

## Developing Students' Collaboration in a Mathematics Classroom through Dramatic Activities

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This paper presents the opportunities that students' participation in playing and acting in dramatic activities offered to them for reflection on their social interaction in mathematics. The research took place in a fifth grade of a typical public primary school. Three different case studies of pairs of students as they are engaged in dramatic activities during their metadiscursive reflection on their interaction in mathematics are presented. The drama seems to be a fruitful environment for the development of students' reflection on their collaboration in mathematics, as it allows them to reflect on issues concerning social and emotional aspects of their collaboration and to become aware of its evolution.

*Keywords:* collaborative learning; drama in education; metadiscursive reflection; primary education; school mathematics

Nowadays, it is widely accepted that learning of mathematics is a product of social activity (Cobb et al., 1997; Sfard, 2001). Moreover, the whole mathematics classroom environment constitutes a micro-community in which learning takes place (Voigt, 1995). The reduction of the student to a "cognitive subject" seems to be replaced by the acknowledgement of the student as a "social subject" that is influenced by his/her history and culture (Lerman, 1998; de Abreu, 2000; Valero, 2004). From a socio-political perspective, one basic research question is how mathematics education is "used by people in particular discourses and of the effects of those discourses on social practices and consequently on people's lives" (Valero, 2004, p.14-15).

In the case of school mathematics these considerations are subversive, because of the dominant belief that learning of mathematics is an individual process and that the social interaction does not play significant role in it (Chaviaris, 2006). In our opinion, this dominant belief creates relationships of power among the students during their interaction in the mathematics classroom. For example, the mathematical ideas of high-achievement students, during a mathematical discussion, seem to be more respectable than the proposals of low-achievement students (Kafoussi et al., 2010). Towards this direction, one question for mathematics educators is how students could change their participation in classroom mathematical practices into a more democratic context, based on the mutual respect of ideas of their classmates and the development of equivalent relationships.

The purpose of this paper is to present the opportunities that students' participation in playing and acting in drama offered to them for reflection on their social interaction in mathematics. This is part of a broader research program that focused on how students in pairs can actively get involved in reforming their behavior as they reflect on their interaction in order to solve mathematical problems (Chaviaris, 2006).

## Theoretical Background

In mathematics education, much research has focused on metacognition for the study of the way that students can become aware of their own thinking and actions during their mathematical activity in the classroom (Goos et al., 2002; Kramarski et al., 2002; Mevarech & Fridkin, 2006). However, as the mathematical activity is considered as a process that takes place in a macro- and a micro-community with concrete socio-cultural characteristics, students' awareness of their actions in mathematics has to be related not only with the reflection on cognitive aspects of their activity but with social and cultural aspects of it, too. Zimmerman (1995) has stressed that the interpretation of students' self-regulation has to be treated as a complex interactive process influenced by students' self beliefs-system. Sfard (2001) has mentioned that students' initiation to mathematical discourse depends on the "meta-discursive rules that regulate the communicative effort" (p.28). These rules are considered as the implicit regulators of interpersonal and intra-personal communication, as they determine the choices of the participants when they act and they embed their values and beliefs. She has emphasized the role of the interlocutors' intentions in a mathematical discussion using the term "meta-discursive intentions" in order to describe the "interlocutors' concerns about the way the interaction is being managed and the issues of the relationship between interlocutors" (p. 39). Therefore, the way that the members of the mathematics classroom develop rules that guide their social behavior by reflecting on their own and their interlocutors' beliefs and intentions about their interaction determine the evolution of their mathematical discourse.

Towards this effort, the study of the development of students' reflection on their socio-mathematical interaction, in order to improve their collaboration, gets into interest. This improvement has mainly been studied in relation to the role of mathematics teacher in the development of social and socio-mathematical norms (Yackel et al., 2000) or patterns of interaction (Voigt, 1995). However, recently the fact that the students have to reflect and regulate their own collaborative learning in mathematics has also received attention (Dekker et al., 2006). Moreover, it has been mentioned that if the students often follow an effective model for interaction, they attain more mathematical level raising (Dekker & Elshout-Mohr, 2004; Pijls et al., 2007). This kind of reflection could be named metadiscursive as it is related with the consciousness of relationships among cognitive, social and emotional components of mathematical discourse. The investigation of learning environments that allow students' self-regulation is an important issue for the improvement of mathematics learning.

These environments could be derived from the domain of arts, as many researchers have shown (Taylor, 1996). In our paper we will focus on drama in education. The drama has been used in education to serve multiple dimensions of it: the students' arts education, the students' social and emotional development, the support of the teaching of other cognitive subjects like language, history, mathematics, etc. (Fleming et al., 2004). Drama in education refers to the dramatic techniques that support and strengthen the learning in the classroom. Some terms like developmental drama, creative dramatics, educational drama, and mantle of the expert have been used to describe different approaches of the engagement of the drama in classroom teaching practices (Andersen, 2004). In this paper, we will use the term dramatic

activity as the students, who participated in this research program, represented their concrete mathematical experiences through a role-play using dramatic techniques.

Much research has been done about the offer of drama in different needs of learning process: optimization and representation of didactical situations (Duatepe, 2005), problem solving process (Bolton, 1985) or metacognitive activities (Johnson, 2002). Furthermore, playing and acting in a drama in education allows the individual and the collective reflection on an experience (Bolton, 1995). Andersen (2002) has stressed that:

learners in role benefit not only from engaging with realistic problems, but also by working and thinking within realistic roles. [...] Simultaneous to thinking as, the learner is thinking about the role, monitoring one's own thinking and action. This self-monitoring is a critical component of metacognition. (p.265)

In mathematics education, little research has been done about the role of dramatic activities for the learning and teaching of mathematics (Duatepe & Ubuz, 2002). The relevant researches have mainly focused on cognitive aspects of mathematical learning and they have indicated the positive influence of dramatic activities on students' mathematical competence. For example, Saab (1987) examined the effects of drama-based mathematics instruction on sixth graders compared to textbook mathematics instruction and the results showed a significant increase in mathematics achievement. Duatepe and Ubuz (2002) pointed out that drama-based mathematical instruction appears to have a significant effect on students' geometry achievement compared to the traditional teaching, promoting students' imagination by improvising a concept or an event, and helping them to experience all aspects of the concepts. Breen and Hannula's work (2003) used playing and acting in a drama as an environment for mathematics educators' reflection on different interpretations of mathematics classroom research findings.

However, as it is mentioned before, students who engage in mathematical activities have to be reflective practitioners on their own and their interlocutors' behavior during their mathematical discussions. The purpose of this study is to investigate how the students' metadiscursive reflection through their dramatic activities influences their self-regulation during their problem solving activity in a mathematics classroom. More specifically, the present study focuses on the following question: which opportunities can students' dramatic activities offer for their reflection on their social interaction in mathematics classrooms?

### **Method**

This research was done in a fifth grade of a typical public school of Athens in Greece, in 2003-2004 and it lasted six months. The participants were 18 students (9 boys and 9 girls). The teacher of the class voluntarily accepted the development of this research program in her classroom. The students worked in pairs four times per week during the mathematics lessons. The mathematical topic in which the students were engaged during the research program concerned the concept of fractions. The activities about fractions have been given by the researchers in order to be meaningful for the students according to the related literature (Kieren, 1992; Streefland, 1991). The designing and the realization of the research program

was based on patchwork case studies method (Jensen & Rodgers, 2001). According to this method a set of multiple cases of the same research entity (in our research we define as entity a pair of students) allows a deeper and more holistic view of the research subject.

Initially, we studied the students' profiles in order to organize them in pairs. The students were interviewed about their beliefs for their own role, others' role, the general nature and the goals of mathematical activity (e.g., When do you feel really pleased in mathematics? How do you feel when you make an error in mathematics? Do your classmates help you in mathematics?). Furthermore, the teacher of the class was asked to assess her students in mathematics based on her personal evaluation by using the criterion of the students' need for help in order to solve a mathematical problem. Finally, we recorded their parents' beliefs about the mathematical activity of their children in school as well as at home. We distributed a questionnaire to the students' parents in order to complete it at home (e.g., When do you feel pleased with your child's mathematical performance? What difficulties do you experience when you help your child in mathematics at home? What is, according to your opinion, the role of collaboration among students in a mathematics classroom?).

We formed nine pairs of students, based on the following criteria:

Table 1

*Criteria for pairs*

Criteria for the organization of pairs	Symbols
Negative beliefs about collaboration in mathematics	N
Positive beliefs about collaboration in mathematics	P
He/she managed in mathematics	1
He/she managed in mathematics but sometimes with help	2
He/she managed in mathematics only with help	3

In Table 1 the notions "negative beliefs" and "positive beliefs" were used to describe the students' responses like: "I would like to solve the problems in mathematics alone" (representing N), "I would like the teacher to help me in order to solve the problems" (representing N), "I would like to solve the problems with my classmates because in this way I can know if my opinion is right or wrong" (representing P). If, for example, a student had a negative belief about collaboration in mathematics and he/she was assessed by the teacher as someone that "he/she managed in mathematics but sometimes with help", he/she was coded as N2. The different cases of pairs formed are presented in Table 2.

According to our criteria for the organization of the pairs in this classroom it was not possible to cover all the different combinations (for example, the pair N2-P3 didn't exist). Moreover, we tried to organize the students in pairs in such a way that in each pair the partners held similar or different beliefs about collaboration, but they didn't have large discrepancies in their mathematical competence (except of one case, P1-N3). During their mathematical activity in the classroom, the collaboration of each pair of students was videotaped once a week. Totally, 54 videotaped sessions were gathered. The discourse analysis of the group's engagement in classroom mathematical activities was based on interactivity flowcharts that Sfard and Kieran (2001) have developed (see Appendix for

details). The mathematical discussion of the group was analyzed according to the way that the members negotiated their mathematical activity (who offered the solution, what kind of solution they offered, how they explained their thinking, how every member of the group was influenced by the other, etc., for more details of the above analysis see Chaviaris & Kafoussi, 2005 and Chaviaris et al., 2007).

Table 2

*Pairs' profiles*

Case	Pairs' profile	Number of pairs
1	N1 - P2	3
2	N1 - N2	1
3	N2 - N2	1
4	P1 - N3	1
5	P2 - P2	3

After a session of collaboration, the members of each group participated in an out of class meeting with the researcher (in the school library). During this meeting, the students watched their videotaped session together and discussed the issues concerning their experience. The discussions of these meetings were tape-recorded.

Each pair was asked to design, play and act in a drama in the classroom based on the experiences of their collaboration in mathematics. Especially, they were asked to use their own observations on their videotaped collaborations for the design of their dramatic activity in order to share them with their classmates. This activity was developed in three phases: a) designing their drama text and their role in a meeting outside of the classroom, b) presenting their drama to their classmates, and c) discussing their roles with their classmates. All the phases of this activity were videotaped.

In this study, we will present samples of students' activity as they designed and played their drama (metadiscursive level) and as they discussed with their classmates on their play (collective level). The following Table 3 presents the elements of the analysis of the students' dramatic activity:

Table 3

*Analysis of a dramatic activity in the mathematics classroom*

<i>Metadiscursive level</i>	- the roles that each student chose to play (e.g., a student chose to play one who didn't manage to solve the mathematical problem)
	- the relationship between the students' scenarios and their experiences from their collaboration in mathematics (e.g., the moments of their collaborations in mathematics that they decided to represent)
<i>Collective level</i>	- the student-actors' comments about their role playing experience as partners in a mathematical discussion
	- the other students' comments about the collaboration in mathematics motivating by the presented dramatic activities

## Results

We will present three different representative case studies that illustrate the opportunities that the students had to reflect on their social interaction in mathematics as they are engaged in dramatic activities. We should mention that at the beginning of the program all the students had participated in a traditional context of mathematics teaching and this fact provides a context about their beliefs on their collaboration in mathematics as well as their mathematical activity in general. All the names used below are pseudonyms.

### 1<sup>st</sup> Case: Stavroula and Alexia (N1-P2)

The first case concerns a pair of students (Stavroula and Alexia) that they had different beliefs about the role of collaboration in mathematics and different levels of achievement in this subject. The goals posed by both students for their mathematical activity concerned the result of their effort (right or wrong) and not the process. For example, during the initial interview Stavroula said, "I don't want the others to help me, because they will think that I'm not good in mathematics." Alexia said, "I feel nice, when I solve the problem right. If I do an error, I feel angry and I am depressed." Nevertheless, they had different conceptions about the role of collaboration in mathematics. Stavroula considered the collaboration to be an obstacle in the understanding of mathematics, because she believed that "if someone doesn't work on his own, he cannot understand mathematics." On the contrary, Alexia believed that collaboration could help her to check on her thoughts before she announced them in the classroom and so she could "avoid mistakes." Their parents attributed to the collaboration in school mathematics a social role and not a cognitive one, as they conceived the collaboration as a means for students' socialization. As for the two students' abilities in mathematics, Stavroula was a student that managed to find solutions on mathematical problems on her own (coded as N1) and Alexia was a student that, most of the times, need some help to complete a mathematical activity (coded as P2).

In their first dramatic activity, both students voluntarily chose to represent the opposite roles of these that they experienced during their collaboration in mathematics. Alexia played the student who managed to solve the problem alone (N1) and Stavroula chose to play the student who needed help (P2). Stavroula justified her choice as following: "I want to know how one feels when she asks for help in mathematics." Their drama text was the following:

- [1] *Alexia:* A fruit-bowl contained 21 apples. George ate  $\frac{2}{3}$  of the apples. How many apples were remained in the bowl?
- [2] *Stavroula:* Ah! It seems difficult!
- [3] *Alexia:* Think again about it. It is easy!
- [4] *Stavroula:* Help me a little.
- [5] *Alexia:* What do you mean "a little"?
- [6] *Stavroula:* Such a little! [She shows with her hands.]
- [7] *Alexia:* If I help you such a little, the problem will be solved by myself and not by yourself!
- [8] *Stavroula:* It doesn't matter at all!
- [9] *Alexia:* It doesn't matter at all? It matters a lot, because you will not learn it.

- [10] *Stavroula*: Oh! You talk like my mother! She told me the same things.
- [11] *Alexia*: She is right! You should solve it alone.
- [12] *Stavroula*: Come here now!
- [13] *Alexia*: What do you want?
- [14] *Stavroula*: Do you want to solve it together, to discuss about it?
- [15] *Alexia*: OK.
- [16] *Stavroula*: How did you solve it?
- [17] *Alexia*: Look here, we can divide the apples in three parts.
- [18] *Stavroula*: Ah! Three times seven ...21, every part has 7 apples.
- [19] *Alexia*: Yes, what about the  $\frac{2}{3}$ ?
- [20] *Stavroula*: 7 plus 7... Ah! It will be remained, 7.
- [21] *Alexia*: OK.

The above scenario was developed in two scenes: at the first scene (phrases 1-13) the students chose to represent difficult moments of their collaboration and at the second scene (phrases 14-21) they presented a productive collaboration. More specifically, Stavroula's belief that mathematical learning is only an individual process was mentioned by Alexia (phrases 7, 9, 11). The continual effort of Alexia to challenge Stavroula's participation during their real collaboration in mathematics was illustrated in Stavroula's dramatic activity (phrases 4, 8). The change of Alexia's behavior during the dramatic activity was related to the evolution of their collaboration in mathematics.

Moreover, an interesting moment in their dramatic activity was the comments made by the students for their parents' beliefs about their mathematical activity (phrases 10, 11). As they tried to connect the way of their interaction in mathematics with their parents' behavior, they revealed their experiences in their broader cultural context in relation to this issue.

The following discussion took place in the classroom after the above dramatic activity:

- [1] *Researcher*: Do you want to talk about the roles? How do you feel about your role?
- [2] *Alexia*: I don't think that I am egoist, because I finally helped Stavroula. I felt nice, because I helped her, I did not solve the problem for her, I just helped her.
- [3] *Stavroula*: I felt a little upset at the beginning, when I asked her to help me. When I persuaded her to discuss, I felt nice.
- [4] *Researcher*: Who wants to talk about the collaboration that your classmates showed to us?
- [5] *Student 1*: At the point where she told her "solve it alone," she felt upset. Then, they began to discuss and they solved the problem together, it was good. I said Kostas the same thing. [*Kostas was his partner*]
- [6] *Stavroula*: Do you want me to explain to you why I chose this role?
- [7] *Researcher*: Yes.
- [8] *Stavroula*: I chose this role because I usually solve the problems quickly and then I help Alexia, but I did not know how it is if someone does not understand the problem.

At this level, both students had the opportunity to express their feelings about their social interaction (phrases 2, 3). As Stavroula put herself in Alexia's place, she began to be conscious of the consequences of her actions to her partner's feelings, when they worked together in mathematics. The presentation of their drama gave the opportunity to the other members of the classroom to recall and reflect on their own behavior with their partners during their collaboration in mathematics. For example, Student 1 (phrase 5) recalled his experience of his collaboration in mathematics and he pointed out that he faced with the same situation with his partner.

The students' reflection on their feelings during their collaboration in mathematics was connected with a special norm of their social interaction, that is, one of the students solved the mathematical problem alone and quickly in spite of his/her partner's needs. Yackel and colleagues (2000) have mentioned that the development of socio-mathematical norms is based on the establishment of concrete social norms in pairs' interaction. From this point of view, the students' awareness of the necessity to change this norm which hindered their collaboration is important for the future development of socio-mathematical norms during their problem solving activity. Stavroula had the opportunity to regulate her intentions about her participation in a mathematical discussion with her interlocutor, as she seemed to change her initial intention (solving the problem alone) to another one (discussing the problem with Alexia).

## **2<sup>nd</sup> Case: Apostolos and Elsa (N2-N2)**

At the beginning of the program, Apostolos and Elsa expressed both their desire to cooperate only with their teacher in mathematics (a typical N2 situation). Elsa justified her view as follows: "I have to try alone and only the teacher can help me when I have difficulties." She declared that she didn't want to help her classmates in mathematics because "If I do not know it right, I will say it to the other students in a wrong way." On the other hand, Apostolos justified his own view as follows: "I have the impression that my classmates will think that I do not do well in mathematics and I don't like this." Although, both students expressed negative beliefs about collaboration in mathematics, their intentions differed. Apostolos wanted to protect his self-image and Elsa had low self-confidence in mathematics. Moreover, these views were connected with the targets that both students posed for their mathematical activity. Apostolos declared that he felt happy in mathematics when he could solve a problem that his classmates "don't manage." In contrast Elsa said, "I feel happy when I answer correctly to my teacher's questions." In relation to the students' abilities in mathematics, their teacher commented that both students very often needed help to complete a mathematical activity. Moreover, Apostolos' mother declared that the participation of her son in collaborations in a mathematics classroom "indicates his strong self-estimation, as in such a situation he will not scare to expose his thoughts to the others." Elsa's mother declared that "Elsa's decision to collaborate or not with her classmates in mathematics depends on her."

In the following dramatic activity, Apostolos and Elsa decided to announce to their classmates their experience from their discussion about the interlocutors' responsibility for a mathematical error. The students chose to represent the discussion between two children in



two scenes. At the first scene the two characters are speaking on the phone and at the second scene they are in the class checking their solution on the blackboard.

### Scene #1.

- [1] *Apostolos*: Hello, how are you?  
 [2] *Elsa*: Fine. I'm doing my mathematics.  
 [3] *Apostolos*: That's why I am speaking to you, for the problem that we have for tomorrow.  
 [4] *Elsa*: The problem about the pizzas.  
 [5] *Apostolos*: I think that in the first table they will eat  $1/2$ .  
 [6] *Elsa*: Yes.  
 [7] *Apostolos*: In the second  $2/4$ .  
 [8] *Elsa*: In the third  $3/8$ .  
 [9] *Apostolos*: Yes, ok. I'll see you tomorrow.  
 [10] *Elsa*: We didn't speak about the last... Oh! He hanged up!

### Scene #2.

- [11] *Elsa*: The results are correct?  
 [12] *Apostolos*: I think so.  
 [13] *Elsa*: Let's see at the blackboard.  
 [14] *Apostolos*: Oh! [*with surprise*]  
 [15] *Elsa*: What happened? Everything is wrong!  
 [16] *Apostolos*: Everything. [*disappointed*]  
 [17] *Elsa*: Why is it  $1/4$ , how did you find it?  
 [18] *Apostolos*: We had to discuss about it yesterday, when we spoke over the phone.  
 [19] *Apostolos & Elsa (together)*: Now, it's late.

This pair of students chose to incorporate in their drama text issues concerning the treatment of their errors in mathematics as well as the significance of argumentation of a decision during their mathematical activity (phrases 15-19). The students, in the first scene, represented a mathematical discussion, through a phone conversation, which focused on the results of a mathematical activity. In the second scene, as they found out that they have made an error, they pointed out that they had to discuss more about the solution of the mathematical problem. The students seemed to consider two questions being crucial for the development of their mathematical discussion and the avoidance of errors, the "why?" and the "how?" Both questions give rise to explanation and justification of a mathematical solution and this process helps their mathematical learning (Pijls et al., 2007).

The significance of the students' choice to represent the role of argumentation in a mathematical discussion was revealed during the discussion that took place among all the students in the classroom, after the presentation of the above drama.

- [1] *Researcher*: Do you want to talk to us about your roles?

- [2] *Elsa*: We would like to show you what happened to us when one of us spoke, the other agreed without either of us thinking whether it was right, and we got it all wrong.
- [3] *Apostolos*: We would like to show you that if you don't ask your classmate why what he says is so, if you don't understand it well enough, you should not accept it, because both of you might make a mistake. What we mean to say is that it isn't possible for one student to say something and the other to write it. You must ask him how he found it, why it is  $1/2$ . We were both led to making mistakes.
- [4] *Student 1*: I like that, because I agree with what they said.
- [5] *Student 2*: It's not only how he solved it, it could actually be wrong.
- [6] *Apostolos*: Yes, but if you don't ask and you are too confident, you will not find the error.
- [7] *Student 3*: I agree with Apostolos, because in this way you can think about what the other said.

This dramatic activity gave the opportunity to the students to reflect on the importance of justification of a mathematical answer (a crucial mathematical norm to be negotiated), especially for the prevention of an error (phrases 4-7). The students discussed about two important elements of a mathematical discussion: a) the consequences of the acceptance of a mathematical proposal without justification (phrase 2) and b) the relationship between the mathematical argumentation and the result of the mathematical activity (phrases 5, 6). It was very interesting when the Student 2 pointed out that only the description of a solution process doesn't ensure the right result. On the other hand, Apostolos underlined that it is through the argumentation process that the students could be led to the exploration of an error in a mathematical activity.

### **3<sup>rd</sup> Case: Paul and Nikos (N1-N2)**

At the beginning of the program, both students reported that they desired to work individually in mathematics. Paul declared that "I never ask help from my classmates, when I don't understand something I ask my teacher" (case of N1). Nikos said, "If I ask some help from my classmates, they will think that I cannot make it in mathematics" (case of N2). Both students considered that they are successful in mathematics when they satisfy their teacher's expectations. Moreover, in relation to the students' abilities in mathematics, their teacher commented that Paul was a student of high achievement and Nikos very often needed help to complete a mathematical activity. Concerning their parents' beliefs, Nikos' father considered that "children are capable to collaborate in mathematics with their classmates better than with a teacher" and Paul's mother declared generally that she believed on collaborations.

In analyzing this case we will discuss about two dramatic activities that the students presented during this research program, as these activities offered different opportunities for reflection on social interaction in mathematics.

During their discussion about the scenario of their first drama, they decided to represent a dialogue between a student who quickly solved a mathematical problem and a student that had difficulties to solve it. The students' idea was related to their mathematical ability and not

to their initial negative beliefs about the role of collaboration in mathematics. Both students expressed the desire to play the role of the “good” student. Finally, Nikos decided to play the student who had difficulties. The concession of Nikos could perhaps be interpreted as a result of the acceptance about the mathematical ability of his partner. The drama-text was the following:

- [1] *Paul:* [he is reading the problem] A boy is thinking, “I am hungry. What is better to eat:  $\frac{3}{4}$  of a pizza or  $\frac{2}{5}$  of it?” Help him to make a decision.
- [They remained thoughtful for a minute]
- [2] *Nikos:* I found it! He has to eat  $\frac{2}{5}$ .
- [3] *Paul:* You’re wrong. He has to eat  $\frac{3}{4}$ . It’s easy, if you think more, you will see your error. Mister! I find it! [He represented that he is addressing the teacher].
- [4] *Nikos:* Why is it wrong?
- [5] *Paul:* Think alone. Mister! [He represented that he is addressing to the teacher].
- [6] *Nikos:* [talking to himself] Where is the error? It is easy and I gave a wrong answer.
- [7] *Paul:* You don’t solve it yet?
- [8] *Nikos:* [upset and nervous] Leave me alone! I will find it alone! Is this your desire?
- [9] *Paul:*  $\frac{3}{4}$  is more than  $\frac{2}{5}$  because...
- [10] *Nikos:* [He is interrupting Paul] Why are you talking to me? I don’t want you to talk to me! Leave me alone with my error!

Paul and Nikos decided to represent those moments when they experienced the absence of collaboration in their mathematical activity. This situation often happened during the beginning of their collaboration and especially because of the existence of an error. When they designed their drama, it is remarkable that they focused on the reactions and the feelings of the different roles that they represented. Especially, Nikos had the opportunity to express emotions like angry and anxiety for his interlocutor’s indifference of his difficulties in a mathematical discussion, like in their real initial collaborations in the mathematics classroom.

In a collective level, when the members of the class talked about the above drama, the following discussion took place:

- [1] *Researcher:* Do you want to talk about your roles?
- [2] *Paul:* I think that he was in an egoistic role, because he didn’t think what happened to his classmate.
- [3] *Nikos:* I felt nervous because I asked him for his help, but he did not care and he only wanted to answer to the teacher.
- [4] *Researcher:* Who wants to talk about the drama that Paul and Nikos presented to us?

- [6] *Student 1*: The children played their roles very well. I would like to be Nikos in this role playing.
- [7] *Student 2*: Me too. If I was Paul, I would discuss with Nikos about his difficulty.
- [8] *Paul*: [*addressing to the Student 2*] I didn't like Paul's behavior, as he didn't help his classmate, but all these happened in the theater [*he means the drama*].

The comments of the students in the above discussion reveal that this drama offered to them the opportunity to evaluate the concrete behavior (indifference for the other's difficulties) according to the negative feelings that it caused to a partner. In our opinion, this discussion helped them to non-inculcate the existence of difficulties in doing mathematics during the collaboration of a pair and, in contrast, to inculcate the absence of collaboration in order to overcome these difficulties. However, as these thoughts were expressed in the drama, Paul had the opportunity to reflect critically on this behaviour and to reject it.

Paul and Nikos's second dramatic activity is representative of the opportunity that the students had to reflect on the evolution of their collaboration in mathematics. Paul and Nikos decided to represent in their play three different moments of the history of their collaboration:

### Scene #1

- [1] *Nikos*: [*He is reading a mathematical problem*] Evi and Nikitas took photos by the same camera. Evi used  $\frac{7}{12}$  of the film and Nikitas used  $\frac{4}{12}$  of it. Who's taken most photos? How much film was left over?
- [*The two boys are in silence looking at their books.*]
- [2] *Paul*: Did you find the solution?
- [3] *Nikos*: I found it!
- [4] *Paul&Nikos*: Mister! Mister! [*they represented that they are addressing to the teacher*].

### Scene #2

- [5] *Paul*: [*He is reading a second mathematical problem*] The next day Evi used  $\frac{2}{4}$  of the film and Nikitas used  $\frac{2}{6}$  of it. Who's taken most photos? How much film was left over?
- [*After a few minutes*]
- [6] *Nikos*: I think that it was left over  $\frac{1}{6}$ . What do you think?
- [7] *Paul*: I think  $\frac{2}{12}$ , it is the same.
- [8] *Paul&Nikos*: Mister! Mister! [*they represented that they are addressing to the teacher*].

### Scene #3

- [9] *Nikos*: [*He is reading another mathematical problem.*] John and Mary were painting two equal walls of their room. John said, "I am better than

- you because I painted  $\frac{2}{3}$  of the wall.” Mary said, “No, I am better than you because I painted  $\frac{5}{6}$  of the wall.” Who’s been right?
- [10] *Paul:* What do you think?
- [11] *Nikos:* I am thinking to draw the walls and to divide them in 6 pieces and in 3 pieces.
- [12] *Paul:* The  $\frac{5}{6}$  is larger. It is obvious.
- [13] *Nikos:* Right. Is there another solution?
- [14] *Paul:* The  $\frac{2}{3}$  is the same with the  $\frac{4}{6}$ .
- [15] *Nikos:* The  $\frac{4}{6}$  is smaller.
- [16] *Paul:* Let’s talk about these solutions to the class!

After the presentation of the above drama, the following discussion took place among the students:

- [1] *Researcher:* Do you want to talk about your drama?
- [2] *Paul:* We wanted to show that in the beginning...
- [3] *Nikos:* In the beginning we didn’t discuss at all and everyone made the operations alone.
- [4] *Paul:* And later we started to talk to each other and now we collaborate and discuss in order to solve the problem and to find different solutions, if there are any.
- [5] *Nikos:* We wanted to show that it is better to collaborate than to work alone.
- [6] *Researcher:* Someone else?
- [7] *Student 1:* The collaboration that we saw was very good. They said their ideas to each other and they continued their work, searching for other solutions.
- [8] *Student 2:* At the end, it was good. In the beginning, one didn’t know how the other found the solution. They talked only for the result.

These comments revealed that the students had become aware about the characteristics of an effective collaboration in mathematical problem solving, like the discussion of their ideas with pleasure in order to find a common solution or the effort to find different solutions for a given mathematical problem (phrases 7-8). Furthermore, the concrete dramatic activity gave the opportunity to all the students to reflect on the history of their own attempts to collaborate with their partners in mathematics and to point out the critical moments of these efforts, like the evolution of the mathematical discussion from focusing on the right or wrong mathematical result to focusing on the exploration of the given proposals. Cobb and colleagues (1991) have commented that this level of interaction, working together to construct a solution, is determinative for a productive collaboration, as it gives rise for reflection and cognitive reorganization.

It was a surprise for us the implicit way that these students presented the evolution of the establishment of the mathematical norms during their collaboration. The mathematical norm of the exploration of different solutions in a mathematical problem and the decision to present them to their classmates as appropriate (scene#3) passed through the changes realized in their

social interaction (scene#1, 2). The students seemed to become conscious of the improvement of their intentions as they participated in mathematical discussions. Through their dramatic activity they pointed out two significant changes: a) from their intention to communicate the solution of a problem only with the teacher to their intention to discuss between them on the result of the problem solving process (scene#1-scene#2), and b) from their intention to focus their discussion on the result of a problem to their intention to explore and to negotiate their proposals for the solution of a mathematical problem (scene#2-scene#3).

### Conclusions

The purpose of our study was to investigate how the students' metadiscursive reflection through their dramatic activities could influence the evolution of their mathematical discourse during their problem solving activity in a mathematics classroom. According to our results, the students' engagement in dramatic activities seemed to offer a lot of opportunities for a reflection on their collaboration in mathematics in a metadiscursive as well as in a collective level. In a metadiscursive level, they gave to them the opportunity to:

- experience their interlocutor's emotions,
- express their emotions provoked by his/her interlocutor's actions,
- correlate concrete behaviours in a mathematical discussion with the generation of concrete emotions,
- evaluate the evolution of their collaboration in mathematics as they thought about the changes happened in it,
- reflect on specific elements of their mathematical discussion, like the treatment of errors and the necessity of justification.

In a collective level, the dramatic activities gave to the students the opportunity to evaluate the behaviors of the students in role and to compare them with their own experiences of their collaboration in mathematics. In this way the separated experiences of each group were taken as shared with the others. The students had the opportunity to discuss on their common and different experiences and to create a consensual domain on what a productive collaboration in mathematics means. In the first and the third cases the characteristics of a productive collaboration in mathematics were discussed among the students. The understanding of a partner's difficulties or feelings during collaboration in mathematics was a critical moment in the students' discussions in the first and the third cases. Moreover, they seemed to understand the important role of argumentation and the treatment of an error during a mathematical discussion, issues that they were mainly discussed among the students in the second case.

We have to mention that at the end of the program, the students' beliefs about cooperation in mathematics had begun to change (Chaviaris & Kafoussi, 2005; Chaviaris et al., 2007). For example, Apostolos expressed the desire to cooperate with his classmates in mathematics, because "you can learn better, when you cooperate." However, he said that the partner in a pair influences the development of the cooperation by declaring that: "It depends on who is your partner. For example, if I cooperate with Paul who is very good in mathematics, he will

talk and I will do nothing. But with Elsa, I can cooperate and I learn better.” Other students declared that “It is better to solve the problems together. You can hear different ideas and finally you find a good solution.” However, we cannot argue that this progress is exclusively a result of the development of their metadiscursive reflection through their dramatic activities, as the students participated in more activities during this program (mathematical activities, students’ discussions on their video-taped collaborations and students’ interaction in the mathematical classroom).

The drama seemed to be a fruitful environment for the development of students’ reflection on their collaboration in mathematics and the development of their mathematical discourse, as it allows them to reflect on issues concerning social and emotional aspects of their collaboration as well as on critical aspects of a mathematical discussion. This environment could help the students to experience the necessity to reform their collaboration in small groups through self-assessment of their behavior and the assessment of others’ actions. The dramatic activities could create a social context in which students could participate in democratic discursive practices for mathematics teaching and learning. The incorporation of dramatic activities as a learning environment for the development of students’ metadiscursive reflection and the improvement of their collaboration in mathematics seem to be a promising context for the teaching of mathematics.

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## Appendix

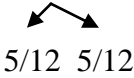
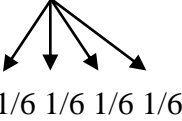
At their first video-taped collaboration, Apostolos (N2) and Elsa (N2) were engaged with the following activity: *The students had to share fairly a chocolate among 2, 3, 4, 5, 6 children and to answer the question: In which case will a child get more chocolate and why?*

During their collaboration the following dialogue took place:

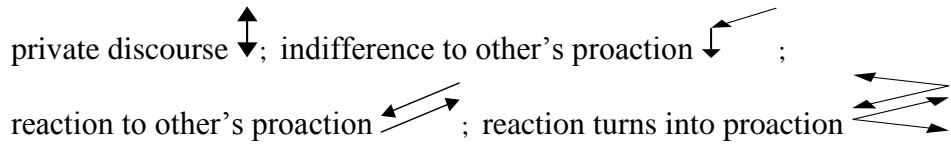
<i>Utterances</i>	<i>Writings</i>
1 A: Here, it will be divided into two parts and each one will eat half of it.	
2a A: Here, it is 1/3.	[2a] He wrote 1/2 and 1/3
2b Is it written in this way?	
3 E: I think so.	[3-5] They wrote the answers without communicating
4a A: ( <i>after personal work</i> ) In which case will a child get more chocolate and why?	
4b In the first case, but why?	
5 E: Wait! ( <i>after personal work</i> )	
6 E: I wrote that the children will get more chocolate when they are six.	
7 A: Ok, we know it.	

At their last video-taped collaboration, Apostolos and Elsa were engaged with the following activity: *2 children share fairly 5/6 of a pizza. How much pizza has each child get? 4 children share fairly 2/3 of a pizza. How much pizza has each child get?*

During their last collaboration the following dialogue took place:

<i>Utterances</i>	<i>Writings</i>
1 E: (She is reading the problem.)	
2a A: 2 children, 5/6.	
2b We can multiply 2 by 5/6.	
3 E: Yes.	
4 A: Will we do it?	
5 E: Just a moment, every child does not take 5/6, but they share 5/6.	
6a A: Ah, yes!	
6b There are 5 pieces. Everyone will get 2 pieces and it left over 1. Right?	
7 E: Yes.	
8 A: This one, don't we share it in two?	
9 E: Yes.	
10 A: Do you want to make a drawing?	[10] They are trying to draw 5/6 in a circle
11 E: It is better.	
12 A: I can't draw it in 6 pieces.	
13 E: It is difficult for me too.	
14 A: If we use operations?	
15 E: Let's try.	
16 A: 5/6 is 10/12 that it can be shared by 2.	
17 E: Let's share it.	[17] They are writing 10/12
18 A: 5/12 everyone.	
19 E: The second one. 4 children share fairly 2/3 of a pizza.	
20 A: Now, how can we share 2/3 in 4?	
21 E: 2/3 is 4/6	
22 A: 4/6...	
23 E: It is 1/6, 1/6 and 1/6 and 1/6.	[23] She is writing 2/3=4/6 

In the following it is presented the interactive analysis of the above collaborations according to Sfard and Kieran's (2001) tool and it is indicated the improvement of the students' collaboration in mathematics. The utterances of interlocutors are characterized as reactions to a target-phrase or proactions demanding a response. This analysis allows the existence of distinct types of interaction among the members of the group. For example, such types of interaction are:



Interactivity flowchart #1

	Apostolos	Elsa
1	●	
2 <sup>a</sup>	●	
2 <sup>b</sup>	●	
3		●
4 <sup>a</sup>	●	
4 <sup>b</sup>	●	
5		●
6		●
7	●	

Interactivity flowchart #2

	Apostolos	Elsa		Apostolos	Elsa
1		●	14	●	
2 <sup>a</sup>	●	●	15	●	●
2 <sup>b</sup>	●	●	16	●	●
3	●	●	17	●	●
4	●	●	18	●	●
5	●	●	19	●	●
6 <sup>a</sup>	●	●	20	●	●
6 <sup>b</sup>	●	●	21	●	●
7	●	●	22	●	●
8	●	●	23	●	●
9	●	●			
10	●	●			
11	●	●			
12	●	●			
13	●	●			

As someone can observe, comparing the two interactivity flowcharts presented above, there were significant improvements in Apostolos and Elsa's social interaction. Initially, the students' mathematical discussion was characterized by private discourses and indifference to other's proactions. At the end of the program the students' collaboration was more productive than at the beginning as the types of interaction that they dominated were reactions to other's proactions and reactions turns into proactions (see Chaviaris et al., 2007 for further details).