Pre-Service and In-Service Teachers' Views of the Sources of Students' Mathematical Difficulties

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This paper examines the views of pre-service and in-service teachers with regard to the sources of students' mathematical difficulties. A group of 40 pre-service mathematics, 15 in-service mathematics and 15 in-service elementary teachers participated in this study. Questionnaires are used as data collection tools to see what the participants think about the sources of student difficulties. The notion of "obstacles to learning" is used as a framework to analyze the collected data. The analysis is carried out on the basis of three main categories to which participant teachers attribute students' difficulties: epistemological causes, psychological causes and pedagogical causes. The data analysis reveals that both pre-service and in-service teachers tend to attribute students' difficulties to student-related factors, namely psychological causes. We discuss the findings in terms of these three sources of learning difficulties, educational implications and note the usefulness of the employing the "obstacles to learning framework" in examining not only students' learning difficulties but also teachers' views of the sources for student difficulties.

Keywords: obstacles to learning, sources of students' mathematical difficulties, teacher views

Students in all grade levels find the learning of mathematics difficult. A close inspection of subject–specific studies conducted within the last four decades in mathematics education literature easily vindicates this proposition. Common to all these studies (Hart et al., 1980; Tall, 1991) are the findings showing lack of students' understanding of and difficulties with mathematical concepts. In other words, these studies collectively show that students have difficulties with many mathematical concepts taught in different level of their schooling. Why is this so? What can be sources of students' mathematical learning difficulties? Equally important is what teachers actually think about sources of students' mathematical difficulties. The latter question is the primary focus of this paper and we aim to investigate how teachers view sources of students' mathematical difficulties and provide insights into sources of these difficulties.

Literature Review

Earlier mathematics education studies (Hart et al., 1980; Tall & Vinner, 1981; Tall, 1991) generally focus on students' conceptions regarding various mathematical concepts. A quick examination of journals and proceedings such as Educational Studies in Mathematics and

Psychology of Mathematics Education can easily justify this observation. The main focus of these studies is the students' understanding of and difficulties with mathematical concepts.

The importance of the influence of teachers and the teaching on students' mathematical learning has started to receive attention, especially within the last two decades. Teachers' attitudes (Philippou & Christou, 1998), beliefs (Stipek, Givvin, Salmon, & MacGyvers, 2001), subject matter knowledge (Linchevski & Vinner, 1988), pedagogical content knowledge (An, Kulm, & Wu, 2004) and technological pedagogical content knowledge (Niess et al., 2009) have all become main research areas in teacher education. Teachers' perspective and competency with regard to all these areas can influence what and how students learn mathematics. Related studies, in fact, have provided empirical evidence that teachers' subject matter knowledge, pedagogical content knowledge, as well as their beliefs related to mathematics, have strong influences on students' mathematics learning (Ball, Thames, & Phelps, 2008; Verschaffel, Greer, & Torbeyns, 2006; Askew, Brown, Rhodes, William, & Johnson, 1997; Brown, Askew, Rhodes, William, & Johnson, 1997; Lamb & Booker, 2004; McClain & Bowers, 2000).

One can infer from these studies that students' difficulties and lack of understanding cannot solely be attributed to their own limitations. Teachers' beliefs, attitudes, subject matter knowledge and pedagogical knowledge can all have dramatic influences on how students learn and why students do not learn. It is, therefore, important to look at to what extent teachers are aware of their own roles alongside some other factors other than students' characteristics or student-related factors in the occurrence of students' learning difficulties and what they regard as sources of such difficulties. This is important to examine because, to us, how teachers view sources of the students' difficulties may shape how they go about teaching.

It appears that teachers' views of sources of the students' difficulties have received less research attention in both general and mathematics education literature. One of these studies comes from Penso (2002) on Biology pre-service teachers. Penso (2002) examines how pre-service teachers identify and describe the causes of pupils' learning difficulties. The findings obtained from 40 pre-service teachers through the use of observations and teaching diaries show that they attribute sources of difficulties to (1) the pupil (cognitive and affective characteristics); (2) the content (aspects of the contents of the lesson); (3) the teacher (the teaching methods); and (4) the lesson (the learning atmosphere). Amongst these sources, the characteristics of the pupils were cited most frequently as a source of the difficulties whereas the content, the teacher and the lesson sources received the least citations.

Although not always reported, such findings are not surely only limited to the participants of Penso's study. Perspectives that the participants in Penso's study hold with regard to the students' learning difficulties are, to us, commonly shared by many teachers. To that end, for instance, Floden (1996) states that when students do not learn or understand, teachers generally tend to attribute the problem to the inadequacy of the students or lack of their motivation, but not to the instruction to which they were exposed. Holding only students responsible for the failure or lack of understanding, nevertheless, is as we believe neither right nor fair. The roles that teachers play in the emergence of students' learning difficulties should not and cannot be neglected.

In fact, Shulman (1986, p.9) has also drawn attention to the importance of the role of teachers in handling student difficulties and that is why he has regarded teachers' "understanding of what makes the learning of specific topics easy or difficult" as a component of pedagogical content knowledge (PCK). With particular regard to PCK, Shulman (1986, p.9-10) states the following:

> "Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. If those preconceptions are misconceptions, which so often are, teachers need knowledge of the strategies most likely to be fruitful in reorganizing the understanding of learners, because those learners are unlikely to appear before them as blank slates."

We agree with Shulman that students' conceptions and preconceptions have deep influence on what and how they learn, and teachers' awareness of the students' preconceptions in this regard is important. Important, however, also is what teachers do think about sources of students' learning difficulties as their views regarding sources can shape how they conduct the teaching.

Theoretical Framework

Although the abovementioned literature points out the teacher tendency to attribute learning difficulties mainly to students and their own limitations, the reality is not necessarily so. The students' difficulties in learning mathematics and sources of these difficulties can be related to many other factors, including teachers themselves and the teaching. For a comprehensive appreciation of sources of students' difficulties, we find the theoretical framework of 'obstacles to learning' helpful. We find this framework useful in examining not only sources of student difficulties but also teachers' views of sources of students' difficulties.

Inspired by the work of Bachelard (1938/2002) on epistemological obstacles, Brousseau (1997) and later on Cornu (1991) introduce epistemological, psychological (cognitive) and pedagogical (didactical) obstacles in an attempt to make sense of students' mathematical difficulties. Cornu (1991), based on the work of Brousseau (1997), describes epistemological obstacles as occurring due to the nature of the mathematical concepts themselves. In elucidating epistemological obstacles, Cornu (1991, p. 159) cites Bachelard (1938/2002) and indicates that "epistemological obstacles occur both in the historical development of scientific thought and in educational practice". To Bachelard, epistemological obstacles have two fundamental features:

- "They are unavoidable and essential constituents of knowledge to be acquired,
- They are found, at least in part, in the historical development of the concept" (cited in Cornu, 1991, p. 159).

As these features suggest, epistemological obstacles may well reside in the nature of the concepts to be learnt. For that reason, an epistemological obstacle is often viewed as a piece of, not a lack of, knowledge, which is interpreted as functioning well within a frequently

experienced, but limited context, and not being generalizable beyond it (Selden & Selden, 2001; Brousseau, 1997). Epistemological obstacles also, as the second feature suggests, were encountered by the scientists during the historical development of the concepts. The difficulties and dilemmas that the scientists encountered during the construction process of the concepts can well be taken as an evidence of epistemological obstacles that the concepts pose.

To exemplify the presence of epistemological obstacles, several researchers focus on the limit concept and its historical development (Sierpinska, 1987; Cornu, 1991). Cornu (1991), for instance, presents several epistemological obstacles regarding the limit concept. One of epistemological obstacles that he presents is related to the idea of whether the limit is attained or not. He notes that the debate around this issue lasted throughout the history of the concept and there were disagreements and different interpretations amongst mathematicians. He provides the views of Robins (1697-1751), Jurin (1685-1750) and D'Alembert (1717-1783) regarding this issue and quotes Robins and D'Alembert actually stating that the limit can never be attained and Jurin stating that the limit can be attained. Herein it is critical to note that Robins' and D'Alembert's interpretation of the limit concept was different from its current interpretation; as it is now accepted that the limits of, for instance, constant functions are attainable.

Interestingly, the studies that have been carried out regarding students' conceptions of the limit concept show similar interpretations amongst the students as well (Williams, 1991; Cornu, 1991; Akbulut & Işık, 2005). These studies clearly show that what has been the problem or obstacle for the scientists may well be a problem or obstacle for the students as well. This similarity can be taken as an evidence for the existence of epistemological obstacles that the limit concept presents due to its nature. Epistemological obstacles can hence be interpreted as causing difficulties for the students and be sources of the difficulties that students encounter. In this paper, we consider students' difficulties related to epistemological obstacles as being epistemological and use the term 'epistemological causes' while referring to the difficulties arising from the nature of concepts.

With regard to psychological obstacles, Cornu (1991) describes them as occurring because of the personal development of the students[1]. Such factors as students' abilities, capabilities, motivation, prior conceptions and knowledge, learning experience regarding the concept to be learnt, ways of thinking and developmental stages all influence how students learn and sometimes explain why they have difficulties in learning. These factors can sometimes be the sources of students' difficulties in learning mathematical concepts. We refer to such factors as psychological causes of student difficulties.

Students' conceptions like 'multiplication always makes bigger' can be given as an example of psychological cause. This conceptualization of multiplication, normally, generates correct responses as far as whole numbers are concerned. It presents, however, an over-generalization as this cannot apply to rational numbers (Graeber, 1993). This example essentially suggests that students sometimes over-generalize what they learn and that can

[1] It should be noted here that Brousseau (1997) describes psychological (ontogenetic) obstacles mainly as occurring due to the limitation of the student at some period of his/her development and he does not provide further information on this issue. In this paper, however, we use the term psychological obstacles in a more comprehensive way for the purpose of our analysis.

cause them to make errors in the learning of successive concepts. In the words of Shulman (1986), students' prior conceptions or misconceptions influence how and what they learn and it is sometimes these conceptions that can cause the learning difficulties for students.

With regard to pedagogical obstacles, Cornu (1991) describes them as occurring as a result of the nature of the teaching and the teacher[2]. Although Cornu (1991) does not provide much detail regarding this issue, we consider pedagogical obstacles in relation to such factors as teachers' teaching approaches, teachers' use of analogies and metaphors, course books and the way concepts and topics are being covered in the textbooks and curricula. To use a more generic expression, we use the term 'pedagogical causes of student difficulties' in explaining pedagogy-caused student difficulties in learning a concept.

An example will be helpful to illustrate a pedagogy-caused student difficulty. Tanner (2000) states that teachers generally use "fruit and salad approach" to introduce the addition of two algebraic expressions, such as, $2a+3b$ in algebra teaching. This kind of expression is often explained to students by teachers through the use of some materials, such as 2 apples and 3 bananas. However, Pimm (1987), as cited in Tirosh, Even, and Robinson (1998), puts forward reservations regarding this approach, warns against its potential role in causing learning difficulties for students and notes that "it leads to confusion between a being apples and a being 'the number of apples...The algebraic expression is not an analog of 5 apples, nor is 5 apples a possible interpretation of 5a… the letters themselves are standing for numbers" (p.132). In fact, some studies have shown the disadvantages of using this approach in introducing algebraic expressions. Booth (1988), for example, shows that some students thought that the expression $2a + 5b$ is equal to $7ab$ on the grounds that '2 apples plus 5 bananas is 7 apples-and-bananas'. Tirosh et al. (1998) also point out that this approach may lead students to think algebraic expressions, such as $2a$ and $3b$, cannot be multiplied, that is one cannot multiply apples and bananas. All these suggest that sometimes the way materials are used and the way the teaching is conducted can be the causes of, or at least play a role in the emergence of, student learning difficulties for mathematical concepts.

Although we have presented these obstacles separately, it is often almost impossible to attribute student difficulties to just one particular obstacle and that the difficulty can stem from any combination of these obstacles. Further to this, in this paper we interpret obstacles more than just only being as 'pieces of knowledge' and regard them as being sources of the student difficulties as well. For us, epistemological, psychological and pedagogical obstacles are the causes of students' learning difficulties and we will employ the terms 'epistemological, psychological and pedagogical causes' while referring to the sources of learning difficulties in this paper. How we employ this framework in this study is detailed in the following sections.

The Study: Background and Data Collection Methods

The theme of this article emerged from two ongoing large projects being conducted on pre-service and in-service teachers in Turkey. The first project is related to integration of

[2] The same situation is the case for pedagogical obstacles too. Brousseau (1997) describes pedagogical (didactical) obstacles mainly as happening due to the choice of the educational system (didactic transposition). Here we, nevertheless, use the term in a more inclusive way for our purposes.

technology into teaching mathematics. More specifically the aim is to develop pre-service secondary mathematics teachers' technological pedagogical content knowledge (TPCK). Broadly speaking, for this aim, the TPCK framework proposed by Mishra and Koehler (2006) was employed in designing a two-semester course for pre-service secondary mathematics teachers to successfully integrate technology into teaching. During the first semester, two workshops were consecutively carried out. The first workshop was on PCK (Shullman, 1986) and the second was on the TPCK where the technology component was brought into the picture. Alongside many other activities, pre-service teachers participated in these two workshops, prepared lesson plans before and after each workshop and a subgroup of them did micro-teaching before their peers after each workshop. After attending the course developed in light of the TPCK framework in the first semester, the participant teachers went to the schools for applications in the second semester under the guidance of their mentor lecturers (for more on the project and the details, see, Ozmantar, Akkoç, Bingolbali, Demir, & Ergene, 2010; Akkoç, Bingolbali, & Ozmantar, 2008).

A group of 40 pre-service teachers participated in this first project. They initially took a three-and-half year mathematics program and then enrolled the secondary mathematics teacher preparation program. The preparation program took one and a half year and only upon successful completion of this program could pre-service teachers obtain the right to teach mathematics at the secondary level (teaching students aged from 15 to 19). The data for this paper were collected in the last two semesters of their preparation program.

Before taking part in the PCK workshop, we wanted to gain insights into the pre-service teachers' PCK. As part of this aim, we developed a questionnaire asking pre-service teachers to respond to the following open-ended questions alongside some other sub-questions:

(1) What can be the causes of the students' difficulties in learning a mathematical concept?
(2) What are the multiple representations of a mathematical concept?
(3) What are the role of teachers with regard to teaching methods and strategies for an efficient mathematics teaching?
(4) What do you think that the purpose of the assessment is?
(5) What kind of mathematics teaching approach that the mathematics curriculum proposes?

These questions were formulated on the basis of the components of PCK suggested by Grossman (1990) and Magnusson, Krajcik, and Borko (1999) (for more on this, see, Ozmantar et al., 2010). For the purpose of this paper, however, we focus our attention only on pre-service teachers' responses to the question, what can be the causes of the students' difficulties in learning a mathematical concept?[3]

The second project, on the other hand, is concerned with the professional development of in-service elementary teachers, elementary mathematics teachers and elementary science and

[3] It should be noted that we intentionally kept the questionnaire items general rather than specific to a particular concept as our aim was to unveil teachers' overall understanding of sources of students' learning difficulties in mathematics. This was due to the conviction that what teachers regard as the sources would appear in their answers and that would tell something about their perspectives on learning and teaching. For instance, if a teacher (be a primary, elementary or university teacher) considers that his way of teaching is important for his students' learning, then this will perhaps appear in his/her answer.

technology teachers. The project aims to create and scale up a professional development program in order to assist teachers to put a recently developed constructivist-oriented curriculum in Turkey into practice and to help them overcome the difficulties that they likely to encounter during the implementation of it. The professional development program aims to equip teachers with the skills and knowledge that is essential for the creation of a learning environment in which students get such skills as problem solving, scientific thinking, creative thinking, communication, and critical thinking. In order to create this learning environment, teachers participated in a training program with the following six areas: (1) establishing classroom norms for the improvement of students' autonomy, (2) overcoming student difficulties and misconceptions, (3) task design and implementation, (4) problem solving and meta-cognition, (5) technology integration, and (6) assessment and evaluation.

A cohort of 45 teachers (15 elementary teachers, 15 elementary mathematics teachers and 15 elementary science and technology teachers) have been taking part in the in-service teacher professional development program[4]. The teachers' teaching experiences range from one to sixteen years and they are selected amongst nearly 200 teachers who wanted to join the in-service training. The selection criteria were that the applicant should be willing to attend the training sessions on a regular basis, that they were prepared to implement new teaching methods suggested during the project, and that enthusiastic to work in close collaboration with the project team. In the time of writing this paper, the teachers have already taken part in the first three training areas (classroom norms, student difficulties and misconceptions and task design) that have lasted for three months. During their training, the participant teachers have joined sessions held by academics, designed lesson plans, conducted teaching along with their plans, and evaluated their instructions by reflecting upon video-recorded classroom practices of their colleagues.

The data that we used for the purpose of this paper from this project came from students' difficulties and misconceptions training area. The teachers participated in training that basically focused on student difficulties and misconceptions, sources of these difficulties (epistemological, psychological and pedagogical) and devised plans to overcome them in their teaching. This training lasted for four weeks and was carried out in the four consecutive sessions during the weekends. Before the training on student difficulties and misconceptions started, we administered a questionnaire including one particular item asking teachers to explain what they think about the sources of student difficulties and misconceptions. All the participant teachers responded to this item. However here in this paper we do not examine the views of science and technology teachers as they were not responsible for teaching mathematics. Yet we focus on elementary mathematics teachers as well as elementary teachers who were responsible for teaching mathematics to the students of early ages (5-12 year-olds). Hence we consider the views of 30 teachers (15 mathematics and 15 elementary teachers)

[4] In Turkey, there is a clear distinction between *elementary teachers* and *elementary mathematics teachers*. Elementary teachers teach grades 1-5 (students aged from 7 to 12) and are generally held responsible to teach five main subjects (Turkish language, mathematics, science and technology, life sciences and social sciences) to the same group of students. Elementary mathematics teachers, on the other hand, teach students aged from 12 to 15 at the middle school level but still considered under elementary education since elementary education in Turkey covers grades 1-8. This second group of teachers are graduates of mathematics education departments in faculties of education.

In sum, our aim in both projects, with regard to the focus of this paper, is to explore how pre-service and in-service teachers view the causes of student difficulties in learning mathematics.

Data Analysis Procedure

To analyze pre-service and in-service teachers' responses to the questions of what can be the causes of the students' learning difficulties, we employ three main categories in the theoretical framework of the study as explained previously: epistemological causes, psychological causes and pedagogical causes. 'Others' category is also employed when the participants' views do not fit into any one of these three categories. We thus use four categories in total to analyze the participants' responses and we now explain what these categories stand for through examples from the participants' responses.

Table 1
The definitions of categories employed for the data analysis

Categories	Definition of categories	Examples from teachers' responses
Epistemological causes	Responses that cite the difficult nature of and abstractness of mathematical concepts	Students' difficulties can be due to; ✓ The difficulties that the concepts pose, ✓ The abstractness of the concepts, ✓ Mathematics not being connected to real life
Psychological causes	Responses that cite student-related reasons	✓ Lack of prior knowledge, ✓ Negative attitudes (prejudice), ✓ Lack of motivation and interest ✓ Lack of ability ✓ Lack of self- confidence ✓ Lack of understanding of concepts
Pedagogical causes	Responses that cite teachers and teaching-related reasons	✓ Teachers' lack of knowledge or competency ✓ Teachers' attitudes ✓ Not teaching in a comprehensible manner for students ✓ Use of inappropriate teaching approaches
Other causes	Responses that cite reasons not fitted into the above three categories	✓ Economic situations ✓ Lack of family interest ✓ Lack of teaching materials ✓ Lack of infrastructure ✓ Unclear responses

In analyzing the data, the responses of the participants were separately examined and then allocated to each of these categories based on their descriptions. For instance, when a participant cites a student-related factor in explaining why students have difficulties with a mathematical concept, then this participant's response is allocated to 'psychological causes' category. However, when participants mention more than one factor, in this case their responses are allocated to more than one category. It should be noted that the allocation of responses to the categories was carried out by the two authors of the paper independently.

Initially, there was over 90% agreement, and later on, the disputed items were discussed until an agreement was reached for the assignment of each participant's response to a category.

After the allocations of the responses, we carried out the data analysis in three stages. First, the frequency analysis of the causes that participant responses generated was performed on the basis of four categories. Second, we focused on the frequency of the suggested causes assigned to only one category and this process generated three sub-categories: only epistemological causes, only psychological causes and only pedagogical causes. Finally, we carried out a further analysis of epistemological, psychological, pedagogical and other causes and developed sub-categories to provide insight into participants' concrete views regarding the sources of student difficulties.

Results

In this section we present our findings regarding pre-service and in-service teachers together. Before presenting the results, as mentioned above, it should be noted that a teacher's response sometimes was allocated to more than one category when the response had causes related to more than one category.

Table 2
The frequency analysis of pre-service and in-service teachers' causes

Categories	Pre-service teachers (n=40)	In-service teachers (n=30)	In-service teachers in groups	
			Elementary mathematics teachers (n=15)	Elementary teachers (n=15)
Epistemological causes	n=7 (17.5%)	n=3 (10%)	n=3 (20%)	n=0 (-%)
Psychological causes	n=32 (80%)	n=25 (83.3%)	n=14 (93.3%)	n=11 (73.3%)
Pedagogical causes	n=20 (50%)	n=11 (36.6%)	n=3 (20%)	n=8 (53.3%)
Other causes	n=5 (12.5%)	n=8 (26.6%)	n=5 (33.3%)	n=3 (20%)

Table 2 shows that 17.5% of pre-service, 10% of in-service (i.e., 20% of elementary mathematics and none of elementary) teachers refer to epistemological causes in explaining sources of the students' mathematical difficulties. Regarding psychological causes, the study reveals that 80% of pre-service and 83.3% of in-service teachers attribute students' mathematical difficulties to the psychological causes. However, it is the group of elementary mathematics teachers that refers to the psychological causes the most (93.3%) and it is the group of the elementary teachers that refers to psychological causes the least (73.3%). The analysis also reveals that 50% of pre-service, 36.6% of in-service (20% of mathematics and 53.3% of classroom) teachers attribute students' difficulties to pedagogical causes.

Further to this analysis, as mentioned above, a frequency of the responses assigned to only one category was also carried out. The findings obtained from this analysis reveal similar patterns. Table 3 shows that both pre-service (35%) and in-service teachers (33.3%) cite the psychological causes alone the most for sources of students' learning difficulties.

None of the elementary mathematics and elementary teachers and only 10% of pre-service teachers refers to epistemological causes alone for sources of students' difficulties. Table 3 also demonstrates that 13.3% of the classroom teachers, none of the mathematics teachers and 5% of pre-service teachers attribute students' difficulties to pedagogical causes alone.

Table 3
The frequency teachers' responses citing only one category of causes

Categories	Pre-service teachers (n=40)	In-service teachers (n=30)	In-service teachers in groups	
			Elementary mathematics teachers (n=15)	Elementary teachers (n=15)
Only epistemological causes	n=4 (10%)	n=0 (-%)	n=0 (-%)	n=0 (-%)
Only psychological causes	n=14 (35%)	n=10 (33.3%)	n=6 (40%)	n=4 (26.6%)
Only pedagogical causes	n=2 (5%)	n=2 (6%)	n=0 (-%)	n=2 (13.3%)
Only other causes	n=0 (-%)	n=0 (-%)	n=0 (-%)	n=0 (-%)

In addition to the above analysis, the following four tables (Tables 4-7) present further analysis of epistemological, psychological, pedagogical and other causes. Sub-categories column in the tables illustrates the categories emerged from the further analysis of the participant responses. 'Examples from participant responses' column, for instance, presents some cited causes from teachers' answers to the questionnaire items.

Table 4
Further analysis of epistemological causes

Sub-Categories	Examples from participant responses	Pre-service teachers (n=40)	In-service teachers (n=30)	In-service teachers in groups	
				Elementary mathematics teachers (n=15)	Elementary teachers (n=15)
Mathematics being abstract and/or not related to real life	Difficulties are due to mathematics consisting of abstract concepts and being unrelated to real life	n=5 (12.5%)	n=2 (7%)	n=2 (13%)	n=0 (-%)
The nature of concepts	Due to the difficulties that the nature of concept poses	n=2 (5%)	n=1 (3%)	n=1 (7%)	n=0 (-%)

Table 4 illustrates the further analysis on epistemological causes. The table shows that both pre-service and in-service elementary mathematics teachers refer to the causes of 'mathematical concepts being as abstract' and of 'the nature of concepts' to explain sources of students' difficulties, which we consider within the scope of epistemological causes. Note that no elementary teacher refers to epistemological causes.

Table 5
Further analysis of psychological causes

Sub-Categories	Examples from participant responses	Pre-service teachers (n=40)	In-service teachers (n=30)	In-service teachers in groups	
				Elementary mathematics teachers (n=15)	Elementary teachers (n=15)
Prior knowledge or its deficiency	Lack of prior knowledge	n=21 (52.5%)	n=12 (40%)	n=8 (53%)	n=4 (27%)
Negative attitudes (prejudice)	Negative attitudes: "maths is difficult"	n=12 (30%)	n=5 (17%)	n=1 (7%)	n=4 (27%)
Lack of motivation and interest	Not being sufficiently interested in concepts	n=6 (15%)	n=11 (37%)	n=8 (53%)	n=3 (20%)
Lack of ability	Lack of intelligence	n= 7 (17.5%)	n= 2 (7%)	n=1 (7%)	n=1 (7%)
Lack of self-confidence	Not being confident	n=3 (7.5%)	n=0 (-%)	n=0 (-%)	n=0 (-%)
Dislike of math or the topics	Maths is seen as boring and not likable by many students	n=2 (5%)	n=1 (3%)	n=0 (-%)	n=1 (7%)
Lack of understanding of concepts	Not comprehending the concept	n=0 (-%)	n=7 (23%)	n=5 (33%)	n=2 (13%)
Lack of efforts	Lack of efforts	n=1 (2.5%)	n=0 (-%)	n=0 (-%)	n=0 (-%)

Table 5 demonstrates that 52.5% of pre-service, 40% of in-service teachers (53% of mathematics teachers and 27% of elementary teachers) referred to 'prior knowledge or its deficiency' to explain why students have difficulties with mathematical concepts. The category of 'negative attitudes (prejudice)' was the second most cited by pre-service teachers whilst the category of 'lack of motivation and interest' was the second most cited by in-service teachers. Elementary mathematics teachers cited 'lack of understanding of concepts' more than elementary teachers, and pre-service teachers did not refer to this category at all.

Table 6
Further analysis of pedagogical causes

Sub-Categories	Examples from participant responses	Pre-service teachers (n=40)	In-service teachers (n=30)	In-service teachers in groups	
				Elementary mathematics teachers (n=15)	Elementary teachers (n=15)
Teachers' lack of knowledge or competency	Mathematics teachers' lack of understanding of math topics	n=5 (12.5%)	n=0 (-%)	n=0 (-%)	n=0 (-%)
Teachers' attitudes	Teacher's not taking his/her lesson seriously	n=4 (10%)	n=1 (3%)	n=0 (-%)	n=1 (7%)
Not teaching in a comprehensible manner	Teaching concepts in an abstract manner	n=6 (15%)	n=5 (17%)	n=1 (7%)	n=4 (27%)
Use of inappropriate teaching approaches	Students' misconceptions can be due to the way the concept is taught	n=3 (7.5%)	n=2 (7%)	n=1 (7%)	n=1 (7%)
Not holding students' attention	Teachers might not be able to draw student's attention	n=0 (-%)	n=2 (7%)	n=1 (7%)	n=1 (7%)
Not doing consolidation	Not presenting sufficient explanations and examples	n=0 (-%)	n=2 (7%)	n=0 (-%)	n=2 (13%)
Not determining students' preparedness	Not determining students' readiness	n=0 (-%)	n=1 (3%)	n=0 (-%)	n=1 (7%)

Table 6 presents a detailed analysis of pedagogical causes. The findings reveal that 12.5% of pre-service teachers and none of in-service teachers cited 'teacher's lack of knowledge or competency.' In-service teachers cited 'not teaching in a comprehensible manner' category the most (17%) to explain the reasons behind the student difficulties. 'Use of inappropriate teaching approaches' is also mentioned by both groups. Further to that, in-service teachers but not pre-service teachers referred to teachers' 'not holding students' attention', 'not doing consolidation', and 'not determining students' preparedness.'

As seen in Table 7, both pre-service and in-service teachers mentioned the economic situation of a student's family, lack of a student's family interest and friends as a reason for

why some students have difficulties with mathematics. Mathematics teachers (20%) refer to this category the most to explain why students have difficulties.

Table 7
Further analysis of other causes

Sub-Categories	Examples from participants' responses	Pre-service teachers (n=40)	In-service teachers (n=30)	In-service teachers in groups	
				Elementary mathematics teachers (n=15)	Elementary teachers (n=15)
Lack of economic situation or family interest or friends	Family, environment, economical situation	n=4 (10%)	n=4 (13%)	n=3 (20%)	n=1 (7%)
Unclear responses	No consolidation	n=2 (5%)	n=3 (10%)	n=2 (13%)	n=1 (7%)

Discussion

The data presented so far show that both pre-service and in-service teachers attribute students' difficulties in mathematics mainly to the psychological causes. It is interesting to note here that these findings show similarities with the results of Penso's (2002) study. In fact, Penso's (2002) study demonstrates that 80% and 55% of pre-service teachers refer to pupil characteristics in explaining sources of students' difficulties respectively at the observation stage and at the teaching stage. Our findings also reveal that in-service elementary mathematics teachers tend to attribute students' learning difficulties to psychological causes more than the other groups do.

Further analysis of psychological causes in Table 5 shows that both pre-service (52.5%) and in-service teachers (40%) referred to 'prior knowledge or its deficiency' the most to explain the sources of students' mathematical difficulties. It is interesting to note that elementary mathematics teachers (53%) referred to 'prior knowledge or its deficiency' more than elementary teachers (27%) did. The role of prior knowledge and its deficiency in the learning is a well-known and well-articulated issue in education (Ausubel, 1968; Resnick, 1983; Shulman, 1986). In this connection, Resnick (1983), for instance, notes that students do not come to classrooms as "blank slates" and that they come to learning environment with already well-established ideas, conceptions and theories. These conceptions, ideas and theories or their deficiencies can sometimes hinder students' learning and even be the causes of students' learning difficulties. It is perhaps this reality that makes Ausubel (1968, p.68) claims that "the most important single factor influencing learning is what the learner already knows." The participant teachers of this study also appear to be well aware of the influence of students' prior knowledge and/or deficiency on their learning.

Among the psychological causes, 'negative attitudes (prejudice)' was the second most cited reason by pre-service teachers while 'lack of motivation and interest' was the second most cited reason by in-service teachers. The reason that in-service teachers referred to lack of motivation and interest more than pre-service teachers was perhaps because they had

material experience of teaching to the students, but pre-service teachers had not. Further to that, elementary mathematics teachers (33%) attributed students' difficulties to 'lack of understanding of concepts' more than elementary teachers (13%) and in fact none of pre-service teachers referred to this category. 'Lack of ability', 'lack of self-confidence' and 'dislike of mathematics' were also cited by both groups of pre-service and in-service teachers.

Similar to Penso's (2002) results, our findings also show that pedagogical causes were the second largest category cited by teachers to explain sources of students' difficulties. Further analysis on pedagogical causes presented in Table 6 also show that in-service teachers referred to 'not teaching in a comprehensible manner' category most (17%) to explain the reasons behind the students' difficulties. This category was particularly cited by the elementary teachers with 27%. Moreover, it was pre-service teachers, but not in-service teachers, who referred to 'the teacher's lack of knowledge or competency' to explain sources of students' difficulties. This suggests that it was possible for pre-service teachers to easily cite teachers' lack of knowledge or competency as they themselves were still students but this was not the case for in-service teachers at all. On the other hand, it was in-service teachers but not pre-service teachers who referred to teachers' 'not holding students' attention', 'not doing consolidation', and 'not determining students' preparedness' to give an account of students' mathematical difficulties. These categories suggest that in-service teachers referred to teachers' classroom practices to make sense of students' difficulties as they were practicing teachers but that was not the case for pre-service teachers who had not yet been in the real business of teaching.

With regard to epistemological causes, the findings reveal that this category was the least cited amongst the three categories. A close examination of these findings also suggests that pre-service and in-service elementary mathematics teachers referred to epistemological causes but elementary teachers did not. We do not have concrete evidence to explain this difference, but we assume that pre-service and in-service mathematics teachers have more material experience with teaching mathematics, which might have prompted them to attribute students' difficulties to epistemological causes more than the elementary teachers did.

On the other hand, with regard to other causes, the findings show that it is actually in-service elementary mathematics teacher group (20%) that attributed student difficulties to such other causes as family and socio-economic situations the most in explaining the sources of the students' difficulties (see Table 7). The mathematics group was followed by pre-service teachers and elementary teachers respectively. Common to all these participants' views was that the lack of family interest in and contribution to a student's education was considered as a reason for learning difficulties with mathematics.

All these findings presented and discussed so far show that both pre-service and in-service teachers tend to attribute learning difficulties primarily to the students and student-related causes. This is, to a degree, understandable and justifiable as teachers continuously see students failing mathematics and experiencing serious difficulties learning it. Some student difficulties, in fact, can be due to their negative attitudes, inability, and lack of prior knowledge, of motivation, of skills and of efforts. Here, the students and their failures are at the center and it is perhaps that makes teachers to mainly 'blame' students for the lack of success and learning difficulties.

Some students' learning difficulties, nevertheless, can very well be due to the teachers and the nature of teaching as stated by Cornu (1991). Studies on both pre-service and in-service teachers, in fact, have shown serious weaknesses in their subject matter and pedagogical content knowledge. In this connection, Goulding, Rowland, and Barber (2002), for instance, reported serious weaknesses in pre-service teachers' understanding of mathematical concepts. Linchevsky and Vinner (1988) reported elementary teachers' difficulties with the mathematical concepts of sets. Ozmantar and Bingolbali (2009) showed that 22% of 216 primary classroom teachers found the computation

$$\frac{7+5}{14+20} = \frac{7}{14} + \frac{5}{20} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

as correct and again 22% of them found the following as incorrect.

$$\frac{7+5}{14+20} = \frac{7}{14+20} + \frac{5}{14+20} = \frac{7}{34} + \frac{5}{34} = \frac{6}{17}$$

In another study, Bingolbali, Ozmantar, and Akkoç (2008) showed that 44% of a group of primary teachers allocated a full grade to a wrong answer on a problem related to the area of a rectangle.

These studies provide evidence that teachers' lack of subject matter and pedagogical content knowledge as well as their beliefs do matter (Ball et al., 2008; Verschaffel et al., 2006; Askew et al., 1997; Lamb & Booker, 2004) as far as students' learning difficulties are concerned. Although the participant pre-service and in-service teachers in our study tend to mainly attribute students' learning difficulties to student-related causes, as the abovementioned studies clearly suggest, teachers' own difficulties, beliefs, instructional approaches and teaching materials can all contribute to the emergence of students' mathematical difficulties. Our study makes it clear that, at least in the case of our sample both in-service and pre-service teachers tend to hold students responsible for the mathematical difficulties but do not pay much attention to the pedagogical causes in accounting for such difficulties. Although both pre-service and in-service teachers do not pay much attention to epistemological causes, which may sometimes hinder students' learning, these causes alone or sometimes together with pedagogical and psychological ones can be the sources for the occurrence of students' learning difficulties. However, our participants seem to ignore (or even perhaps unaware of) this fact in accounting for the reasons of student mathematical difficulties.

As a conclusion, the issue of sources of students' learning difficulties is a complex and multidimensional one. Students' learning difficulties can be due to many causes and attribution of students' difficulties to only one source is deficient. Teachers' awareness in this respect is, therefore, crucial and needs close and further attention.

Educational Implications, Further Research and Conclusions

The findings show that both pre-service and in-service teachers attribute students' difficulties to mainly student-related causes. We below interpret these findings and

theoretical framework used to make sense of the findings as raising two important but closely related issues and some further research areas.

The first issue is related to sources of students' learning difficulties and their causes. The findings of this study clearly show that our participant pre-service and in-service teachers tend to focus mostly on psychological causes in explaining students' mathematical difficulties. Findings also suggest that our participants seem to be largely unaware of or at least to ignore pedagogical and epistemological causes in giving rise to student difficulties. On the basis of these findings, we suggest that pre-service preparation programs and professional development programs designed for in-service teachers should equally emphasize the epistemological and pedagogical causes and potential sources of student mathematical difficulties. Considering that teachers' "understanding of what makes the learning of specific topics easy or difficult" (Shulman, 1986, p.9) is interpreted as constituting a component of their PCK, discussion with teachers in this respect can contribute to the development of their PCK as well. Focusing only on psychological causes, therefore, is insufficient and the attention should especially be drawn to epistemological obstacles that the concepts pose and pedagogical causes of students' difficulties as well.

The second issue is related to the use of the theoretical framework of the study. The notions of epistemological, pedagogical and psychological obstacles seem to have been used in a very narrow manner and actually mainly in terms of the role of epistemological obstacles in the learning of the students (Selden & Selden, 2001; Dorier & Sierpinska, 2001). We have used this framework in a different way in this study for teacher education. We have found this framework useful to examine the views of the teachers regarding sources of students' learning difficulties. This framework, as alluded to above, can be useful for both teachers and teacher educators to analytically examine sources of students' learning difficulties and also to enable teachers to have an awareness regarding why students have difficulties to learn mathematics.

Our findings point to the issues that warrant further considerations. Further research is clearly needed to find out, for instance, whether there is any difference between practices of those teachers who refer to psychological causes more and those who refer to epistemological and pedagogical causes more. Another further research area can be related to the use of our framework adopted in this paper to examine views of teachers of different subject areas regarding sources of students' learning difficulties. This strand of research will help us to understand the extent to which the epistemology of subject areas influences teachers' views regarding sources of students' learning difficulties.

On the other hand, although this study has shed some light on the views of teachers about sources of students' learning difficulties with mathematics, it has still left many questions unanswered. The results have shown that there were some differences between the attributions of pre-service and in-service teachers regarding sources of students' learning difficulties. For instance, elementary mathematics teachers referred to psychological causes more than elementary and pre-service teachers. Again elementary mathematics teachers referred to 'prior knowledge or its deficiency' more than elementary teachers. More of similar differences between the groups of the teacher can be found in the results sections. We do not have concrete data to explain these differences between these groups of teachers.

Further research is, therefore, clearly needed to examine why there was a difference between the views of these teachers as well.

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