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Categories of Intuitive Reasoning in the teaching of parabolas: A structured practice in Didactic Engineering

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ABSTRACT

This work is the result of a pre-experiment carried out as part of a master's course, dealing with the study of the parabola through different mathematical views. It aims to recognize possible didactic obstacles in its teaching, based on intuitive manifestations in the resolution of a didactic situation based on GeoGebra software. The methodology adopted was didactic engineering in its four phases, experimented with a teacher in initial training. The observation and data collection provided us with elements for a posteriori analysis and validation of the experiment in which we verified the need to discuss the parabola, articulating its geometric, algebraic, and analytical views.

Keywords: parabolas, GeoGebra, intuition, initial teacher training

INTRODUCTION

The parabola has great relevance in the development of areas of knowledge such as architecture, physics, engineering, among others. However, its study in basic education in the Brazilian context has been explored in a purely algebraic way, under a fragmented and little contextualized approach, generating difficulties in subsequent stages of studies, such as higher education (Cerqueira, 2015; Siqueira, 2016; Vargas & Leivas, 2019).

Halberstadt (2015) states that a possible epistemological difficulty about learning analytical geometry is due to the fact that it has a two-dimensional character, requiring prior knowledge of both algebra and geometry. In addition to this, another difficulty encountered is the general understanding of the subject, considering that its concepts and properties are abstract. Therefore, we understand that this can be a factor that hinders student learning, as classes run the risk of becoming overly technical.

Based on this, one understands the importance of investigating the link between algebra and geometry, arising from analytical geometry itself, searching an articulation that develops the student's mathematical thinking. So, from this perspective, we also consider it relevant to work on this topic with future teachers, which generally has not occurred in initial training courses in the Brazilian context, or when it occurs, it is very abbreviated and superficial (Siqueira, 2016), which justifies the development of this research. Maioli et al. (2012) explain that, despite the strong presence of mathematics in our daily lives, it is sometimes difficult to show students clearly the real and interesting applications of the subjects studied at school or to demonstrate situations that involve geometric visualizations without a technological contribution. Based on this, we carried out a pilot experiment in initial training, as a way of understanding some of the difficulties that permeate the work of mathematics teachers with this theme, its particularities, and possibilities of exploration with GeoGebra software.

GeoGebra software is a dynamic geometry environment that allows the creation, visualization, and manipulation of representations of mathematical concepts, treating geometry, algebra, and calculus in an interconnected way, among other possibilities. Thus, the software can facilitate the discovery of relationships between the objects that make up a geometric construction. According to Alves (2019), given the potential of GeoGebra software for problem solving, manipulation, visualization and understanding of concepts, the teacher can encourage student involvement in a dynamic exploration of numerical, algebraic, and geometric properties, developing visualization, perception, and intuition, essential for the evolution of their learning.

This work is the result of an excerpt from a master's thesis carried out in Brazil in which we bring the study of the parabola, seeking to explore and articulate its algebraic, geometric, and analytical views. With the contribution of GeoGebra, we seek to discuss its particularities and the interrelationship between mathematical topics that address it. In this sense, we outline the objective of recognizing didactic obstacles in teaching parabolas based on intuitive manifestations in the resolution of a didactic situation with the support of GeoGebra.

For this, the methodology adopted was didactic engineering (DE) in its four phases. According to Artigue (1988), DE is characterized as an experimental scheme, which is based on didactic achievements in the classroom, that is, on the conception, realization, observation, and analysis of teaching sequences. We used together the theory of didactic situations (TDS) (Brousseau, 1986) given the compatibility between DE and TDS and the fact that both are theories of French-speaking origin. In addition to these, we also adopted the categories of intuitive reasoning (CIR) (Fischbein, 1987) as a basis for analyzing the subject's interaction with the parabola associated with technology and the data related to this process. TDS and CIR were necessary to structure the experiment within DE developed.

From the above, in the next sections we bring a discussion about TDS, CIR and an articulation between them, the development of DE phases of this work, as well as our considerations.

THEORETICAL FRAMEWORK

In this section we bring a brief discussion of the theoretical-conceptual contribution, essential to structure the developed pilot experiment. Here we discuss TDS, as a teaching theory, on CIR, as a basic theory in the observation and analysis of data, and we seek to establish a relationship between these.

Theory of Didactic Situations

TDS brings a theoretical model that aims to understand the dialectical relationship established between the main actors in a didactic system–the teacher, the student and knowledge–as well as the environment (milieu) in which the conjuncture of a specific didactic situation develops. Based on this, TSD aims to encourage the student to behave like a researcher, where, from a set of dialectics, he can develop and be able to formulate hypotheses and concepts. Meanwhile, that the professor provides favorable situations for this student, when acting, to transform the information into knowledge for himself.

According to Brousseau (2002), the conception, organization, and planning of a didactic situation by itself demands stages in which the student is alone facing the problem and tries to solve it without the direct intervention of the teacher. This situation is called by the author as an adidactic situation in which the student, when interacting with the proposed problem-situation, manages to solve it, without any help or direct response given by the teacher, doing this only based on their previous knowledge and experiences. It is worth emphasizing that the didactic situations are designed so that they coexist with the didactic situations, characterizing and obeying a predetermined didactic process by objectives, methods, resources, and concepts.

TDS organizes the student's learning process from phases or dialectics, which are action, formulation, validation, and institutionalization, the first three being considered the *adidactic situation*. These dialectics were used to organize the development of experimentation in this research.

For the development of this work, we are interested in the path of mathematical reasoning in the development of TDS dialectics. For a long time, it was considered that in Mathematics, reasoning should be conceived as a presentation of model proofs, taught by the teacher, and faithfully reproduced by the students. However, for teachers today "reasoning as a mental activity is not a simple recitation of a memorized proof" (Brousseau & Gibel, 2005, p. 14). In this way, to build a model of mathematical reasoning of a subject from the notion of situation, it is necessary to understand that reasoning concerns a domain that is not restricted to formal, logical, or mathematical structures. Despite being made up of an ordered set of statements linked, combined, or opposed to each other, these models respect certain restrictions that can be made explicit in the solution of a problem, or even, as explained by Brousseau and Gibel (2005):

> Therefore, to be able to claim that a given observable behavior is a sign of a reasoning whose elements are, for the most part, implicit, it is necessary to go beyond the formal definition and examine the conditions in which a "presumed reasoning" can be considered as an "actual reasoning" (p. 16).

According to Brousseau (1997), reasoning can be characterized by the role it plays in a situation, that is, by its function in that situation. Thus, such a function can be to decide about something, to inform, to convince or to explain. From this perspective, the function of reasoning varies according to the type of situation in which it occurs, having a direct relationship with the dialectical movement within TDS, whether it is a situation of action, formulation, or validation. Thus, Brousseau and Gibel (2005) seek to distinguish the levels of mathematical reasoning, considered more or less degenerate, and that adapt to different types of situations in TSD, as summarized below:

1. **Reasoning of level 1 (L1):** It can be characterized by a type of reasoning that is not formulated as such, however it can be attributed to the subject based on his actions, and constructed as a model of this action, being considered as an implicit model related to the situation of action in TDS.
2. **Reasoning of level 2 (L2):** It can be considered as an incomplete reasoning from the formal point of view, but with gaps that can be, implicitly, filled by the student's actions in a situation, where a complete formulation would not be justified. This type of reasoning appears in situations, where communication is necessary, being related to the formulation phase.
3. **Reasoning of level 3 (L3):** It can be defined as a formal, global, and concluded reasoning, on a set of correctly related inferences, which make a clear mention of elements of the situation or knowledge considered as shared by class, even if it is not yet postulated that such reasoning is absolutely correct. Reasoning at this level is characteristic of validation in TDS.

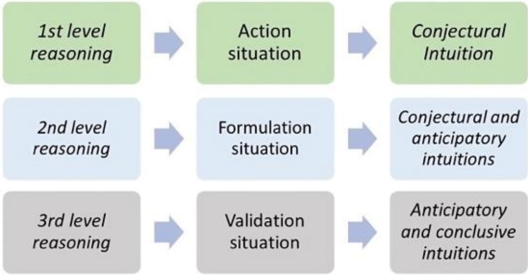

Figure 1. Relation between levels of reasoning & intuitive categories (Source: Authors' own elaboration)

Thus, it can be considered that each stage of reasoning is incorporated into logical and mathematical justifications considered standard in which their validity and relevance seem to be autonomous. Therefore, in the authors' proposal, the interpretation of the students' solutions must consider a larger and more complex system, if the teacher's intention is to challenge them, instigate them or even explain how such forms of reasoning, correct or not, were produced. In this way, it is recommended that the teacher consider the student's prior knowledge to build his reasoning in an objective situation.

As a way to better understand these types of reasoning and their relation with the intuition as an ontological faculty, we bring in the following section a discussion about intuition in the educational field.

Categories of Intuitive Reasoning

In this work, intuition was observed according to Fischbein's (1987) categorization, associated with the development of TDS and its levels of reasoning.

Regarding the categorization of intuition, Fischbein (1987) discusses the existing articulation between the different types of intuition and their relation with problem solutions, separating them into what he classifies as CIR, which are: *affirmative intuitions*, *conjectural intuitions*, *anticipatory*, and *conclusive intuitions*, briefly described in the subsequent paragraphs.

The first of the categories refers to *affirmative intuitions*. These are representations, interpretations or understandings directly accepted by human beings as natural truths, evidently and intrinsically significant (De Sousa, 2022; Fischbein, 1987). For example, if someone asked a student what a straight line is, presumably he would try to sketch a straight line or show an example of a very taut line.

The second category deals with *conjectural intuitions*. Fischbein (1987) considers that in this model of intuition there is an explicit perspective of the solution of a problem, however, the subject is not involved in an effort for its resolution. That is, this type of intuition refers to assumptions linked to the feeling of certainty. They "represent statements about future events or about the course of a certain event, being a preliminary, global view that precedes an analytical and fully developed solution of a problem" (De Sousa, 2022, p. 202).

The third category is *anticipatory intuitions*. Fischbein (1987) explains that this type of intuition provides an absolute point of view, preceding the solution of a problem, which precedes the fully developed analytical resolution. The subject who is solving the problem sees all the steps towards its solution and understands the path he must follow to reach the expected answer. Starting from a global understanding of a possible way to solve a problem, this intuition influences and directs the stages of search and construction of the solution, where there is a concrete application of strategies that effectively help to identify an adequate solution. Furthermore, it can be assumed that *anticipatory intuitions* are stimulated by pre-existing *affirmative intuitions*.

In the case of the fourth category, which are the *conclusive intuitions*, these summarize a globalized and structured view of the basic ideas of solving a problem, previously elaborated, thus depending on the other three types of intuition mentioned above, allowing the generalization of the structure mathematics for the proposed problems and replication of the solution model in similar situations.

Relation Between TDS and CIR

In view of what was exposed in the preceding sections, we can infer a relationship between what Brousseau and Gibel (2005) propose as the different levels of mathematical reasoning in the development of TDS, and what Fischbein (1987) proposes in his classification of intuition, that he calls CIR. In this sense, we propose a correlation between the authors' ideas, as shown in the scheme of **Figure 1**.

When Brousseau and Gibel (2005) propose the *1st level reasoning*, describing it as a model of reasoning not yet formulated, but related to the subject's position in a *situation of action* within TSD, we can see a similarity with the category of *conjectural intuition* by Fischbein (1987). This relationship can be perceived when Fischbein (1987) proposes that the subject, in this model of intuition, has a preliminary view of the problem and superficially of its path of solution, however, he did not, in fact, perform any action to solve it, but rather is conjecturing its hypotheses to then follow a path based on reasoning that makes sense.

The *2nd level reasoning* proposed by Brousseau and Gibel (2005) is considered as an unfinished reasoning from the formal point of view, but with gaps that, implicitly, can be filled from the student's actions in a *situation of formulation* within TDS. This reasoning model can be compared, depending on the level of these gaps, both with the *conjectural intuitions* proposed by Fischbein (1987) in which the student starts his deductions with a starting point, as with *anticipatory intuitions* in which the student has already managed to formulate ideas and establish a path to the solution more explicitly.

From this same perspective, we can understand that *3rd level reasoning* as a model of global and finalized formal reasoning, which is based on the sequential connection of cohesively articulated inferences, even if such reasoning does not be absolutely correct, as being a format presented in validation situations in TDS. So, we can relate *3rd level reasoning* to what Fischbein (1987) proposes as *anticipatory intuition* and *conclusive intuition*, depending once again on how this reasoning was produced by the student. We consider the *anticipatory intuition*, given the fact that the student, in this course of reasoning, can envision a fully developed analytical solution, presenting a coherent logic for its solution. And conclusive, because if this student has a full understanding and articulation between his previous knowledge and the development of new knowledge from the proposed didactic situation, this student can establish a standardization and a generalization of his solution for situations that present similarity with the exposed. Such generalization can be validated by the teacher in a later situation of institutionalization.

In this way, we understand that both the levels of reasoning proposed by Brousseau and Gibel (2005) and the categories established by Fischbein (1987) show that the learning path of a new reasoning occurs when it is promoted from just one particular means of solving a certain problem for a “universal” means of solving all problems of a certain type and integrates as such with the knowledge of the subject. In an autonomous situation, the reasoning is based on induction, but this induction is supported by a chain of inferences that can be made explicit.

METHODOLOGY: DIDACTIC ENGINEERING

According to Artigue (1988), DE is characterized by an experimental scheme based on didactic achievements within the classroom, that is, in the design, realization, observation and analysis of teaching sessions. In addition, DE can also be considered as an experimental research methodology, due to the record in which it is located, and the validation mode associated with it: the comparison between a priori and a posteriori analysis (Almouloud, 2007). Planning and execution of a DE can be structured in four stages, which are

(i) preliminary analysis,
(ii) conception and *a priori* analysis of didactic situations,
(iii) experimentation, and
(iv) *a posteriori* analysis and validation.

In the case of this research, DE was considered as a research methodology because it was a methodology developed in France, as well as TDS, both being compatible and used together in a harmonic way.

The pilot experiment carried out had as subject one student of the degree course in mathematics, female, 21 years old, belonging to the 6th semester of a Brazilian university, in the face-to-face format. It is important to note that this student was not a direct participant in the field research of the dissertation, but a voluntary participant, who was invited to collaborate by providing data that would enable an analysis of the didactic variables of the study. The pre-established didactic variables in this DE were tested to verify the viability of the experiment. Data were collected in the form of photographs, audio and video recordings, construction records in GeoGebra and written material. The course of the research followed the four phases of DE, these being the first two phases of DE focused on the construction of the experiment and the last two phases directed to the practical part, collection, and analysis of the experiment data.

1st Phase: Preliminary Analysis

According to Almouloud (2007), the preliminary analysis of a study considers a general didactic theoretical framework about a certain mathematical object, consisting of an analysis that considers the epistemological, historical and didactic prism. Based on this premise, the preliminary analyzes of this study are subdivided into the different mathematical perspectives on the parabola concept, in the survey of historical, epistemological, and didactic aspects in its teaching and in the role of GeoGebra in the construction of this concept. The concept of parabola according to Lima (2014, p. 115) says: “let d be a line and F be a point outside it. In the plane determined by d and F, the set of points equidistant from d and F is called a parabola with focus F and directrix d”. This definition is common in basic education textbooks. However, it generally does not explore the geometric meaning of these elements, possibilities with the use of technology or real applications (Bermúdez & Mesa, 2018; Cerqueira, 2015). This reverberates in the students’ difficulty in understanding the subject at the higher level in disciplines such as analytical geometry, linear algebra, and differential and integral calculus.

Based on the above, we understand the importance of teacher development in the epistemic scope, reinforcing the search for means for a clear presentation of content, with possibilities for practices, reflecting on student learning. Common national curriculum base (BNCC)[1] highlights the articulation between geometry and algebra, building meaning for student, recommending the non-approach of equations dissociated from their geometric interpretation and suggesting the use of softwares for their teaching (Ministry of Education of Brazil, 2018). In addition, it is worth mentioning the relevance of addressing this topic in initial training, which rarely occurs in mathematics degrees in Brazil (Siqueira, 2016).

Cerqueira (2015) and Siqueira (2016) reflect on the approach to this subject and its exploration in abbreviated form, in a summarized analytical/algebraic view and the non-exploration of geometric characteristics and possibilities of the parabola, or even the use of technology in the study of its elements. Based on this, we bring the parabola in this work, seeking to understand it from a geometric view and the possibility of manipulating its parameters with support from GeoGebra software.

[1] Guiding document for the curriculum of basic education in Brazil. Acronym in Portuguese.

With the aid of a ruler, draw a straight-line *r* and outside it marks a point *F*. Choose at *r* equidistant points *A1*, *A2*, *A3*, *A4*, ... and fold the paper so that *A1* coincides with *F*. To facilitate the visualization of the curve, draw the line that coincides with the fold. Repeat this operation for all other points marked in *r*. See that the folds obtained are tangential to a curve. Draw this curve and answer:

1. What kind of curve was described?
2. What roles do the line *r* and the point *F* play in the described curve? Sketch this situation in GeoGebra and present your solution.

Figure 2. Proposed teaching didactic situation (Source: Authors' own elaboration)

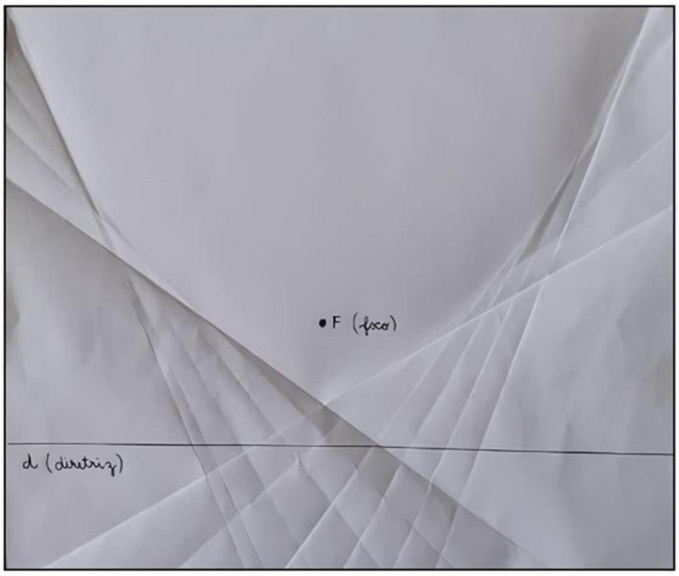

Figure 3. Parabola with folding techniques (Source: Authors' own elaboration)

2nd Phase: Conception and *A Priori* Analysis

In this section, we seek to structure a didactic teaching situation that addresses the parabola beyond the algebraic/analytical view, exploring it from a geometric perspective, based on a construction in pencil and paper and its transposition to GeoGebra. The software, by allowing the manipulation of its elements by the student, provides an environment in which they can demonstrate their mathematical reasoning for the solution.

From the prepared situation, we delimit the possible didactic variables (local), as more specific hypotheses focused on the scope of the classroom. These hypotheses refer to the attitudinal prediction of the student, about his behavior in the face of the proposed situation that, at the end of the entire course, were essential for the validation of engineering and progress of the other situations developed during the master's research. In this case, as local variables, we consider:

1. Possible difficulties in the development of the didactic situation in GeoGebra (transposition from paper to software).
2. The student's prior knowledge about the subject is not sufficient for understanding and solving the didactic situation.
3. The student does not clearly present a manifestation of CIR.

In this stage, we elaborated a didactic situation aiming that the student (teacher in training) recognize the parabola curve from paper folds and its main elements–focus, vertex, and directrix–, as well as use geometric design techniques and materials such as paper, ruler, and pencil. Next, we aim to build and discuss the proposed activity in the software. We believe that their resourcefulness depends directly on their school/academic career, whether or not the student has studied this topic, as well as how it was approached. The didactic situation proposed was (**Figure 2**).

Its description is based on theories presented in the theoretical framework explained and discussed in the previous section.

In the situation of action, we hope that the student, after carefully reading the question and, in case of prior knowledge of the definition of parabola, manifests an *affirmative intuition*, identifying the elements "line *r*" and "point F" as the directrix and focus of the parabola, respectively. Already in the situation of formulation, we expect the student to carry out the procedures described in the question and arrive, from *conjectural intuitions*, at construction of parabola with folds in the paper, as shown in **Figure 3**.

In the situation of validation, with the contribution of GeoGebra, we hope the student to formally express the concept of parabola and a preview of its solution in the software, expressing *conjectural* and *anticipatory intuitions*.

The construction using pencil and paper, in our conception, can provide subsidies for the student to visualize the elements of the parabola and look them for different ways to construct it in GeoGebra, as requested in the question. According to Fischbein (1999, p. 16), "intuitions are not absolute. They depend on context–in the present case, the perceptual context". Thus, in GeoGebra environment, from a visual perception, the student could arrive at the construction shown in **Figure 4** (or a similar construction).

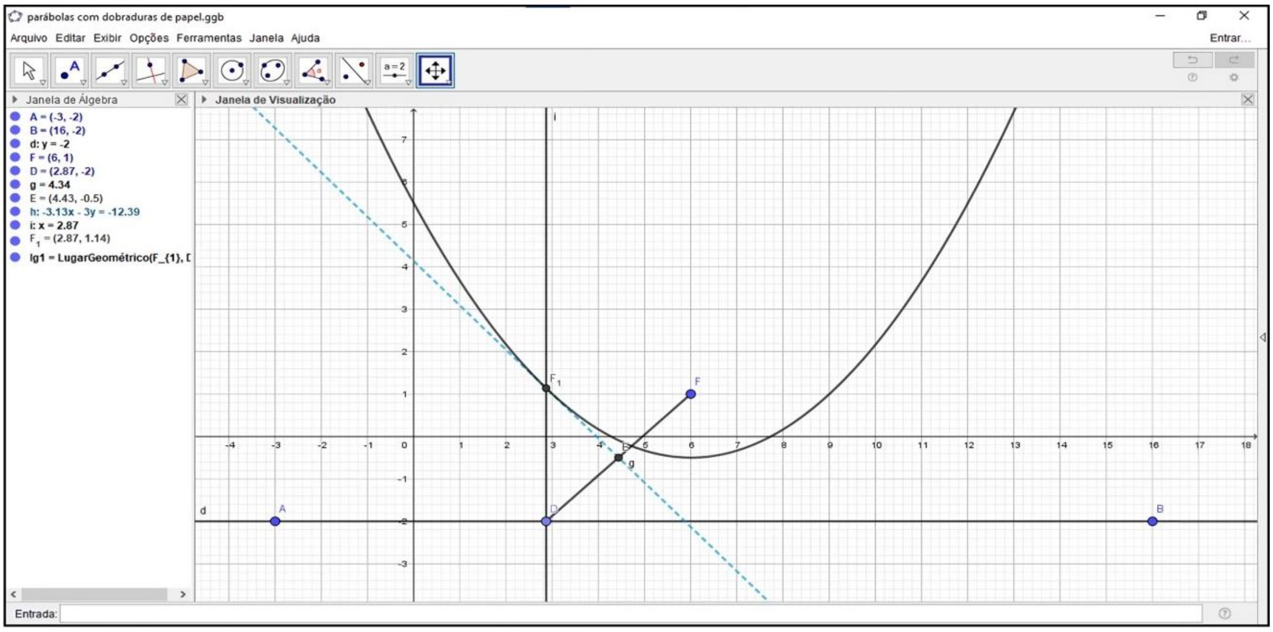

Figure 4. Parabola with folds in GeoGebra environment (Source: Authors' own elaboration)

Table 1. Student's information (Research Data, 2022)

Question	Student's answer
How old are you & what semester are you studying in undergraduate course?	21 years old. 6th semester
Have you ever taken any course involving analytical geometry during undergraduate course?	Analytical vectorial geometry
Have you ever worked or had any professional experience as a teacher in classroom?	No.
Do you know GeoGebra software? Have you used it during undergraduate course?	Yes, but on very specific occasions. I already used it once in calculus II classes with teacher's supervision. I have basic knowledge.

Finally, for institutionalization, we aim for the resumption of the didactic situation by the teacher, as well as the discussion of the definition of parabola and demonstrations of the solution on paper and in software, considering the work of Lima (2014), mentioned in our preliminary analysis.

PRACTICAL DEVELOPMENT AND DATA COLLECTION

In the following subsections, we present two final phases of DE, which provide us with results of implementation of teaching didactic situation, based on the collection and analysis of structured data in the outlined theories, as well as their discussion.

3rd Phase: Experimentation

The proposed didactic situation was developed with a single subject, female 21 years old, a teacher in initial training, a student of the 6th semester of the mathematics degree course at a Brazilian public institution and non-participant in the official research experiment. This student collaborated voluntarily so that we could collect data and evaluate the pre-established didactic variables in the a priori analysis and, if necessary, make the necessary adjustments so that we could proceed to the experimentation and data collection stage of the research.

During the interaction between the student and the didactic situation, we observed aspects such as the clarity of the problem, the time proposed for its solution, the level of difficulty of the question associated with the course of TDS dialectics and the intuitive manifestations presented by this student, both when using the pencil and paper environment as well as the computational environment, as well as the relevance of the collected data.

At first, the didactic contract was established in which the teacher researcher asked the student to read the proposed situation, sketch his solution using pencil and paper and transpose it to GeoGebra, pointing out his conjectures and observations. It was specified that all material (notes, video and audio recordings, and photographs) would be collected for analysis. The researcher, in an initial conversation, collected some information about the student, written in a summarized way in **Table 1**.

The student in question declared that he was aware of his collaboration in the experiment, agreeing that it would occur, by signing a consent form[2]. After that, the researcher handed in a form with the didactic situation and the questions regarding the subject that she wanted the student to answer.

[2] Informed consent form (TCLE, acronym in Portuguese) ensures preservation of your identity, safety, & benefits when participating in a research, being a legal document approved by the Ethics & Research Committee of the Institution in Brazilian territory to which the work is linked.

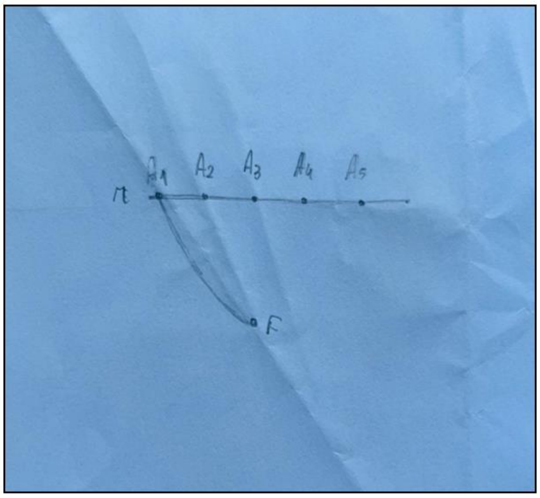

Figure 5. Initial sketch of participant in action situation (Source: Field study, 2022)

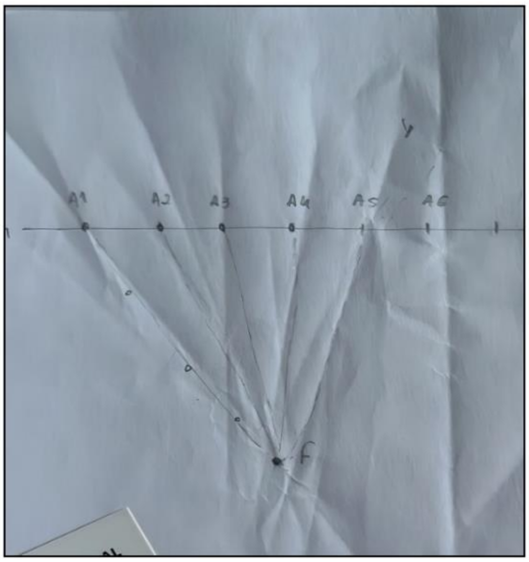

Figure 6. Second sketch of the participant in formulation situation (Source: Field study, 2022)

With regard to the dialectic between the researcher and the student, this experiment has characteristics of an action research (Thiollent, 2007). An action research needs to meet two basic purposes: practical and knowledge. The first is understood as the contribution of the research in the solution of the problem in question and the second as the knowledge generated from the solution of the problem (Marconi & Lakatos, 1996). Then, here the researcher sought to identify the particular difficulties and needs about parabola with this pilot experiment, to later plan actions to work in the improvement of a collective difficulty, represented by the gap in the teaching of parabolas in the context of the degree in mathematics. In this way, the experiment of the didactic situation occurred, as follows:

In the *situation of action*, the student read the problem and spent some time thinking about how to proceed. Initially, there was difficulty in understanding how the folds should be made on the paper. At first, the student drew a short ray, with several points spaced apart, as shown in **Figure 5**.

Initially, the student made a drawing that was not the right size for the folds, demonstrating an *affirmative intuition*, even if partially inadequate. According to **Figure 5**, there was an attempt to sketch the folds that was not successful. When the didactic situation proposes that *"with the aid of a ruler, draw a straight-line r and outside it marks a point F. Choose from r equidistant points A1, A2, A3, A4, ... and fold the paper so that A1 coincides with F"* there is no mention of the length of this line.

During the situation of action, "the student can improve or abandon his model to create another one: the situation thus provokes a learning by adaptation" (Almouloud, 2007, p. 37). In this way, when the student understood that he would not be able to make the folds that way, he reworked his initial reasoning and redid his sketch, in a larger size, as shown in **Figure 6**.

In the *situation of formulation*, when carrying out the folds, the student sketched with a pencil a connection between the points drawn on the straight line and the point outside it (point *F*), which for the teacher-researcher, as an observer, was something incomprehensible. However, when reading the statement again and observing its outline, through a *conjectural intuition* and still with remnants of doubts, the student stated that the curve "resembled a parabola", if we considered the more "open" points (referring to to the most distant points).

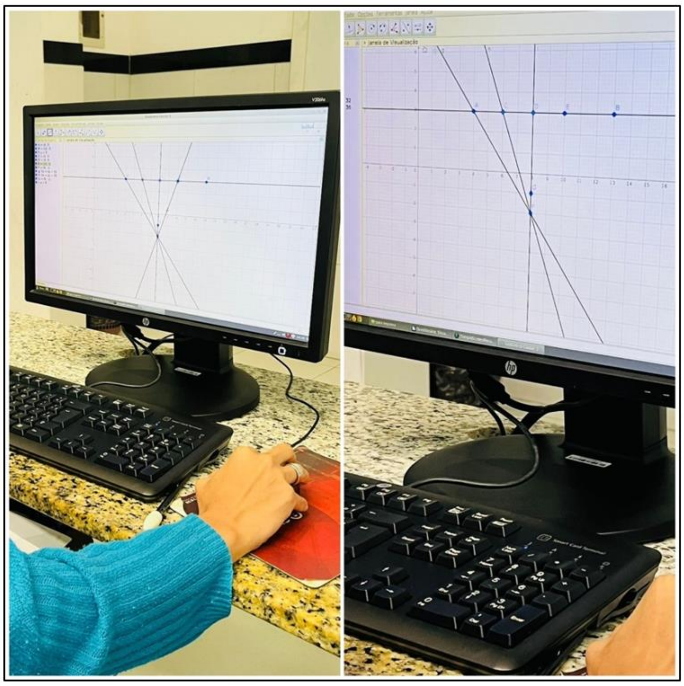

Figure 7. Transposition from paper to software (Source: Field study, 2022)

In the second part of the solution, the student proceeded to outline his strategy in GeoGebra. But we observed that in its construction he did not immediately resort to resources that could provide a parabola, such as using parabola tool, in guide *conics*. Intuitively, he tried to sketch his own drawing, as it was being visualized on paper, transposing it to the software (**Figure 7**).

In **Figure 7**, we can observe that the student's partial construction shows that his *conjectural intuitions* still do not point to paths for *anticipatory intuitions*, because of the way the construction course took place, it was not possible to visualize the parabola. The lines that connect the point F outside the line and the other points, apparently were created to represent what would be the folds on the paper. Thus, the student stated again that it would be a parabola, but that he had difficulties in reproducing it in GeoGebra environment, because he "*did not remember the subject well*". In this sense, one of the didactic variables foreseen in the *a priori* analysis occurred: the student's previous knowledge was still insufficient for a global development of the solution, which also confers validity and relevance to this study.

The identification of difficulties inherent to the understanding of a mathematical object is of great influence to guide the teacher's work, with regard to the structuring of a *milieu* that provides the minimum conditions for the apprehension and construction of new knowledge to occur. In this sense, Brousseau (1976) explains the importance of understanding the notion of obstacle in mathematics teaching for the construction of meaning by the student, stating that:

> We will admit, therefore, that the constitution of meaning, as we understand it, implies a constant interaction of the student with problematic situations, a dialectical interaction (because the subject anticipates, finalizes his actions), where he engages previous knowledge, revises, modifies, complements, or rejects them to form new designs. The main objective of didactics is precisely to study the conditions that the situations or problems presented to the student must fulfill in order to favor the appearance, functioning and rejection of these conceptions [...] We can deduce from this discontinuous pattern of acquisitions that the informational characteristics of these situations must also vary by leaps and bounds. (Brousseau, 1976, p. 104, translated by the authors).

In the *situation of validation*, when trying to answer the question in item b: "What roles do the line r and the point F curve described play?", the student stated, according to the audio transcription record of the meeting:

> It's as if the line *r* were the *x* axis and the point *F* any point, but when I draw the lines, it seems that the *F* is the vertex of a parabola, in this case the minimum point. And the r can be the *x*-axis or a line parallel to the *x*-axis. I know that it is supposed to be a parabola, that the parabola is concave up or down depending on *a*, *b*, and *c*, and in this case, in this drawing here in GeoGebra, it was supposed to be upwards, but I cannot draw it the curve.

Note that the student realizes that the curve is a parabola but cannot describe it more accurately on paper or reproduce its outline with the help of the software. When asked by the researcher about his knowledge about parabolas, the participant stated that he had studied them only in high school, but that he never got to approach them during graduation. Then, when asked about his knowledge of conics, he said he knew what it was, but had never studied them.

We noticed in his speech that he does not use terms such as *focus* or *directrix*, which are basic elements in the geometric construction of a parabola. However, based on his *affirmative* and *conjectural* strategies and intuitive manifestations, the student was able to perceive the existence of the curve through his sketches.

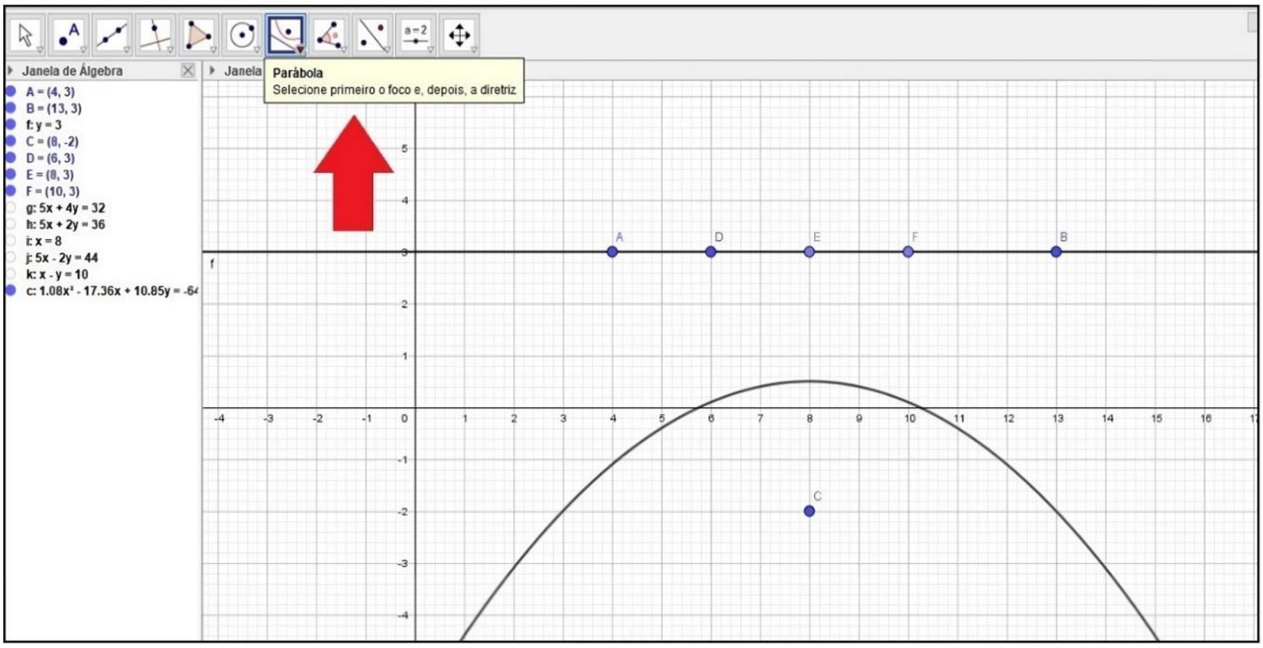

Figure 8. Parabola built in the pre-experiment (Source: Field study, 2022)

After this discussion, the teacher-researcher presented the institutionalization from Lima (2014, p. 115), already presented in the *a priori* analysis of this work, clarifying the concept of parabola as a geometric place and demonstrating the technique for its construction with folds. In addition, he presented his construction in GeoGebra (see **Figure 4**).

RESULTS AND DISCUSSION

4th Phase: Analysis *A Posteriori* and Validation

After dialoguing with the teacher-researcher and understanding the concept of parabola presented in the institutionalization, the student returned to the construction in GeoGebra and stated that the line *r* would be the directrix and the point *F*, outside the line, would be the focus. When exploring the guides and looking for a tool that would help him to draw the parabola, the student found the parabola tool and, observing the orientation provided by GeoGebra "*select the focus first, then the directrix*" (**Figure 8**), clicked on the point *C* and the line *f*, finding:

Note that the previously drawn lines were then hidden by the student, for a better visualization of the parabola. Subsequently, as instructed in the didactic contract, the student saved the .ggb file of his construction and the computer screen recording video and delivered all the written records, aware that these materials would be the basis for analysis of this pilot experiment, nodding once more its use.

That said, we analyze each of the points listed above for the continuity of the research and development of the other didactic situations of the master's research in the pre-established molds. We verified that the didactic situation was clear to the participant, despite the initial misconception of interpretation, with regard to the size of the line to be drawn. We also consider that its level of difficulty was acceptable, given the fact that the participant did not have in-depth knowledge on the subject, enabling the development of TDS in a well-structured way.

The participant's knowledge of GeoGebra, although basic, was sufficient to perceive its intuitive manifestations. Even if the participant did not reach the final solution (only after institutionalization), what hindered his resolution process was the lack of prior knowledge about the subject and not his skills with the software. In addition, the time for the development of the didactic situation and the resources provided were sufficient. The recording of the computer screen and the .ggb file, the written materials and the audio recordings and photographs also provided us with important data about the feasibility of the research, as we were able to perceive intuitive manifestations during the process of building the solution.

Among the didactic obstacles that can be identified, we perceive the limitation in the previous knowledge about the parabola restricted only to the topic of quadratic functions in which the subject associates the parabola in a natural/intuitive way to the graph of a function of the type $f(x) = ax^2 + bx + c, \forall\, a \neq 0$. It is recommended a reflection and observation of these results in the official experiment of the master's thesis, because if this scenario lasts, we understand that it can limit the work of the mathematics teacher in teaching the topic of parabolas through the geometric and analytical prism, which consequently reverberates in student learning.

If this student intends to be a teacher, he needs to have this knowledge and adjust it to BNCC matrix, according to required from the mathematics teacher in his professional practice. BNCC reinforces the importance of the articulation between geometry and algebra. In BNCC document we can find one of the skills (code EM13MAT402 in the document list) that corroborate the pertinence of knowledge about 2nd degree polynomial functions and that can be extended to the study of parabolas:

> (EM13MAT402) Convert algebraic representations of 2nd degree polynomial functions to geometric representations in the Cartesian plane, distinguishing the cases in which a variable is directly proportional to the square of the other, resorting or not to algebra and dynamic geometry software or applications. (Ministry of Education of Brazil, 2018, p. 534).

Thus, when recognizing the value of this subject of study, it is also worth mentioning the relevance of working on this topic with teachers working in Basic Education. For this, BNCC suggests the use of technological resources through software or applications, as a way to facilitate teacher planning and streamline classes, promoting an environment conducive to student development (Ministry of Education of Brazil, 2018).

After the pilot experiment carried out, we followed the path of research and construction of the dissertation with other structured didactic situations, continuing this engineering.

CONCLUSIONS

The starting point for this work started from a problematization that observed, in a bibliographic survey, some gaps that permeate the initial formation of the mathematics teacher, with regard to the teaching of the parabolas. The recurrent model of traditional approach and the fragmentation of the subject, disconnected from reality and related topics, such as quadratic functions are an example of these pre-existing gaps found in this survey.

For an observation of reality and in order to devise strategies to contribute to the training of future mathematics teacher in teaching this topic, we rely on theoretical references from the field of didactics of French mathematics, such as Brousseau (1976, 1986, 1997, 2002) in which we use TDS, Almouloud (2007) and Artigue (1988), in the conception of DE as a research methodology.

Given the need to understand how the teacher in training brings the topic of parabolas as prior knowledge, from an epistemic point of view, as well as develops it (or supposedly would develop it in the locus of the classroom), we feel the need to understand the functioning of their mathematical reasoning when interacting with the topic in question. For this, we used another theoretical contribution, which were CIR, by the Romanian psychologist and researcher Efraim Fischbein (1987), being a theory for our empirical observation, with regard to the interpretation and recording of intuitive manifestations of the student's participation in this research. In complementarity, GeoGebra was the technological contribution capable of providing us with part of these empirical records.

In this way, we developed a DE to verify the possible didactic obstacles that cause difficulties in the way that the teacher in initial training understands and, in a future vision, would approach the parabola in the locus of the classroom. In order to better understand this panorama, we are anchored in TDS and in CIR, which structured the teaching session of this pilot experiment.

In the preliminary analysis, we outlined some epistemological and didactic aspects in the teaching of parabolas. In the *a priori* analysis, we structured a didactic situation for its teaching and developed it in a pilot experiment. In experimentation, we realized that the student knew the software in an elementary way. However, the didactic situation allowed an improvement in the use of many tools, as well as triggered geometric and algebraic perceptions through the manipulation and visualization of elements within the constructions. As a consequence, with the contribution of TDS, GeoGebra and the structure elaborated in the a priori analysis, we were able to capture moments in which intuitive manifestations occurred, in addition to providing a discussion environment to understand the existing gaps in the study of the parabola. TDS was fundamental in structuring the didactic session, with regard to interpreting intuition and the different levels of reasoning produced, recognizing them within the course of their dialectics.

The observation and collection of data took place through the voluntary participation of a student in initial training, who provided us with elements for correcting the routes of didactic situations, a posteriori analysis and validation, as well as conjectures for a future validation of the official experiment in the course master's degree.

We found, for now, the need to discuss the parabola in the classroom, articulating the geometric, algebraic, and analytical views, establishing a relation between the parabola as a conic section, graph of a quadratic function and geometric locus with the contribution of technology.

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