Language: A cultural capital for conceptualizing mathematics knowledge

Nosisi Feza-Piyose
Human Sciences Research Council, South Africa

Mathematics education in South Africa is in crisis. Students continue to perform at a lower level compared to other nations including those with low GPD compared to them. Two factors have been highlighted in research that impedes mathematics learning: teacher content knowledge and irrelevant teaching strategies. This study contributes to this literature by investigating five African (from a former White school) fifth grade students' learning of length measurement with the aim of eliciting the students' thinking levels by using a length learning trajectory. Clinical interviews and teaching experiments were employed for a comprehensive description of these students' processes. The findings reveal that students' mother tongue is a psychological tool that enriches their mathematics learning process, learning trajectory assisted in analysing students developmental processes with language and poor number development impeded abstraction in learning of length measurement concepts.

Keywords: multilingualism, number development, internalization, learning trajectory

Introduction

Mathematics performance of African students in South Africa remains poor even after liberation from the apartheid era. One of the contributing factors to this performance is quality of instruction received by the majority of South African children. The majority of students in the country lives below poverty level and school in poor resourced schools. Mji and Makgato (2006), Adler (2001), and Howie (2003) investigated factors that contributed to poor mathematics performance in these schools and discovered that teaching strategies used do not cater for students' needs. The second factor that affects the quality of mathematics education received by these students is poor mathematics content knowledge demonstrated by teachers (Mji & Makgatho, 2006; Van der Sandt & Niewoudt, 2003; Wessels, 2008). Addressing these challenges, black parents of South Africa send their children to previously white schools to seek quality education. However, little is known about how these students perform mathematically, and if the multilingual research by Setati and Adler (2001) and Setati (1998, 2005) applies in this new context.

This study aims to contribute to the already growing literature on multilingualism and also to test the relevancy of the learning trajectories in the South African context. Therefore, this study describes how five fifth grade Black students internalize length measurement concepts, and what mathematical experiences have they accumulated. In achieving these objectives the study employed clinical interviews in eliciting mathematical experiences about length and individual teaching experiments in describing internalization of length measurement concepts. Clements and Sarama's (2009) learning trajectories was employed to analyze the clinical interviews. The learning trajectories are used in this study because they are informed by theory and research. The theories that influence these learning trajectories are cognitive theories that have evolved over time to the current research.

Length measurement is employed in this study for its strength to bring together a variety of mathematical concepts. Also, measurement creates opportunities for in-depth understanding of space, number and algebra. Measuring involves spatial understanding, unit iteration that involves continuous counting, conversion of units that uses multiplication and decimal fractions. These concepts develop into pattern recognition that develops algebraic thinking.

Theoretical Framework

> If we wish to provide learning opportunities for students, we must first reflect on what it means for the student to "learn mathematics" and how the student goes about that task meaningfully (Reynolds and Wheatley, 1996, p.564).

Reynolds and Wheatley explicitly described the importance of giving students' opportunities to show and tell us how they learn. In learning how the students internalize length measurement concepts, the literature on learning trajectories of length informs this study. Also, addressing the psychological tools (the cultural capital) these students bring into their learning, the literature on multilingualism in mathematics classroom is employed.

Learning Trajectories

Learning trajectories hypothesized by Sarama and Clements (2009) on learning of length measurement concepts were designed from research-based theories that have been developed from Piagetian and Vygotskian contributions of young children's learning (Piaget, 1960, 1967); Vygotsky's (1934, 1986) theory to the present research by (Sarama and Clements 2009, Clements, 2010, Clements et al., 2011). The timeline covered in this theory supports historical and continuous reforms in learning of measurement concepts by young children. This study employs this theory as an analysis tool for investigating these students' thinking processes. The three parts that compose this theory are: mathematical goal, developmental path in reaching the goal, and activities that are designed towards reaching the mathematical goal.

Mathematical goal. Mathematical goals are the big ideas of mathematics. Clements and Sarama (2009, 2004) describe the mathematical goals as grouped concepts and skills that are mathematically essential and logical, customary with children's thinking and generative of future learning. For example the length measurement is composed of these big ideas that include understanding the distance that involves space, then how to measure this distance such as deciding on the unit, aligning the measuring tool correctly to the object of measure, using repeated units to measure, being able to calculate overlaps or gaps using knowledge of units and likewise. All these concepts form the cluster of the big ideas towards understanding length measurement.

Developmental path. Developmental path is more about students' levels of thinking (Clements & Sarama, 2004). These levels are like a ladder as the next level become more complicated than the previous one. Knowing these developmental levels allow the educator to plan instruction tailored at the students' level (Vygotsky, 1978). For example, from knowing "big" a child begins to use gestures in explaining the meaning of big at a different context. If it means tall, the child will show it by spreading his/her harm high, whereas if it means wide two hands will be used parallel to each other. This indicates that the understanding of "big" is getting richer with more sophistication and requires more vocabulary to be expressed. Clements and Sarama (2009) suggest that interpreting this development should be at the child's point of view. Learning trajectories recognize those innate abilities and stretch them further through experiences to develop these measurement skills and concepts more. The length measurement concepts themselves require internalization as they are meaningless as skills. This theory recognizes the power of mediation in developing these concepts and therefore provides activities that enrich the learning experiences.

Instructional activities. Measuring distance demands reorganizing space that needs to be measured (Lehrer, 2003). Re-organization leads to the development of units that need to be used in representing the successive distance of multiple tiled units. These units need to be the same and iterated. Therefore, once the student selects the unit of measure, s/he has to put the unit end to end until s/he can cover the distance. Then the next step is to count the units. This process reflects conceptualization of iteration. Instructional activities should be directly focused at the students' level of thinking. Developmentally appropriate activities need to be designed to achieve this outcome. Lehrer et al. (1999), Clements (1999), and Barrett and Clements (2003) divulged that the ability to iterate the same units and to use the zero point correctly in measuring length typically develops with students' age. In addition, Hiebert (1986) argued that there is a big difference between conceptual knowledge and procedural knowledge. Conceptual knowledge refers to the thoughts and intuitions about how mathematics works. Procedural knowledge refers to skills, procedures, and formal symbols. The major struggle in learning measurement for students is to link procedures with understanding. For example, the results of studies conducted by Bragg and Outhred (2000a, 2001) indicate that most students could not demonstrate what a centimeter looks like in a ruler. These results confirm that being able to use a ruler does not mean students understand what they are measuring. Using a ruler to measure is a skill, and counting the lines or spaces on the ruler while measuring is a procedure. Conceptualization occurs internally when a mental ruler develops. This mental ruler can be used anytime anywhere as a conceptual object.

Clements (1999), Lakoff and Nunez (2000), Lehrer, Jaslow, and Curtis (2003), and Bragg and Outhred (2004) suggest that teachers need to mediate the meaning of numbers on a scale and length as movement on a scale from the point of origin, that is zero. Wilson and Rowland (1992) and Van de Walle and Thompson (1985) highlighted the importance of building conceptual understanding of zero as the point of origin and length as continuous not discrete quantity. Therefore, students need to make a distinction between counting discrete objects and counting continuous length. Also it is vital to provide instruction that will nurture students, enabling them to rename zero when a line is not aligned with zero or when using a broken ruler (Clements, 1999). Developing a strong conceptual understanding of length will create opportunities for developing area and volume concepts effectively. It will also attend to the misconceptions that are created when using inappropriate units.

Length Measurement Learning Trajectory

The theory suggests eight hierarchical developmental progression levels for the length measurement trajectory.

Pre- length quantity recognizer (PLQR). In pre-length quantity recognizer a student cannot identify length as an attribute. In this level students see everything as long regardless of the attribute.

Length quantity recognizer (LQR). In this level the student identifies length as an attribute but cannot compare. This is the level when tall becomes a big thing. For example, "my mom is tall, my daddy is tall, and my brother is tall". No comparisons at this level.

Length direct comparer (LDC). In this level the student aligns objects physically to compare their lengths.

Length indirect comparer (LIC). In this level the student uses a third unit to compare two objects' lengths.

Serial orderer to 6+ (SO+). Sarama and Clements (2009) suggest that this level develops parallel with the next level, End-to-end measurer. Students order lengths in ascending or descending order from 1 to 6.

End-to-end length measurer (EELM). Student lays units end-to-end when comparing or measuring length. In this level using the same unit repeatedly might not have developed yet.

Length unit relater and repeater (LURR). Uses similar units repeatedly to compare and measure length. However, sometimes might still use a different unit in cases of a distance left that is shorter than the repeated unit. Students might change the unit to fit the distance. In other cases a student might be able to break the unit mentally into a fraction and finish up iteration.

Length measurer (LM). The student understands the need for using the same unit for measuring length. He/she understands the relations between units. She recognizes and lays the object from zero when measuring.

Conceptual ruler measurer (CRM). This is a level of abstraction. Units become mental units that a student can use anywhere anytime. Estimation skill is attained in this level.

Multilingualism in Mathematics Classrooms

African students go through different processes in conceptualizing mathematical concepts compared to their peers whose mother tongue is English. It becomes a marginalizing treatment if the educators treat them similarly (Khisty & Morales, 2004). Students from diverse cultures bring more different backgrounds and experiences to the classroom than their peers. They bring their initial cultures and languages that influence their frame of reasoning (Bishop, 1985). Educators need to tap on this richness to create opportunities for learning (Raborn, 1995). Recognizing individual learning differences, educators can shape curricula and instruction for all learners and give them opportunities to reach higher levels of mathematics. Setati and Adler (2001) in their results of studying language practices in multilingual mathematics classrooms of South Africa indicated that code switching between English and the native language of a student enriched mathematical discussions. This mathematics discussion becomes more conceptual and results in personalization of meaning (Vygotsky, 1978). Carignan et al. (2005) revealed that this code-switching was not allowed in an ex-Model C school they studied. They observed that in this urban school parents were required to speak English at home with their children instead of their mother tongue to be enrolled in this school.

Research has proven that use of native language in mathematics classroom enriches students' understanding of mathematical concepts (Nicol, 2005; Matang, 2006; Adler, 1998; Setati, 1998; 2002; 2005; Setati and Adler, 2001; Setati and Barwell, 2006). Therefore, instead of native languages becoming stigmatized languages in the classroom they should be treated as cultural capital students' bring and be acknowledged as their point of reference. Literature on multilingualism reveals that mother tongue is used for sense making, understanding of new ideas and conceptual discourses (Setati, 1998, 2006; Setati and Adler, 2001). The three uses of native language indicated in the studies are fundamental for learning of mathematics. Sense making integrates known to the unknown that lead to internalization of ideas. When ideas are internalized they become personalized and applied in different contexts (Vygotsky, 1978). Understanding new ideas is a process that requires cultural tools. The cultural tool in this case is language that students attained socially and use in learning new ideas. Denying them this opportunity of using their cultural tools then denies them access in learning of new ideas. These ideas need to be internalized and become mental structures that can be used in solving problems. Justifications, explanations and argumentations are elements of conceptual discourse. Language is a tool for conceptual discourse, students need language they are confident to justify, explain and argue for their reasoning.

Methodology

Research Design

This study reports data from my dissertation supervised by Prof Julie Sarama. A case study design was employed to focus on in-depth understanding of how black fifth grade students learn length measurement. The case study focused on having an in-depth understanding of students' conceptualizing processes, learning tools they use in learning measurement. Classroom observations were used to understand the kind of instruction received by these students in the former White school, followed by clinical interviews of five selected students to determine their thinking levels on measurement concepts (length, perimeter, area and volume). The analysis of the clinical interviews was used to inform the creation of teaching experiments. There were four main teaching experiments for each measurement concept with episodes for each teaching experiment. The number of episodes was determined by the needs of each student for each teaching experiment. This paper reports only the pre and post clinical interviews and teaching experiments of the length concept. The total number of length episodes were 8, however not all five learners needed all of the eight episodes depending on their progression the episodes were more for those who were taking longer route to understand the concept. The instructional episodes for the length teaching experiment included, practical activities measuring lengths using different measuring instruments, using broken rulers measuring lines, reading of calibrated measuring instruments and meaning of numbers in a ruler, activities involving conversion of units of length, conversion activities, number development, basic operations, and fraction activities. Each length episode session lasted 20 minutes totalling to 160 minutes per students. As mentioned previously this 160 minutes was used differently for different students. These individual length teaching episodes were conducted on daily basis over 8 school days. After the 8 days of teaching post-clinical interviews were conducted to support the claims that emerged from the analysis of the teaching experiment weather students' conceptual understanding of length progressed, regressed or remained the same.

Participants

This study collected data from 5 black fifth grade students. The selection of participants focused on different levels of performance with relevant gender representation. Out of two fifth grade classes, 16 students represented gender and mathematical performance based on students' grades among all fifth graders in the school. From the 16 students they were grouped into three performance levels, high, average and low. The students' grades ranged from 74-84% were 7 girls only, 52-58 % was 2 boys and 2 girls, and 37-48 % was 2 girls and 3 boys. The 5 participants selected were selected from the 16 students using the performance and gender representation. The representation was 2 girls from the high performance, 1 boy and 1girl from the average group and 1 boy from the low performing group.

Research Site

The site used to collect this study's data was a former White school in the Eastern Cape Province, South Africa. The majority of students in this school are African with 3% White students. The teacher - student ratio in this school is 1:55. The majority of teachers are White Afrikaans speaking with 3 African teachers. The medium of instruction is English.

Data Collection

Clinical interviews. Structured clinical interviews were administered to elicit the levels of thinking attained by the students in their prior learning experiences. These structured interviews were administered before teaching experiments were conducted to inform teaching experiments. At the end of the teaching experiments the same structured interviews were administered as an assessment tool to support the claims that come out of the teaching experiments and to measure students' developmental progressions. Measurement studies that are influenced by Piaget and Inhelder's cognitive development theory fall short in informing the literature about the mediation needed to develop understanding (Lehrer, 2003), because their focus has been on using clinical interviews over a period of time to describe developmental epistemology. Therefore, conducting clinical interviews solely denies researchers' access to pedagogical needs of students. Hence, this study employs clinical interviews in combination with teaching experiments.

Teaching experiments. With the results from pre-structured clinical interviews, teaching experiments were developed to instruct at the students' actual level and stretching them to the next level. Teaching experiments allow access to prior conceptions and how they are used to make meaning of new ideas (Thompson, 1979). Teaching experiments give access to both students' mathematical reasoning and mathematical learning (Steffe & Thompson, 2000).

Data coding. Data analysis was inseparable with data collection in this study to inform continuous data collection and to engage with data. Analytical memos assisted the researcher in making dialogue and making sense of the data (Ely et al., 1991). After conducting the pre-structured clinical interviews, analysis assisted in determining the actual levels of thinking the students demonstrated. Each student's transcripts from the video recorder with non verbal cues was typed and grouped according to the relevant developmental progression level of the length learning trajectory (Clements and Sarama, 2009), then the audio data from the tape recorder was also grouped using the developmental progression, and the researcher's notes were grouped too using the developmental progressions. A table with three columns of audio, video and field notes were created. Under each column developmental progressions were tabled and color-coded according to similarities and differences. The visual picture assisted the researcher in creating a common teaching experiment with different levels of instruction determined by individual student's performance on the interviews. This triangulation gave credibility and trustworthiness to the claims about students' thinking levels. The similar analysis for post-structured interviews occurred with a different focus. That analysis supported the teaching experiments and indicated the amount of growth students attained after the teaching experiments.

Teaching experiments were analyzed differently. After each episode of the teaching experiment, an analysis was conducted to inform the next episode. The video, audio and field notes were all typed and annotated. The annotations were tabled for each source under the source column. These annotations were color-coded and analytical memos were used to make sense of the visual display. The triangulation assisted in preparing for the next episode. When the tables did not give a clear picture, other visual displays were used to display relationships and differences. A total of 14 Descriptive codes emerged from the teaching experiments of all students. Four themes emerged from the 14 descriptive codes.

Findings

The findings of this study are reported in three parts: the pre-clinical interview results, teaching experiments and the post-clinical interviews. In pre-interviews the findings report the thinking levels of the five black fifth grade students before they were involved in teaching experiments. It is important to note that South African schools have quarterly terms and this data was collected during the last two terms of the year.

Pre-structured Interview Results

All five students demonstrated that measuring any length requires alignment of the measured object accurately from the Zero origin. That is, on all occasions all started from zero with accurate alignment of the ruler. However, four students when required to measure in millimetres measured accurately but erroneously in centimetres, with only one of the students measuring accurately in millimetres. Thus, all but one of the students measure with limited conceptual understanding of units. For example, one of the students was measuring a distance between the shelf and the beginning of the window in the library room the researcher was using. The following episode presents the dialogue:

R: Can you measure the distance from the shelf to the beginning of the window.
Lulama: (takes the tape-measure. Measure the wall from zero to the end of the distance). 138
R: 138 what?
Lulama: 138 millimeters, no kilometers.

According to the Learning Trajectories that are hypothesized by Sarama and Clements (2009) these students are not conceptual measurers. They do not have mental images of millimetres nor centimetres. Lulama here mentions "kilometres" that is longer than the length of a classroom. He does not show mental understanding of how long is a kilometre. He is reading numbers only in the measuring tool but cannot measure length. The learning trajectories place these students on End-to-end length measurer as they are able to align the measured object with the measuring instrument from the zero. These students are able to use the ruler when measuring but do not recognize units of measuring (Sarama and Clements 2009).

Teaching Experiment Themes

Curry et al (2006) suggest five principles that should be applied in measurement instruction. These were used to plan these teaching experiments. These principles include:

> "The need for repeated units, the appropriateness of a selected unit, the need for the same unit to be used to compare two or more objects, the relationship between the size of the unit and the number required to measure, and the structure of the repeated units" (p. 377).

Figure 1 indicates the developmental paths the five students took from end-to-end length measurer to the conceptual ruler. This figure presents the overall diagram of the five students that presents the two themes of number development and mother tongue use. However, individual diagrams for the five students will present individual paths taken by each of the five students. The path is similar for all five students but the end points are different and tools used to get to the next level are unique to each student's level of thinking about number and use of language.

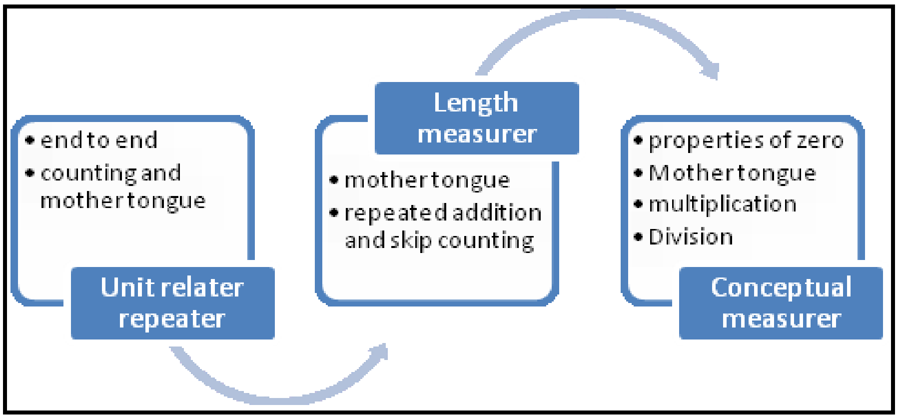

Figure 1. Summarized learning paths of the five students.

In teaching experiment one, episode 1 focused on the appropriate selection of a unit and the need for a repeated unit. However, each student had unique needs and therefore the teaching experiment was adapted to address such needs. Episode 2 of teaching experiment one focused on differentiating between millimeters and centimeters and creating an understanding of the relationships between the two units.

The following themes emerged from the teaching episodes: mother tongue use and number development. Episodes of each student will be presented under the two themes. Gloria's developmental path starts like all others at the end-to-end length measurer because she has no conceptual understanding of measuring units, could not measure with a tool that is not calibrated, however was able to use the skill of aligning the ruler correctly with measured object. Figure 2 presents Gloria's developmental path in conceptualizing length measurement.

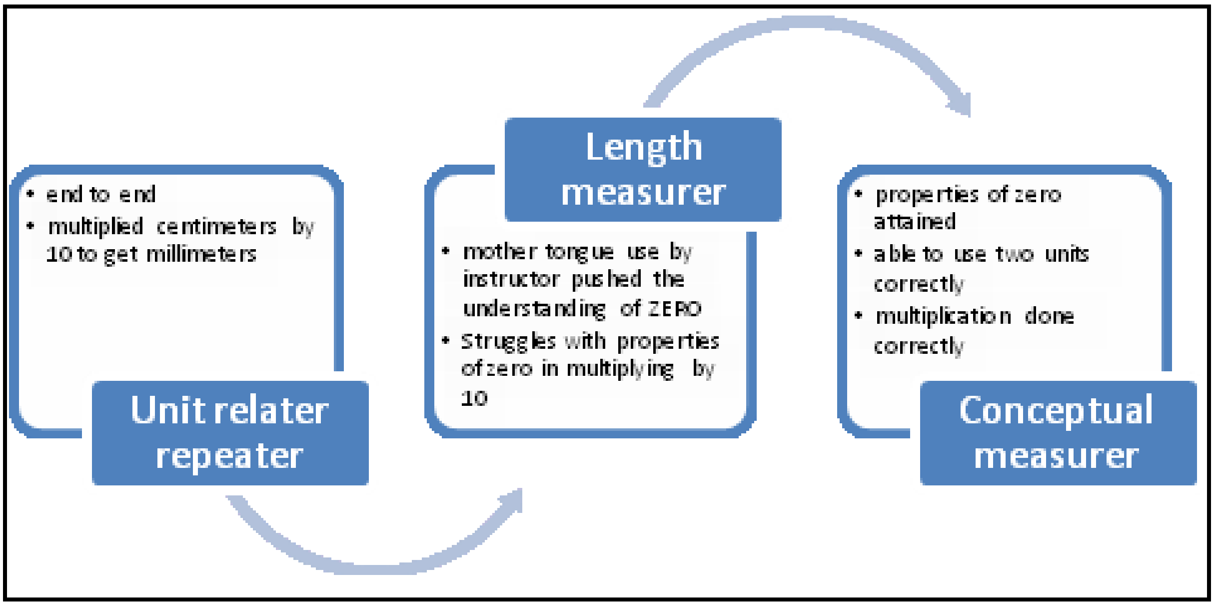

Figure 2. Gloria's developmental path.

The following episode present the process of development presented by the diagram when Gloria decided to convert centimeters into millimeters and multiplied 158 by 10 and got 1048

R: Does the 8 become just 8? What is 0×8
Gloria: 8
R: You are multiplying not adding. Do you understand what it means?
0×8 means 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 =
0 + 0 is
Gloria: 1
R: Ukuba ndingadibanisa into engekhoyo kwengekhoyo. (If you want to add nothing to nothing) What do you get? (Exact language used in the teaching experiment)
Gloria: Nothing.
R: 0×5
Gloria: 0
R: So when we multiply by 0 what do we get?
Gloria: 0
R: So to get millimeters what do we do?

Gloria: No juffrou we must multiply by 10. (Calculating the following)

158
×10
000
1580
1580 mm

In this selected episode Gloria was challenged by properties of zero in both addition and multiplication. Conceptual meaning of zero was not realized and therefore her mother tongue pushed her very fast to realizing zero. When she had to apply the new attained knowledge it became easy for her to convert "14cm 8mm" to "148 mm."

Figure 3 presents Simphiwe's developmental path indicating his unique needs during mediation. Simphiwe is one of the students who did not demand or require mother tongue instruction during teaching episodes and his path does not include mother tongue as presented in Figure 3. Siphiwe was able to convert using counting as the conversion tool even for fractions. Below is the episode that Simphiwe and researcher experienced when Simphiwe measured a 3½ cm strip with his ruler correctly.

R: Ok let's see, how many millimeters in the 3½ centimeters?
Simphiwe: (counts) 10, 20, 30, and 5 mm.
R: If we want to present 137 cm you measured in millimeters what should we do?
Simphiwe: 10, 20, 30, 40, 50, 60, 70, -------1370mm (counting in tens throughout)
R: How long is the length of this pencil?
Simphiwe: (measures the pencil accurately) 14½ cm.
R: What will it be in millimeters?
Simphiwe: 140 mm ½ cm

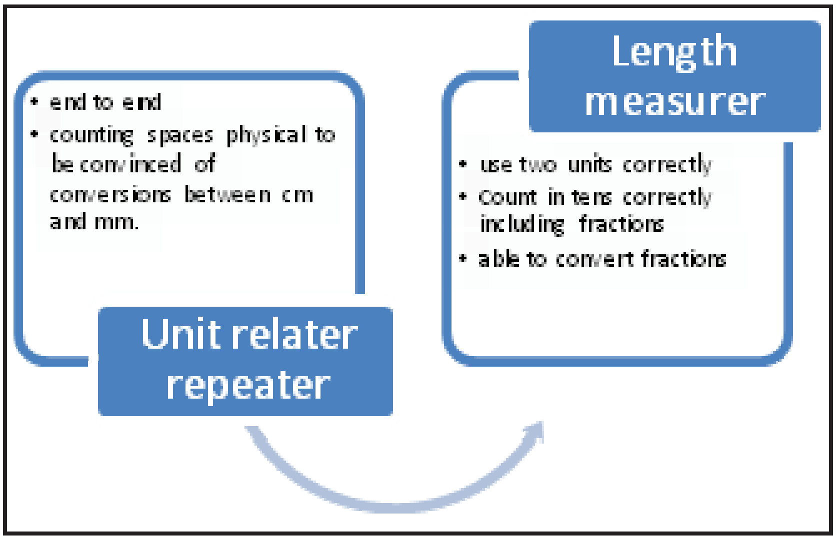

Figure 3. Simphiwe's developmental path.

Figure 4 presents Siziwe's developmental path that was not easily observable. In her processes Siziwe did not demonstrate any of the strategies she used in converting units.

Instead she converted units efficiently including fraction units. Siziwe used mother tongue for expressing her reasoning compared to her peers.

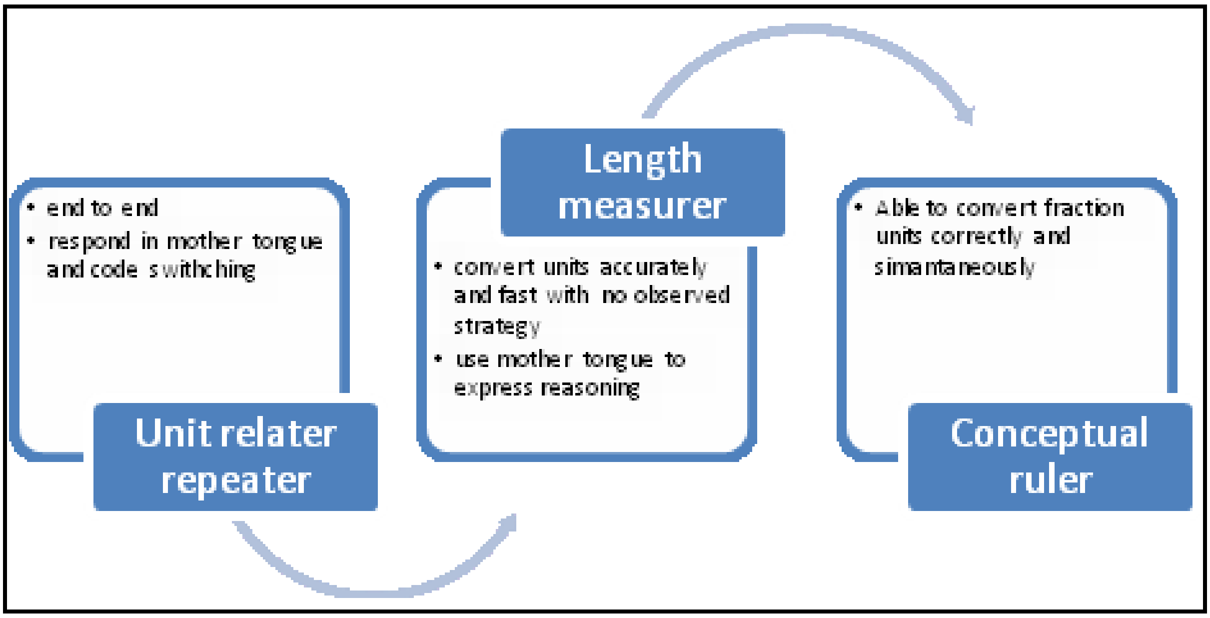

Figure 4. Siziwe' s developmental path.

The episode on Siziwe supports Figure 4 in drawing the picture of her path in attaining length measurement. When Siziwe was measuring the length of the classroom she explained her actions using mother tongue.

Siziwe: (picks the tape-measure)
R: Why are you taking that one?
Siziwe: Inde iklasi nayo inde (it is long and the classroom is also long)
R: Ok what about the strip?
Siziwe: You can use it for into ezincinci (you can use it for small things)
R: How long is this pencil?
Sinesipho: 14 ½ cm.
R: Can you tell me how long is it in mm?
Siziwe: yi 145 mm.

In her transition to English, she started by mixing both languages in a sentence and then continued with English only. Her mother tongue dialogue was about the length concepts like “ubude bale klasi” meaning the length of the classroom, “inde iklasi nayo inde” meaning the meter stick is long as the classroom and “into ezincinci” meaning for small things. These “isiXhosa” phrases give clear meaning of her conversation conceptually. Then “into ezincinci” refer to something smaller than short to her. Meaning she wanted to say ‘tiny” in English. Thus, Siziwe’s use of mother tongue assisted her in clearly expressing her thinking. Figure 5 presents Lulama’s path that is not progressive like his peers. He could not move from his level to the next level.

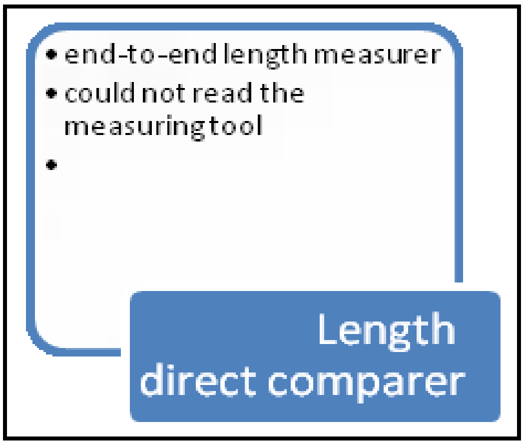

Figure 5. Lulama's developmental path.

R: From here to here how many 10's (0-----2cm).
Lulama: 20.
R: From here to here how many 10's (0------3cm).
Lulama: 40.
R: Can you count in tens for me?
Lulama: 10, 20, 40.
R: If you change 36 cm to mm what will that be?
Lulama: 1, 2, 3, 4.
R: Does it really get to 4. How many mm in 1 cm?
Lulama: 10.
R: If there are 2cm how many will be those in millimeters?
Lulama: 19.

Lulama demonstrated understanding that 1-centimeter is equal to 10 millimeters. However, his counting skills were becoming a barrier for progress. He struggled to count in tens beyond 20. He also made an obvious mistake like adding 10 and 10 and getting 19. In addition Lulama showed that he skip counts when counting in tens as the researcher engaged with him. He responded:

R: If they are 3cm. How many mm?
Lulama: 40.
R: Can you count in tens for me?
Lindisipho: 10, 20, 40.

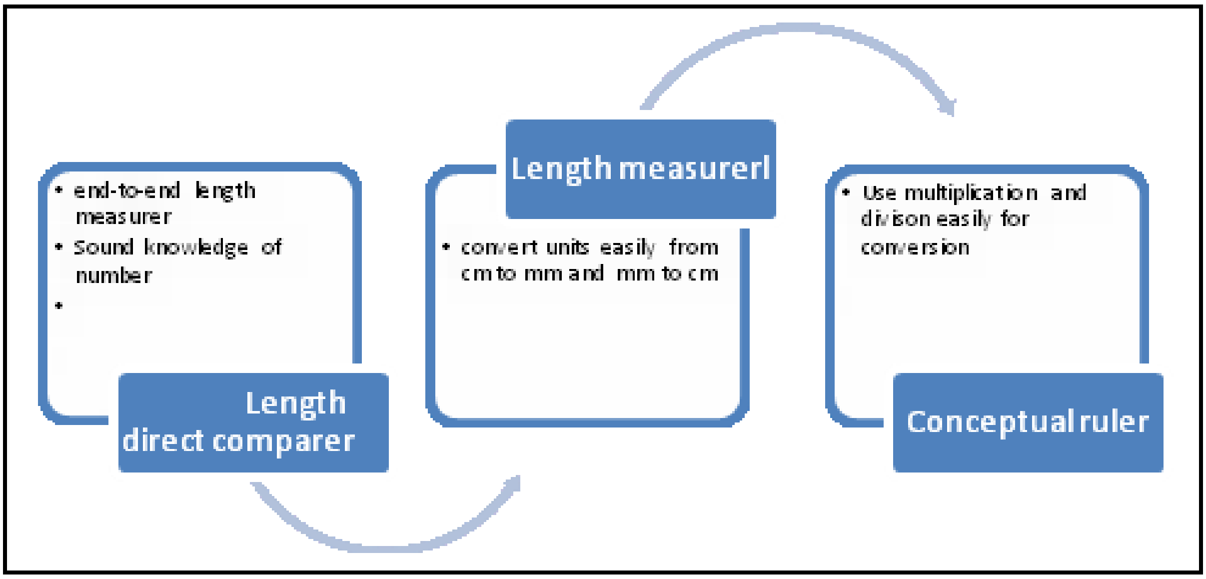

Figure 6. Mimi's developmental path.

Mimi's number sense nurtures her length concepts. For example her knowledge of fractions, and multiplication accelerated her understanding of length measuring units. The following present Mimi's learning process:

R: It's 140 what?
Mimi: 140 + 5 mm=145 mm.
R: Can you change it into cm?
Mimi: 14cm and 5mm.
R: That 5mm is what to a cm?
Mimi: 1/2 cm.
R: Can I call this 14½ cm?
Mimi: Yes, it is 14½ cm.

Mimi easily converted 140 mm to 14 cm without a struggle because of her division and multiplication knowledge. She went further to covert numbers to fractions successfully and vice-versa.

Post Clinical Interviews

All five students were able to accurately draw 10 centimetres, 12 millimetres, and a 29 centimetres line segment. When they also were required to draw a straight path of 29 centimetres with two turns; all five students drew an accurate path of 29 centimetres but without two turns as instructed. Teaching experiments were able to mediate conceptual understanding of units of measuring length and were able to mediate the relationships between these units. According to the Revised National Curriculum Statement (2002) fifth grade students are expected to be able to use appropriate measuring units, instruments and formulae in a variety of context. Figure 6 presents the development of each learner at different times of the learning process.

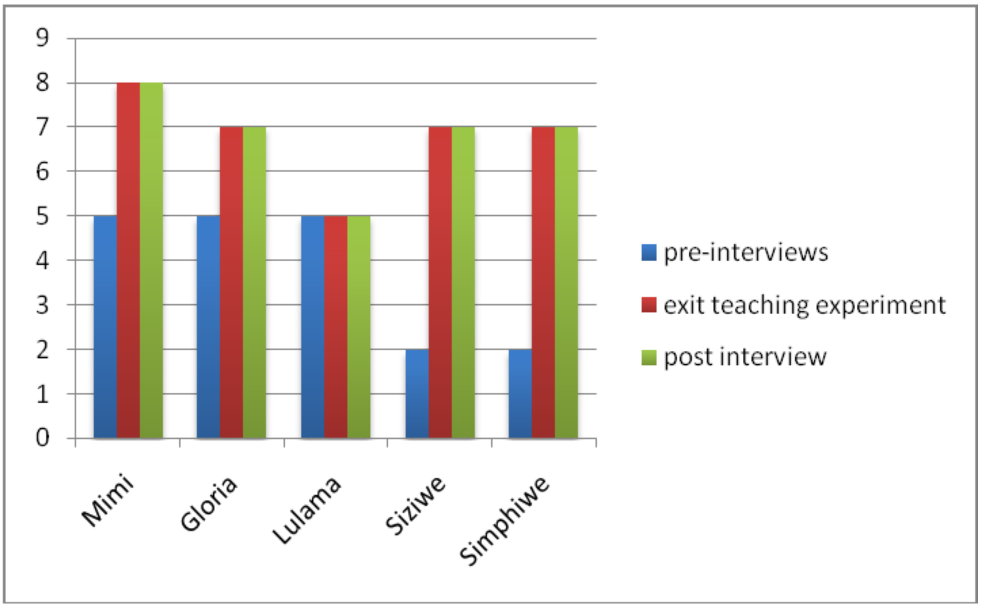

Figure 7. Eight length development progressions.

Discussion

The findings of this study reveal that (1) number development of these students lacked foundation and created a barrier for abstraction. Gloria was challenged by the properties of zero and just needed conceptual meaning of zero as a number and once she got that through mother tongue explanation she progressed very fast. Only one student was not challenged by number concepts and that allowed her to reach the abstract developmental progression of length measurement; (2) mother tongue of these students play a significant role to some learners in conceptualising number and length concept. While one student used mother tongue to express her conceptual thinking. (3) Mediation that was on the students actual thinking levels nurtured students' levels of thinking about length (Setati and Adler, 2001; Vygotsky, 1978); (4) both procedures and understanding are needed during mediation to develop higher order thinking levels, length learning trajectory guided understanding of conceptual development of length. All four students lacked understanding of skip counting, reading numbers, and properties of 0. This reflects to their foundation phase experiences on number that did not develop conceptually. One of the students demonstrated that he has not attained cardinality after counting. He could not determine how many centimetres the length of the pencil was. Lulama did not demonstrate development or regression. His teaching episodes demonstrated his lack of number sense. He could count in tens further than 20, could not add 10+10. The teaching experiments were at a higher level for him and measurement concept was far ahead of him. All he could do during the study was to align the ruler from 0 to the end with no understanding. In fact he began as a direct comparer and ended as a direct comparer. His number development closed the possibilities for observable growth.

South African studies on mathematical performance of students report performance at lower levels than expected (Mji & Makgatho, 2006; Van der Sandt & Niewoudt, 2003 and Wessels, 2008). This study revealed an additional component to teacher lack of content knowledge, and poor teaching strategies. This component is the lack of strong mathematical foundation of the students in the beginning years of schooling before Grade 5. South African early childhood in mathematics begins in grade R leaving behind the developmental stages of children when they are young, hence one of them still struggles with cardinality that they should start developing before Grade R. Research has proven that exposing young children as early as 3 years to quality mathematical experiences predict success in literacy and mathematics learning in their elementary years (Barnett, 1995; Cross et.al., 2009; Sarama and Clements, 2004). These results confirm the call for formalisation of early childhood learning to account for gaps of knowledge children enter schools with. South African research needs to inform policy on quality early childhood mathematics curriculum and practice. This study proved that Setati's research on multilingualism has implications for all multilingual students regardless of their learning environment.

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Authors

Nosisi Feza-Piyose, Postdoctoral fellow, Education and Skills Development Research Program, Human Sciences Research Council, South Africa; nosisi.piyose@gmail.com.