• Identify and describe spatial relationships using geometrical coordinates and other representational systems;

• Apply transformations and use symmetry to analyse mathematical situations;

• Use visualization, mathematical reasoning and geometric modelling to solve problems. (p. 42)

So, the relevance that orientation has with respect to the geometry block is clear. Furthermore, in the Pre K-2 period (from 4 to 7 years old), if we centre our interest on the ability "*Identify and describe spatial relationships using geometrical coordinates and other representational systems"*, this capability must enable child to:

• Describe, name and interpret relative positions in space and apply ideas about relative position;

• Describe, name and interpret the direction and distance when traveling in space and apply these notions;

• Find and name "places" with simple relations as "near" and coordinate systems such as maps. (p. 100)

In the Spanish case, the official curriculum for Childhood Education (form 0 to 6 years old), in the second cycle (from 3 to 6 years), makes explicit mention of orientation.

Some of the aspects that students should learn are "location of themselves and of the objects in space; relative positions, basic topological notions (open, closed, inside, outside, near, far...)" and "performing oriented displacements". The criterion to evaluate their development is: "The knowledge that children exhibit about spatial concepts (up, down, inside, outside, near, far...) will be considered " (Ministerio de Educación y Ciencia, 2008).

About research on Orientation in early years, Clements (1998) claims that

… it is unclear what kind of "mental maps" young children possess. Some researchers believe that people first learn to navigate only by noticing landmarks, then by routes, or connected series of landmarks, then by scaled routes, and finally by putting many mutes and locations into a kind of "mental map". (p.13)

…children must learn to deal with mapping processes of abstraction, generalization, and symbolization. Some map symbols are icons, such as an airplane for an airport, but others are more abstract, such as circles for cities. Children might first build with objects such as model buildings, then draw pictures of the objects' arrangements, then use maps that are "miniaturizations" and those that use abstract symbols. Some symbols may be beneficial even to young children. (p. 16)

To better understand how the previous guideline can be carried over to the classroom, Clements and Sarama (2009, p. 118) describe the learning trajectories for spatial thinking and spatial orientation depending on the age of the children as (we will only describe the period 0-6 years, corresponding to Spanish Early Childhood Education period):

whole activity, concluding that gestures are essential in the learning process of early mathematics.

On the other hand, DeLoache (1991) believes that visual representations are important for mathematics. Also Carruthers and Worthington (2005) devote their efforts to analysing mathematical marks made by children in different teaching contexts. In our case, we want to see which types of visual representations are used by 5-year-old children (characterized by symbols) when trying to understand the 3-dimensionality property of space and translate it to a map; because, as far as the authors of this paper know, there is no previous research on what kind of comprehension Childhood Education children have of the 3-dimensionality of space.

Principles of the Realistic Mathematics Education (RME)

We remind the reader that, according to Freudenthal (1973), the RME has six principles:

• *Activity principle:* the students, instead of being receivers of ready-made mathematics, are treated as active participants in the educational process, in which they develop all sorts of mathematical tools and insights by themselves;

• *Reality principle:* the overall goal of mathematics education is to make the students capable of using their mathematical understanding and tools to solve problems. This implies that they must learn 'mathematics so as to be useful';

• *Level principle:* learning mathematics means that students pass through various levels of understanding: from the ability to invent informal context-related solutions, to the creation of various levels of shortcuts and schematizations, to the acquisition of an insight into the underlying principles and a discernment of even broader relationships;

• *Intertwinement principle:* it is also characteristic of RME that mathematics, as a school subject, is not split into distinctive learning strands. From a deeper mathematical perspective, the chapters within mathematics cannot be separated;

• *Interaction principle*: Education should offer students opportunities to share their strategies and inventions with each other. The students can get ideas for improving their strategies by listening to what others find out and by discussing these findings. Moreover, the interaction can evoke reflection, which enables the students to reach a higher level of understanding;

• *Guidance principle:* mathematics education should give the students a 'guided' opportunity to 're-invent' mathematics. [...]teachers have to provide their students with a learning environment in which the constructing process can emerge. (extracted from Van Den Heuvel-Panhuizen, 2000, p. 5-9)

Objectives of the Research

The three main objectives of this paper are:

(1) To analyse the ability of the five-year-olds to depict in a map a change of level produced in the space.

Table 1 Symbols and contextualization as landmarks

Symbols	Meaning	Children's actions
	Go up (the stairs) /go straight the corridor	
	Go into (the classroom)	
 	Look under the table	

Participants

The participants have been five-year-old children of a public Early Childhood Education school of Galicia (Spain). The school is in Sigüerio, a small rural village, near Santiago de Compostela. The group consisted of 20 students (11 girls and 9 boys), and none of them required any special adaptation of the educational program.

Instruments

As we have said before, we have used several tools to calculate the impact of the teaching experiment on our research:

1. To analyse the interpretation of the children about the itinerary and the location of the treasure, we have used the handmade productions (maps) and the video recording archives of their verbal explanations;

2. To measure the spatial abilities, we have use the test "Pruebas de Diagnóstico Preescolar" (De la Cruz, 1988). This test defines 4 different factors/variables to determine some of the children's capabilities: relative locations/positions, orientation, perception and visual coordination. This author uses the sum of the first three variables to speak about the global spatial skill.

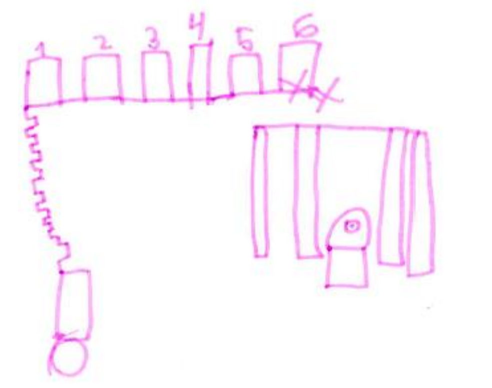
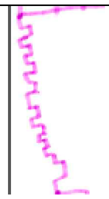
<table>
<tr><th>Produced map</th><th>Symbol used to depict the stairs</th></tr>
<tr><td>
</td><td>
The child describes the action of going up the stairs as a polygonal line, associating the climbing movement with the vertical direction of the paper</td></tr>
</table>

Figure 3 Vertical stairs map

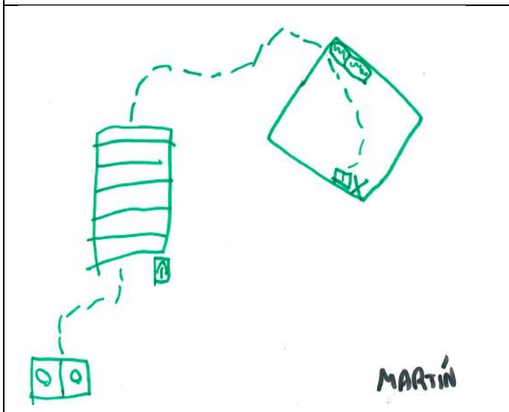
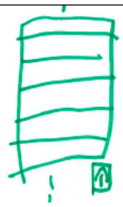
<table>
<tr><th>Produced map</th><th>Symbol used to depict the stairs</th></tr>
<tr><td>
</td><td>
The child describes the action of going up the stairs as a different symbol, where all the steps of the stairs are depicted in the map.</td></tr>
</table>

Figure 4 Plane projection map

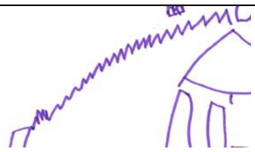
<table>
<tr><th>Produced map</th><th>Symbol used to depict the stairs</th></tr>
<tr><td></td><td>
The child describes the action of going up the stairs as a polygonal line, associating the climbing movement with a diagonal line across of the paper.</td></tr>
</table>

Figure 5 Diagonal stairs map

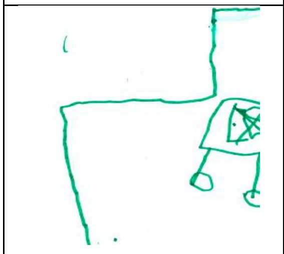
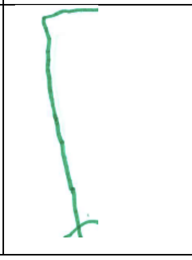
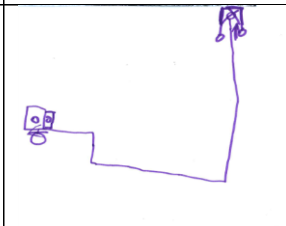
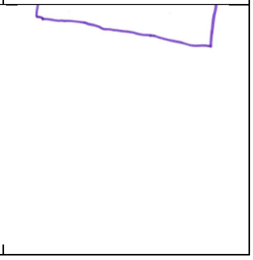
Produced map	Representation using vertical line	Produced map	Representation using horizontal line
			

Figure 9 Different representations of stairs using vertical/horizontal lines

Finally, we should point out that not all the children have used these previous representations; at Figure 10 some other interesting symbols used by kids are shown.

Figure 10 Other different symbols to represent the change of level

Some Parts of the Explanations of the Children about their Maps

In this section, we transcribe some parts of the conversation between the teacher and the children relative to the meaning of the map done, with the purpose of analysing the importance that children give to the change of vertical dimensionality through the representation used.

In short, we centre our interest on 4 cases to show in greater detail their interpretation (the names are fictitious), showing the transcription of the conversations extracted from the video record (translated from Galician into English), and after them, the conclusions that can be extracted:

First case: Ron.

-Teacher: Ron, describe me your map! (Ron ¡cóntame o teu plano!)

-Ron: This is the door of the classroom, (esta es la puerta del aula)

and we go up the stairs (y subimos las escaleras)

to the door, to the 6th door (hasta la puerta, hasta el aula 6)

and bend down, and we see the cards (y bajamos, miramos las cartas)

and after we find the treasure behind the table (y despues debajo de la mesa encontramos el tesoro.)

-Teacher: and What was? (¡ah! ¿e qué era?)

we go up (the stairs) subimos (las escaleras)

and we find the treasure (y buscamos el Tesoro)

and it is under the table (y (está) debajo de la mesa)

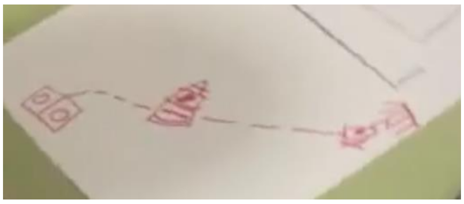


Figure 14 The map done by the child

In this case, it is clear that for the child the relevant parts of the itinerary are the main classroom, the stairs (painted with a different symbol, as a triangle) and the treasure. The rest of the parts of the itinerary are not relevant for him. This is a special case mentioned in Figure 10.

Forth case: Anne.

-Teacher: Anne, describe me your map!

Anne: you are here (aquí tás tí)

then go up the stairs (subes as escaleiras),

The door is the 6th (na porta seis)

Under the table there is the treasure (debaixo de unha mesa está o mapa)

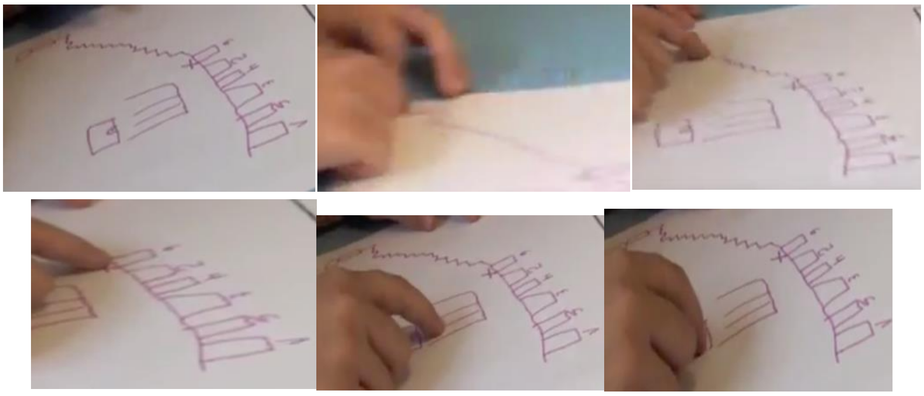


Figure 15 The map, where is the classroom, going up the stairs, the door they should find, the table inside the classroom and the position of the treasure (from left to right respectively)

In this case, the child is concerned about the way to describe the whole itinerary, the stairs, and the corridor where the classrooms of the first floor are located, and about determining the correct door that should be opened to find the treasure. In addition, she describes how the treasure is behind a table found at the 6th door. The representation of the stairs is mentioned in a previous section, in Figure 2.

Summarizing, as described before, in all the previous examples, it is obvious that the children need to describe the movement related to the fact that

2	14	10	7	6	30
3	12	11	9	7	30
12	12	13	10	6	31
11	8	15	8	9	32
18	14	16	8	9	39
10	13	14	7	6	33
15	13	16	8	6	35
16	14	15	8	8	37

Table 3 Statistics for the different factors of spatial skill

	Locations	Orientation	Visual coord	Perception	Total sum
Percentile 1	8,75	8,75	6,75	6	25
Median	11,5	10,5	7	6,5	28,5
Percentile 3	13	15	8	9	31,25
Minimum value	6	4	4	0	17
Maximum value	14	16	10	9	39
Average	10,7	11,2	7,3	6,75	28,65
Deviation	2,75489994	3,736026597	1,34164078	2,40339671	5,15317990

Taking this into account, we can obtain the total results of the class depending on each variable. This part gives us a perspective about the ability of each child with respect to the different factors. In this sense, we centre our analysis on the variables: perception, location, orientation and sum of them.

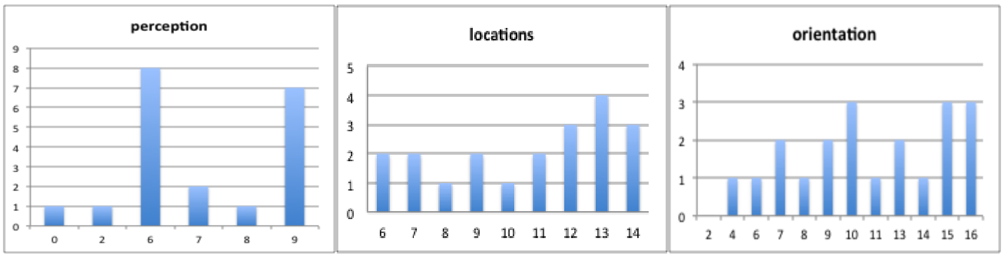


Figure 16 Number of children depending on correct answers on perception, location or orientation variables (left, middle and right respectively)

In order to better understand the aim of this test, we describe here what each variable measures, the maximum punctuation that can be obtained, and afterwards, we analyse the results yielded by our students.

In the case of perception variable, the maximum value that children can obtain is 9, and the minimum 0. The variable measures the ability of children to recognise specific objects from a picture. They should fix their vision on the picture and mark the objects asked by the teacher. As we can see above, a big percentage of the class (35%) has obtained the maximum score, that is, 7

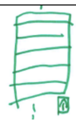
problems with respect to location or orientation or both. We will refer to this union of groups as the group with medium spatial skill.

Third, children of Group 4 can do all the exercises correctly, with high scores, and in general they have no problem with the perception variable, and some errors, if any, are related with either orientation or location variables. Children of this group will be regarded as children with high spatial skill.

Analysis of the Possible Relation between the Spatial Abilities and the Representation Used

According to the total results and the type of representation used by children, as an exploratory study, we wondered if there could be any relation between them. For this purpose, we classify the symbols used depending on the spatial ability skill of the children, which results in the following table (Table 4):

Table 4 Skill and representations

Group	Polygonal line	Vertical or horizontal line	Other	
			quantity	Symbol
Low skill	3	1	1	
Medium skill	5	4	1	
High skill	4	0	1	

Even though the present research is not based on a huge quantitative study, it seems that, in general, when children have a bigger spatial ability, they stop using a simple line to try to depict the change of level (the stairs) by a more sophisticated symbol (in this case, the polygonal line). This pushes us to think that there is a relation between the spatial ability and the way of visual representation.

Conclusions

As we have seen before, at the age of 5, we can reach the following conclusions:

First, respect to the analysis of the ability of the 5 year-old-children to depict in a map a change of level produced in the space, it is clear they can realize the need to highlight a 3-dimensional change in a 2 dimensional representation (according to the learning trajectory described by (Clements and Sarama, 2009), but, furthermore, this research allows us to conclude that children have implicitly learned the notion of the space as a 3-dimensional mathematical object and they feel the need to describe any change on the vertical axis in the 2-dimensional representation.

So, even though it is difficult to speak about semiotic representation in Early Childhood Education, it is clear that children use an image to represent this spatial change, so it can be considered as a representation type, according to Elia, Gagatsis, & Demetriou (2007).

Furthermore, in the case studies we have analysed, children have used the landmarks in macro-space as a reference system to describe perfectly to another person the itinerary they should follow to find the treasure (Clements, 1998). In these dialogues children have explained correctly the directions involved in this process and the relevance of the symbol used for the 3d-change.

Second, we have seen that the way to represent this dimensional change is not completely a standard one; and, in fact, this depends on the development of children's mobility and the way they understand the space around them. Anyway, we have classified the symbols used in the planar representations, determining that the most common representation is a polygonal line, but there are many other representations, as vertical/horizontal lines, or others.

Third, we have used the test of De la Cruz (1988) as a tool for the description of the main characteristics of our students on spatial skills and to try to determine a possible relation between this spatial ability and the sophistication of the symbol used to depict the 3-dimensional change. After the analysis of test results, we have been able to describe which types of errors are more common depending on this skill, where low ability involves a huge number of errors related to the three variables. Children thus show some difficulties in identifying objects, they have not correctly interiorized the notion of position, and they have problems to point to an object they have been asked about; children with medium ability do better in exercises related to perceptions, but, in general, they have problems with respect to location or orientation or both; and children with high ability have no problem with variable perception, and some errors, if any, are related to orientation or location variables.

On the possible relation between this spatial ability and the sophistication of the symbol used to depict the 3-dimensional change, we conclude that it seems that when the spatial ability is bigger, the symbols used are more complex, thus is, they stop using a simple line to try to depict the change of level (the stairs) by a more sophisticated symbol (in this case, the polygonal line), and the reverse. This aspect gives us an opportunity to continue in the future with this research line, where we hope to be able to determine clearly if there is more to this possible relation than found so far or if it is only a matter of mere chance.

Finally, this study has helped us to understand how children "see" the space they have around them and how it is possible to work this complex notion, 3- dimensionality, in their early years. Anyway, an interesting research field is open in front of us, where we must investigate how complex mathematical notions (their intuitive meaning) are understood by kindergarteners.

Disclosure statement

No potential conflict of interest was reported by the authors.

Notes on contributors

Ainhoa Berciano - Universidad del País Vasco/Euskal Herriko Unibertsitatea, Spain.

Clara Jiménez-Gestal - Universidad de La Rioja, Spain.

María Salgado - CEIP Sigüeiro & Universidade de Santiago de Compostela, Spain.

References

Alsina, A. (2012). Hacia un enfoque globalizado de la educación matemática en las primeras edades [Towards a holistic approach to mathematics education at the early ages]. *Números*, *80*, 7-24.

Carruthers, E. & Worthington, M. (2005). Making sense of mathematical graphics: the development of understanding abstract symbolism. *European Early Childhood Education Research Journal*, *13*(1), 57-79.

Clements, D. H. (1998). *Geometric and spatial thinking in young children*. Retrieved from ERIC database (ED436232).

Clements, D. H. (1999). Geometric and spatial thinking in young children. In J. V. Copley (Ed.), *Mathematics in the early years* (66-79). Reston, VA: National Council of Teachers of Mathematics.

Clements, D. H. & Sarama, J. (2009). *Learning and Teaching Early Math. The learning trajectories approach*. NY, New York: Routledge.

De la Cruz, M. V. (1988). *Pruebas de Diagnóstico preescolar*. Madrid: TEA Ediciones.

DeLoache, J. S. (1991). Symbolic functioning in very young children: understanding of picture and models. *Child Development*, *62*(4), 736-752.

Elia, I., Gagatsis, A. & Demetriou, A. (2007). The effects of different modes of representation on the solution of one-step additive problems. *Learning and Instruction*, *17*, 658-672.

Elia, I., Gagatsis, A., Michael, P., Georgiu, A. & Van den Heuvel-Panhuizen, M. (2011). Kindergartners' use of gestures in the generation and communication of spatial thinking. In M. Pytlak, T. Rowland, & E. Swoboda (Eds.), *Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education* (1842-1851). Rzeszów, Poland: University od Rzeszów.

Fernández Blanco, Mª. T. (2011). Una aproximación ontosemiótica a la Visualización y el Razonamiento Espacial [An ontosemiotic approach to visualization and spatial reasoning] (Doctoral dissertation). Retrieved from TESEO (947559).

Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht, The Netherlands: Reidel Publishing Company.

Hershkowitz, R., Parzysz, B. & Van Dormolen, J. (1996). Space and shape. In A. J. Bishop (Ed.), *International handbook of mathematics education 1* (161-204). Dordrecht, The Netherlands: Kluwer.

Kotsopoulos, D., Cordy, M. & Langemeyer, M. (2015). Children's understanding of large-scale mapping tasks: an analysis of talk, drawings, and gesture. *ZDM Mathematics Education*, *47*, 451-463.

Landau, B., Gleitman, H., & Spelke, E. (1981). Spatial knowledge and geometric representation in a child blind from birth. *Science*, 213, 1275-1277,

Ministerio de Educación y Ciencia. (2008). ORDEN ECI/3960/2007, *Currículo español de Educación Infantil* [Spanish Early Childhood Education Curriculum]. Retrieved from https://www.boe.es/boe/dias/2008/01/05/pdfs/A01016-01036.pdf

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Organisation for Economic Cooperation and Development. (2011). *Pisa in Focus 2011/1*. Retrieved from http://www.oecd.org/pisa/pisaproducts/pisainfocus/PiF1_esp_revised.pdf

Resnick, I., Verdine, B. N., Golinkoff, R. & Hirsch-Pasek, K. (2016). Geometric toys in the attic? A corpus analysis of early exposure to geometric shapes. *Early Childhood Research Quarterly 36*, 358-365.

Sarama, J. & Clements, D. H. (2009). *Early Childhood Mathematics Education Research. Learning Trajectories for Young Children*. New York, NY: Routledge.

Van den Heuvel-Panhuizen, M. (2000). *Mathematics education in the Netherlands: A guided tour*. Utrecht, The Netherlands: Utrecht University.

Yuzawa, M., Bart, W. M., Yuzawa, M., & Junko, I. (2005). Young children's knowledge and strategies for comparing sizes. *Early Childhood Research Quarterly 20*, 239-253.