE, 2011c, p. 17).



Figure 5. Vuyo's drawing made in response to Question 3f

Although many students realised that the answer was supposed to be six, they did not necessarily relate the six to halves. Numerous answers depicted one apple divided into six portions, which apparently satisfied the need to have six elements in the answer.

Peter realised that an answer consisting of 6 whole units was incorrect, and that fractional portions were called for, and thus changed his answer to reflect these (Figure 6).

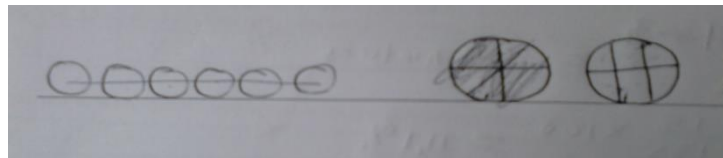


Figure 6. Peter's drawing in response to Question 3f

Peter hence realised that the fractions in his first circular drawing were unequal, and attempted to correct it. He however ended up with four parts equal in size, but unequal in size to the other two parts.

Sam in turn reported the answer as six halves, but Sam's drawing (Figure 7) did not correspond to his written answer, and contained six parts, but not six halves. The drawing he made was similar to Peter's.

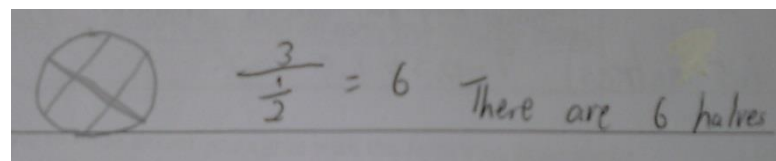


Figure 7. Sam's drawing in response to Question 3f

Sam was able to find the correct answer, but did not display accurate conceptual understanding of the answer.

Table 13. Scores for Question 3f

Description	Mainstream		Extended Stream	
	Electrical Engineering (20)	Civil Engineering (17)	Civil Engineering (8)	Analytical Chemistry (32)
Scores for Question 3f				

and therefore could not make the correct decision. An example of such a method is to first convert the price for the 2.5 kg pack of sugar to price per kg, and then to stop. The second step, which was missing, would have been to also convert the price for the 500 g pack to a price per kg, and then to compare the prices.

A study conducted in Italy on consumer choice (Graffeo et al., 2015) used a field experiment that showed marked similarities to Question eight in this study. During the experiment a product was made available, with different initial prices and discounts, at two shops. One of the deals was better than the other, and consumers had to pick the better one and describe the arithmetic operations used in their decision. The researchers classified the approaches used by the consumers as either “complete” or “partial”. “Complete” refers to decisions taken after all the arithmetic operations required to solve the problem were calculated. “Partial” referred to decisions taken after only some of the operations were calculated. The researchers came to the conclusion that higher levels of numeracy were associated with the “complete” decision approach, which enabled the consumers to make a better quality purchase decision. The students involved in the current study, who had incomplete answers to Question eight, therefore used the “partial” decision approach, possibly demonstrating lower levels of numeracy. Wilson and MacGillivray (2007) assert that success rates fall rapidly when answers require multiple steps to be performed.

Table 15. Scores for Question 8

Question 8		Mainstream		Extended Stream	
Scores for Question 8	Electrical Engineering (20)	Civil Engineering (17)	Analytical Chemistry (17)	Civil Engineering (8)	Analytical Chemistry (32)
correct answers	11	8 (65%)	13 (76%)	4 (75%)	15 (38%)
correct answers per stream	32 (59.3%)			19 (47.5%)	
sample	51 (54.3%)				

A summary of the scores for Cohort on B3 (Operations combined with SI units) is given in Table 16.

Table 16. Contingency Table - Cohort and B3 Score

B3 Score	Cohort						Total	
	Analytical Chemistry		Civil Engineering		Electrical Engineering			
0 to 39	18	37%	9	36%	6	30%	33	35%
40 to 80	17	35%	8	32%	7	35%	32	34%
81 to 100	14	29%	8	32%	7	35%	29	31%
Total	49	100%	25	100%	20	100%	94	100

If I can walk $1\frac{1}{5}$ kilometres in twelve minutes, how long will it take me at that rate to walk five kilometres? Answer in minutes.

The results are summarised in Table 18. Fewer than half of the students (46.8%) answered correctly. Some students attempted to convert the rate to a rate per hour, by multiplying by 5. Few of these students however proceeded to answer the question correctly. Another common mistake, especially amongst the extended stream students, was to translate a mixed fraction incorrectly to a decimal fraction, that is, $1\frac{1}{5}$ was incorrectly converted to 1.5. Most of the students, who presented incorrect answers for this question, showed no steps, and it was therefore difficult to analyse thought processes without interviewing the students.

Table 18. Scores for Question 10

Question 10		Mainstream		Extended Stream	
	Electrical Engineering (20)	Civil Engineering (17)	Analytical Chemistry (17)	Civil Engineering (8)	Analytical Chemistry (32)
correct answers	7 (35%)	13 (76.5%)	8 (47.1%)	3 (38.5%)	13 (40.6%)
correct answers per stream	28 (51.9%)			16 (40%)	
sample	44 (46.8%)				

Question 12 tested ratio and direct proportion (DoBE 2011a, p. 14). A girl's height was provided together with the length of the girl's shadow. Also, the length of a nearby pole's shadow was provided, and students had to calculate the height of the pole.

Very few of the students could present a correct answer (Table 19). Most students who presented incorrect answers, attempted to solve this problem by means of subtraction, i.e. they calculated the difference between the length of the girl's shadow and the girl's height, and then subtracted that amount from the length of the pole's shadow to obtain the pole's height, a process that yielded the incorrect answer of 6.5 m, offered by 17 (18.1%) of the students.

Table 19. Scores for Question 12

Question 12		Mainstream		Extended Stream	
	Electrical Engineering (20)	Civil Engineering (17)	Analytical Chemistry (17)	Civil Engineering (8)	Analytical Chemistry (32)
correct answers	2 (10%)	11 (64.7%)	3 (17.6%)	3 (37.5%)	4 (12.5%)
correct	16 (29.6%)			7 (17.5%)	

“meaningful participation in society as citizens of a free country” (DoBE, 2011c, p. 18). These students will have to enter the free market and will encounter price increases on a daily basis as part of their work and personal environment. One of the common mistakes made was to calculate the difference between the original price and the raised price, (R15) and to assume this amount to be a percentage (incorrect answer 15%). In some instances, students divided the difference (R15) by the new price (R70) when calculating a percentage, instead of dividing by the original price. Another common incorrect answer was 80%. These students presumably divided the original price by the raised price (R60/R75) and then multiplied by 100 to convert to a percentage.

Table 22. Scores for Question 13

Question 13		Mainstream		Extended Stream	
	Electrical Engineering (20)	Civil Engineering (17)	Analytical Chemistry (17)	Civil Engineering (8)	Analytical Chemistry (32)
correct answers	9 (45%)	7 (41%)	7 (41%)	5 (63%)	6 (15%)
correct answers per stream	23 (43%)			11(27.5%)	
sample	34 (36.2%)				

The final question (Q14) was:

The price of fuel has dropped by to R10.89. What was the price of fuel before the price decrease?

This question yielded by far the worst results (4.3% correct answers, Table 23), which is disturbing, since the topic of financial mathematics features in the Grade 8 syllabus as Finance and Growth, emphasising how ‘to solve problems, including annual interest, hire purchase, inflation, population growth and other real-life problems’ (DoBE, 2011a, p. 18). Furthermore, the topic is repeated in the Grade 10 Mathematics syllabus, which prescribes that learners should understand the implication of ‘fluctuating foreign exchange rates, for example on petrol price, imports, exports, overseas travel’ (DoBE, 2011b, p. 26). The topic is examined in the first of two compulsory mathematics examination papers in Grade 8 and the question should be worth 10 ± 3 of the 100 marks for Grade 10, and 15 ± 3 marks for Grade 11 and 12.

Table 23. Scores for Question 14

Question 14		Mainstream		Extended Stream	
	<i>Electrical Engineering</i>	<i>Civil Engineering</i>	<i>Analytical Chemistry</i>	<i>Civil Engineering</i>	<i>Analytical Chemistry</i>
correct answers	1 (5%)	1 (6%)	1 (6%)	0	1 (3%)
correct	3 (5.6%)			1 (2.5%)	

support programs should include quantitative skills. A study by Campbell (2009) in mathematics, and another by Kremmer et al. (2010) in the field of business, showed that remedial programmes at tertiary education institutions had a positive impact on students' success at university.

Data collected pointed to lack of conceptual understanding of operations on fractions. This is a common problem internationally (Siegler & Lortie-Forgues, 2015). Especially in the case where no calculators are allowed, procedural mastery does not necessarily imply conceptual understanding. Students may remember the procedure without ever having understood the theoretical underpinning for the procedure. This may also apply to teachers (Ma, 1999). Consistent with this interpretation, Ma (1999) found that most United States teachers in her study could not generate any explanation of what $1\frac{3}{4} \div \frac{1}{2}$

means, or resorted to explaining a different problem, i.e. $1\frac{3}{4} \div 2$. A study by Ball (1990) revealed similar results. Teachers in other local and international studies have demonstrated weak conceptual understanding of fraction arithmetic (Lin et al., 2013; Ma, 1999; Rizvi & Lawson, 2007). A study conducted at five South African universities reported that prospective teachers enter university programmes with reasonable procedural knowledge of mathematics but poor conceptual knowledge (Bowie, 2014).

Similar problems occur when students have to multiply by fractions with magnitude smaller than one. Teachers should emphasize that multiplication and division produce different outcomes depending on whether the numbers involved are greater or lesser than 1, and should discuss why this is true. 'Chinese textbooks include such instruction' (Siegler & Lortie-Forgues, 2015). To strengthen their case, these researchers referred to an example given by Sun and Wang (2005) in which Chinese students were asked to solve and discuss answers to the following three problems: $4.9 * 1.01$; $4.9 * 1$; $4.9 * 0.99$.

Jukes and Gilchrist (2006) observe that a lack of retention may be to blame. Some of the students involved in this research may have understood the concepts at the time when these were explained to them, but may have forgotten at the time of this study. Johnson and Johnson (2002) claim that the educational institution and the student should share the responsibility for both attainment and retention of skills.

Reports of studies done by various researchers (Siegler et al., 2011; Booth & Newton, 2012; Torbeyns et al., 2014; Wu, 2001) emphasise the importance of magnitude representations of fractions on number lines when teaching fractions, as opposed to the part-of-a-whole approach. Some countries focus almost exclusively on the part-whole approach and neglect the number-line approach. South African teaching seems to fall into the latter category. Both methods should be incorporated into the pedagogy of teaching fractions in order to supplement and complement each other.

Conclusions and Suggestions

The current study revealed that entry-level students enrolled for engineering and science diplomas performed poorly in a test of numeracy skills.

researcher would then be able to reach more detailed conclusions regarding the levels of conceptual understanding that cause university entry level students' incorrect answers.

The sample in this study was relatively small, but there is no reason to assume that the findings cannot be generalised to other diploma students in science and engineering, provided that the admission requirements for the diplomas are compatible.

The current study revealed that entry-level students enrolled for engineering and science diplomas performed poorly in a test of numeracy skills. The average score (47.8%) was regarded as a cause for concern, especially considering that the test was pitched at Grade 8 level. The average score was far below the researchers' expectations. Furthermore, students displayed a lack of conceptual understanding of the procedures. This study also revealed a marked difference between the performance of the mainstream and the extended stream students.

Mathematics lecturers at universities and mathematics teachers at secondary schools should take note of the results of this study. First year university lecturers at universities need to offer remedial action, especially for students in the extended stream. Furthermore, problems with conceptual understanding of fractions could possibly be ascribed to conceptual problems that teachers might have had with fractions. Hence, one approach to addressing students' difficulties with fractions would be to conduct in-service workshops to refresh practicing teachers' subject content knowledge and pedagogical content knowledge in order to enhance teachers' conceptual knowledge of fractions so as to promote effective teaching and learning activities.

Even people who are proficient in fraction procedures often possess weak conceptual understanding of multiplication and division of fractions less than 1. Literature highlights that specific teaching approaches may yield improved results, especially in this instance. Teachers should point out to students that multiplication and division produce different outcomes depending on whether the numbers involved are greater than or less than 1, and should discuss why this is so.

Lastly, the importance of magnitude representations of fractions on number lines, as opposed to the part-of-a-whole approach to teaching fractions, does need emphasis. Both methods should be incorporated into the pedagogy when teaching fractions.

Further research should examine whether a cause-effect relationship exists between reflective reasoning and mathematics scores in general, and if so, whether it is possible to improve students' cognitive reflection in mathematics. Also, various studies have found fraction knowledge to be an early and accurate predictor of later mathematics achievement. These predictions may extend to predictions of tertiary mathematics success. Possible links between fraction test scores and tertiary students' pass rates in mathematics should be explored. It may well transpire that scores on fraction tests could be used as a predictive measure for the eventual academic success in mathematics service courses at university. If a positive correlation does exist, further research should be conducted on whether remedial measures on fraction skills will positively

- Bone, A. A., Carr, J. A., Daniele, V.A., Fisher, R., Fones, N .B., Innes, J. I., Maher, H. P., Osborn, H.G. & Rockwell, D.L. (1984). *Promoting a clear path to technical education*. Washington DC: Model Secondary School for the Deaf.
- Booth, J.L. & Newton, K.J. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemporary Educational Psychology*, 37(4), 247-253.
- Booth, J.L., Newton, K.J. & Twiss-Garrity, L.K. (2014). The impact of fraction magnitude knowledge on algebra performance and learning. *Journal of Experimental Child Psychology*, 118, 110-118.
- Bowie, L. (2014). *Report on mathematics courses for intermediate phase student teachers at five universities*. Johannesburg: JET Education Services. Retrieved 24 March 2016 from http://www.jet.org.za/publications/initial-teacher-education-research-project/copy_of_bowie-report-on-maths-courses-offered-at-5-case-study-institutions-18-feb.pdf.
- Bowie, L. & Frith, V. (2006). Concerns about the South African Mathematical Literacy curriculum arising from experience of material development. *Pythagoras*, 64, 29-36.
- Brousseau, G., Brousseau, N. & Warfield, V. (2004). Rationals and decimals as required in the school curriculum. Part 1: Rationals as measurements. *Journal of Mathematical Behavior*, 23, 1-20.
- Cai, J. (1995). *A Cognitive Analysis of U. S. and Chinese Students' Mathematical Performance on Tasks Involving Computation, Simple Problem Solving, and Complex Problem Solving* (Vol. monograph series 7). Reston, VA: National Council of Teachers of Mathematics.
- Campbell, A. (2009). *Remediation of First-year mathematics students algebra difficulties*. (MSc), University of KwaZulu-Natal, KwaZulu-Natal, South Africa. Retrieved 30 April 2014 from http://researchspace.ukzn.ac.za/xmlui/bitstream/handle/10413/761/Campbell_A_2009.pdf?sequence=1
- Case, J. (2006). Issues facing engineering education in South Africa. Paper presented at the *Engineering Education for Sustainable Development: Proceedings of the 3rd African Regional Conference*, 26-27 September 2006, University of Pretoria, Pretoria, South Africa, 26-27 September 2006.
- Cetin, H & Ertekin, E. (2011). The relationship between eighth grade primary school students' proportional reasoning skills and success in solving equations. *International Journal of Instruction*, 4(1), 47-62.



- Clarke, D. (2006). Fractions as division: The forgotten notion. *Australian Primary Mathematics Classroom*, 11(3), 4-10.
- Coetzee, J. & Mammen, K. J. (2016). Challenges Faced by Entry-level University Students in Word Problems Involving Fractions Terminology. *International Journal of Education Sciences (IJES)*, 15(3), 461-473.
- DoBE, (2011a). *National Curriculum Statement (NCS): Curriculum and Assessment Policy Statement (CAPS) Further Education and Training Phase MATHEMATICS GR 7-9*. Pretoria: Department of Basic Education. Retrieved 30 April 2014 from <http://www.education.gov.za>.
- DoBE, (2011b). *National Curriculum Statement (NCS): Curriculum and Assessment Policy Statement (CAPS) Further Education and Training Phase MATHEMATICS Grades 10-12*. Pretoria, South Africa: Department of Basic Education. Retrieved 30 April 2014 from <http://www.education.gov.za>.
- DoBE, (2011c). *National Curriculum Statement (NCS): Curriculum and Assessment Policy Statement (CAPS) Intermediate Phase MATHEMATICS GR 4-6*. Pretoria: Department of Basic Education. Retrieved 30 April 2014 from <http://www.education.gov.za>.
- DoBE, (2012). *National Senior Certificate 2012: National diagnostic report on learners' performance*. Pretoria, South Africa: Department of Basic Education. Retrieved 15 October 2015 from <http://www.education.gov.za>.
- Dorko, A & Speer, N. (2014). *Calculus Students' Understanding of Units*. Paper presented at the 17th Annual Conference on Research in Undergraduate Mathematics Education, February 26 - March 2, 2014, Denver, Colorado.
- Duffin, J. (2003). Numeracy in Higher Education. In J. K. Peter Kahn (Ed.), *Effective Learning and Teaching in Mathematics and Its Applications*: Routledge, London.
- Fonseca, K & Petersen, N. (2015). Online supplementary mathematics tuition in a first-year childhood teacher education programme. *South African Journal of Childhood Education*, 5(3), 9 pages.
- Gabaldon, T.A. (2015). Strength in Numbers: Teaching Numeracy in the Context of Business Associations. *St. Louis University Law Journal*, 59, 701-709.
- Graffeo, M., Polonio, L. & Bonini, N. (2015). Individual differences in competent consumer choice: the role of cognitive reflection and numeracy skills. *Frontiers in Psychology*, 6.

- Gravetter, F.J. & Wallnau, L.B. (2009). *Statistics for the Behavioral Sciences* (8th ed.). Belmont, CA: Wadsworth.
- Houston, J., Tenza, S.P., Hough, S., Singh, R. & Booyse, C. (2015). The rationale for teaching Quantitative Literacy in 21st century South Africa: A case for the renaming of Mathematical Literacy. *The Independent Journal of Teaching and Learning*, 10. Retrieved 6 April 2016 from <http://hdl.handle.net/11622/53>
- Johnson, A.W. & Johnson, R. (2002). Cooperative Learning Methods: A meta-analysis. *Journal of Research in Education*, 12(1), 5-14.
- Jukes, L & Gilchrist, M. (2006). Concerns about numeracy skills of nursing students. *Education in Practice*, 6(4), 192-198.
- Kremmer, M., Brimble, M., Freudenberg, B. & Cameron, C. (2010). Numeracy of First Year Commerce Students: Preliminary Analysis of an Intervention. *The International Journal of Learning*, 17(1), 1-13.
- Lamon, S.L. (2001). Presenting and Representing: From Fractions to Rational Numbers. In A. Cuoco & F. Curcio (Eds.), *The Roles Of Representations in School Mathematics-2001 Yearbook* (pp. 146-165). Reston: NCTM.
- Lesh, R., Post, T. & Behr, M. (1988). Proportional Reasoning. In J. Hiebert & M. Behr (Eds.), *Number Concepts and Operations in the Middle Grades* (pp. 93-118). Reston, VA: Lawrence Erlbaum & National Council of Teachers of Mathematics.
- Lin, C.Y., Becker, J., Byun, M-R., Yang, D.C. & Huang, T.W. (2013). Preservice Teachers' Conceptual and Procedural Knowledge of Fraction Operations: A Comparative Study of the United States and Taiwan. *School Science and Mathematics*, 113(1), 41-51.
- Livy, S. & Herbert, S. (2013). Second-Year Pre-Service Teachers' Responses to Proportional Reasoning Test Items. *Australian Journal of Teacher Education*, 38(11), 17-32.
- Long, C., Dunne, T. & De Kock, H. (2014). Mathematics, curriculum and assessment: The role of taxonomies in the quest for coherence. *Pythagoras* 35, 35(2), 14.
- Lortie-Forgues, H., Tian, J. & Siegler, R.S. (2015). Why Is Learning Fraction and Decimal Arithmetic So Difficult. *Developmental Review*, 38, 201-221.
- Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' knowledge of fundamental mathematics in China and the United States*. Mahwah, NJ: Erlbaum.



- Naureen, D. & Vicki, N.T. (2012). The role of numeracy skills in graduate employability. *Education and Training*, 54(5), 419-434.
- NMAP, (2008). *Foundations for Success: The final report of the National Mathematics Advisory Panel*. Washington, DC: National Mathematics Advisory Panel. Retrieved 27 February 2016 from <http://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf>.
- OECD, (2013). *OECD Skills Outlook 2013: First Results from the Survey of Adult Skills*. Paris: OECD Publishing. Retrieved 4 April 2016 from www.oecd.org/skills/.
- Pienaar, E. (2014). *Learning About And Understanding Fractions And Their Role In The High School Curriculum*. (Master of Education), University of Stellenbosch, Stellenbosch. Retrieved 15 October 2015 from <https://www.google.co.za/#q=Learning+About+And+Understanding+Fractions+And+Their+Role+In+The+High+School+Curriculum>
- Pinker, S. (1998). *How the mind works*. London: Penguin Books
- Reyna, V.F., Nelson, W.L., Han, P.K. & Dieckmann, N.F. (2009). How Numeracy Influences Risk Comprehension and Medical Decision Making. *Psychological bulletin*, 135(6), 943-973.
- Rivera-Batiz, F.L. (1992). Quantitative Literacy and the Likelihood of Employment among Young Adults in the United States. *The Journal of Human Resources*, 27(2), 313-328.
- Rizvi, N.F. & Lawson, M.J. (2007). Prospective teachers' knowledge: Concept of division *International Education Journal*, 8(2), 377-392.
- Roohr, K.C., Graf, E.A. & Liu, O.L. (2014). Assessing Quantitative Literacy in Higher Education: An Overview of Existing Research and Assessments With Recommendations for Next-Generation Assessment. *ETS Research Report Series*, 2014(2), 1-26.
- Schneider, M. & Siegler, R.S. (2010). Representations of the magnitudes of fractions. *Journal of Experimental Psychology: Human Perception and Performance*, 36(5), 1227-1238.
- Schollar, E. (2008). *Towards Evidence-based Educational Development in South Africa*: Eric Schollar and Associates c.c. Retrieved 21 March 2016 from <https://www.ru.ac.za/media/rhodesuniversity/content/sanc/documents/Schollar%20-%202008%20-%20Final%20Report%20Short%20Version%20The%20Primary%20Mathematics%20Research%20Project%202004-2007%20-%20Towards%20evidence-based%20educational%20de.pdf>.

- Siegler, R.S., Duncan, G.J., Davis-Kean, P.E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M.I. & Chen, M. (2012a). Early Predictors of High School Mathematics Achievement. *Psychological Science*, *23*(7), 691-697.
- Siegler, R.S., Fazio, L.K., Bailey, D.H. & Zhou, X. (2012b). Fractions: the new frontier for theories of numerical development. *Trends in Cognitive Sciences*, *17*(1), 13-19.
- Siegler, R.S. & Lortie-Forgues H. (2015). Conceptual Knowledge of Fraction Arithmetic. *Journal of Educational Psychology*, *107*(3), 909-918.
- Siegler, R.S. & Thompson, CA. (2014). Numerical landmarks are useful-except when they're not. *J Exp Child Psychol*, *120*, 39-58.
- Siegler, R.S., Thompson, C.A. & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, *62*(2011), 273-296.
- Spaull, N. & Kotze, J. (2015). Starting behind and staying behind in South Africa: The case of insurmountable learning deficits in mathematics. *International Journal of Educational Development*, *41*, 13-24.
- Sun, L.G. & Wang, L. (2005). *Mathematics: Spring, Fifth grade*. Nanjing, Jiangsu Province: Phoenix Education.
- Titus, J. (1995). The concept of fractional number among deaf and hard of hearing students. *American Annals of the Deaf*, *140*(3), 255-263.
- Torbeyns, J., Schneider, M., Xin, Z., & Siegler, R.S. (2014). Bridging the Gap: Fraction understanding is central to mathematics achievement in students from three different continents. *Learning and Instruction*, 1-9.
- Torbeyns, J., Schneider, M., Xin, Z. & Siegler R.S. (2015). Bridging the gap: Fraction understanding is central to mathematics achievement in students from three different continents. *Learning and Instruction*, *37*, 5-13.
- Watts, T.W., Duncan, G.J., Siegler, R.S. & Davis-Kean, P.E. (2014). What's Past Is Prologue: Relations Between Early Mathematics Knowledge and High School Achievement. *Educational Researcher*, *43*(7), 352-360.
- Wilson, T.M. & MacGillivray, H.L. (2007). Counting on the basics: mathematical skills among tertiary entrants. *International Journal of Mathematical Education in Science and Technology*, *38*(1), 19-41.
- Wu, H. (2001). How to prepare students for Algebra. *American Educator*, *25*(2), 10-17.