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MODESTUM

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Analyzing elementary students' access to cognitive-oriented positions in mathematics

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ABSTRACT

The effectiveness of group problem-solving in mathematics depends on the extent to which meaningful participation is distributed across all group members. One way to explore how participation is distributed within groups is by examining how students are positioned within group interactions. In this study, we explore the social instructional factors that cause elementary students to move in and out of positions that support cognitive engagement during collaborative problem-solving in mathematics. Using a case study analysis of three elementary students working in a group, we found five social instructional factors that caused students to move in and out of cognitive-oriented positions during group work in mathematics: (1) building an ally through common language, (2) physical access to the chalkboard and resources, (3) tone of voice, (4) teacher intervention, and (5) contestation from peers. The findings promote implications for effectively facilitating group work in mathematics.

Keywords: positioning theory, mathematics, group work, collaborative learning

INTRODUCTION

There has been an increasing emphasis on collaborative problem-solving in mathematics classrooms over the last three decades (Chan et al., 2018; National Council of Teachers of Mathematics [NCTM], 2000; Nieminen et al., 2022; OECD, 2017; Zhang et al., 2022). Collaborative problem-solving refers to an instructional practice, wherein students learn mathematics through working on complex problems in collaborative teams (e.g., Chan et al., 2018). The teacher acts as a facilitator under this approach, allowing groups to create unique strategies and providing support when needed. Shifts in mathematics pedagogy that privilege collaborative problem-solving have largely been influenced by a changing society (OECD, 2017). Whereas individual knowledge acquisition was appropriate for the 20th century workforce, the 21st century requires students to develop new and unique skills. Namely, collaborative problem-solving is becoming more important than ever in STEM, with research revealing that teams often promote better working outcomes (e.g., Kniffin & Hanks, 2018). In addition to a changing society, recent pedagogical shifts have been motivated by research revealing that collaborative problem-solving can support student learning and engagement in mathematics (e.g., Boaler & Staples, 2008; Capar & Tarim, 2015). For all such reasons, policymakers, researchers, and practitioners are increasingly advocating for collaborative problem-solving in mathematics classrooms.

While collaborative problem-solving *can* support student learning, it often does not lead to intended outcomes (Barron, 2003; Kotsopoulos, 2010; Sfard & Kieran, 2001; Wood & Kalinec, 2012). The effectiveness of collaborative problem-solving depends on the extent to which meaningful participation is distributed across all group members. Participation is meaningful when it benefits students' academic growth. Meaningful participation includes explaining ideas, listening to others' ideas, and genuinely contributing to problem-solving. Oftentimes, meaningful participation is inequitably distributed within groups in mathematics, with some students experiencing meaningful opportunities to participate and others having few opportunities to cognitively participate (Bishop, 2012; Esmonde & Langer-Osuna, 2013; Wood, 2013; Wood & Kalinec, 2012). Such inequitable participation supports the learning of some students and limits the learning of others. This is especially troubling since collaboration is often promoted as a strategy for increasing equity in mathematics classrooms (e.g., Esmonde, 2009; Weissglass, 2000).

One way to explore how participation is distributed within groups is by examining how students are positioned within group interactions (e.g., DeJarnette & González, 2015; Esmonde, 2009; van Langenhove & Harré, 1999). Positioning theory, first developed by van Langenhove and Harré (1999), is an analytic and conceptual lens for exploring the positions that people play in an interaction. Students position themselves and others in group interactions in a variety of ways (e.g., leader, follower, note-taker), and these positions influence who has access to meaningful participation (Esmonde, 2009). Many researchers have examined students' participation in groups through the lens of positioning theory (e.g., Campbell & Hodges, 2020; DeJarnette &

González, 2015; Kotsopoulos, 2014; Tait-McCutcheon & Loveridge, 2016). However, there is much to learn about the social instructional factors that support or limit students' access to meaningful positions.

In this study, we explore the social instructional factors that cause elementary students to move in and out of positions that support cognitive engagement during collaborative problem-solving. Social instructional factors simply refer to the social and instructional aspects that influence group work, such as peer interaction, teacher interventions, and so forth. Exploring elementary students' positioning is vital because these students are in the early stages of learning how to work with and learn from their peers. Elementary students need to develop productive group habits so that they will continue this trajectory throughout schooling. Thus, this study seeks to uncover some of the social instructional factors that support elementary students to work constructively in groups.

Drawing from a larger research project that examined collaborative problem-solving with 48 elementary students, we use a case study design to explore the position shifts of one group of sixth-grade learners while working on a complex task. The case study analysis provides rich insight into how meaningful positions are "won," as well as the role that the social instructional context plays in students' access to meaningful participation. Below, we describe the study's framework, which provides a conceptual lens for the case study analysis.

Positioning Theory

Positioning theory is a theoretical framework that explains how people interact with one another and negotiate their rights and obligations. It was first introduced by van Langenhove and Harré (1999). van Langenhove and Harré (1999) suggested that when people interact, there are three constructs at play: storylines, positions, and actions-acts. The storyline refers to the narrative that is being portrayed by individuals within a conversation. The positions refer to the parts, or roles, that individuals play within a conversation. Actions refer to the utterances or gestures that the individuals use in a storyline, and acts refer to the meanings that actions convey. These three constructs allow researchers to make sense of how people behave in social interactions and how power is distributed.

Positioning theory is widely used as an analytic lens for exploring how students negotiate their rights and obligations within group work in mathematics. One can imagine several different storylines playing out during a group interaction in mathematics. For instance, some groups exhibit a *follow the leader* storyline, wherein one student leads the group and others follow (Campbell & Hodges, 2020). In this storyline, one student assumes the position of "leader," while other students assume positions of "follower." The leader gains authority within the group based on certain actions, such as using a dominating voice or providing multiple intellectual contributions. Followers, on the other hand, are subjected to the leader's authority. Positions are fluid in the sense that students may assume multiple positions over time during group work (Wagner & Herbel-Eisnenmann, 2009). Students positioned as a leader at the beginning of an interaction may or may not retain that position over time, depending on how and whether students contest their initial positions. Positions can also be considered from multiple vantage points (Wagner & Herbel-Eisnenmann, 2009). An outside observer (e.g., researcher) may construct a different storyline (and positions) than students within the interaction would perceive themselves. Therefore, it is important to describe the vantage point by which positions are described and analyzed.

To frame our analysis of student positioning in this study, we rely on Barnes' (2004) classification of the different types of positions that students assume during group work in mathematics. **Table 1** describes the 14 different positions that Barnes (2004) identified. Five positions relate to the group's organization and cohesion: manager, helper, facilitator, humorist, and spokesperson. For instance, managers organize the group by inviting ideas and providing suggestions about how the group should organize itself. Four positions relate to cognitive aspects of the group's collaboration: expert, outside expert, critic, collaborator, and in need of help. For instance, experts retain authority within the group to determine what is mathematically valid, and they often provide multiple intellectual contributions. Four positions relate to being distracted: entertainer, audience, networker, and outsider. For instance, entertainers tell jokes and distract the group from the primary objectives of the task.

Table 1. Barnes' (2004) positions

Position	Description
Group organization & cohesion	
Manager	Manages group work by giving orders and providing suggestions to group members
Helper	Provides help to other group members by receiving directions and doing mundane tasks
Facilitator	Helps the group function by resolving conflict and making sure all are involved
Humorist	Provides brief humorous moments that keeps the group's interaction light
Spokesperson	Speaks on behalf of the group to the teacher
Cognitive aspects of collaboration	
Expert	Decides how group will solve problem, holds mathematical authority within group, & other group members come to this person for help
Outside expert	Provides expertise from outside of classroom that can support group in understanding task & making progress
Critic	Criticizes solution proposals, seeks alternative solution proposals, and points out flaws
Collaborator	Works collaboratively with others to solve problems; actively engages in discussion toward solving the problem
In need of help	Asks for help from other group members or receives help from other group members without asking
Being distracted	
Entertainer	Entertains the group by telling jokes, gossiping, or engaging in off-task talk
Audience	Provides an audience for the entertainer; listens to off-task talk and laughs at or with entertainer
Networker	Engages with or listens to talk from other groups
Outsider	Does not participate with group either because they are ignored or because they choose not to participate

For participation to be meaningfully and equitably distributed in groups, all students need opportunities to assume cognitive-oriented positions (expert, outside expert, critic, collaborator, and in need of help). Not only do students need access to these cognitive-oriented positions, but they also need to fluidly move in and out of these positions (Barnes, 2004). Students need to share access to the position of expert. If one student maintains the position of expert throughout group work, then the group will lose opportunities to hear a range of ideas. Furthermore, the non-experts will not have opportunities to meaningfully participate in mathematics. It is also productive for students to share positions of collaborator, critic, and in need of help because each of these cognitive-oriented positions support different aspects of learning in groups.

In this study, we are interested in how students move from cognitive-oriented positions (expert, outside expert, critic, collaborator, in need of help) to non-cognitive oriented positions and vice versa. We emphasize the cognitive-oriented positions in this study because these positions provide access for students to meaningfully participate in mathematics. Positions related to being distracted are clearly antithetical to the goals of group work, and while positions related to group organization and cohesion are vital for the success of group work, the present study centrally focuses on students' meaningful participation in the cognitive aspects of mathematics.

Researcher Positionality

Before discussing the research question and methods, it is important to discuss our positionality as authors. Author 1 is a teacher educator and formerly worked as a schoolteacher. He is a white male from North America. Author 2 is an elementary school teacher and graduate research assistant. She is a white female from North America. This study was conducted in a high-poverty school with diverse characteristics in Eastern Canada. Author 1's and Author 2's personal assumptions and biases undoubtedly influenced findings of this study. We encourage reader to criticize our interpretations and read findings in light of our positionality.

Research Question

The research question motivating this study is: What are the social instructional factors that cause elementary students to move in and out of cognitive-oriented positions during group work in mathematics?

METHOD

To answer the research question, we employed a multiple case study design, which is a variant of the traditional case study design. In a traditional case study, researchers analyze and describe a bounded system (e.g., person, classroom, group, etc.) (e.g., Merriam, 1998). In a multiple case study design, researchers examine and describe multiple bounded systems (Yin, 2009). The bounded systems in our study are three sixth grade students (to be described shortly). While scholars use multiple case study designs for many purposes, our purpose is to generate tentative theory. Examining a small sample, we seek to understand the social instructional factors that influenced the elementary students' access to cognitive-oriented positions. The purpose is not to pursue generalizability, but to provide a descriptive analysis that informs research and pedagogy and also paves the way for larger scale analyses in the future.

Introducing Cases

This article focuses on three cases that highlight social instructional factors that may support or limit students' access to cognitive-oriented positions. The participants were drawn from a larger research project exploring how elementary students worked in groups on engaging mathematics and science tasks. In the larger research project, video recorders captured elementary students' engagement as they worked in groups on non-routine mathematics tasks. We chose to examine the social processes of three sixth-grade students who exhibited multiple position shifts while working in a group together. These cases were ideal for our research purposes because it provided many opportunities to explore position shifts and analyze the underlying causes of those position shifts.

The focal cases included Arturo, a boy who is racialized as Asian, Gary, a boy who is racialized as White, and Donesha, a girl who is racialized as Black. These three students were working together in a larger group of seven students on the following task:

> Five people are standing on one side of a bridge. They want to cross the bridge. Without a flashlight, they cannot proceed. Only one flashlight is available. The battery in the flashlight has a remaining life of only 30 seconds. Only two people can go over the bridge at one time. The flashlight needs to be returned to the remaining persons. The five people take different times to cross the bridge. One takes one second to cross the bridge. The others take three seconds, six seconds, eight seconds, and 12 seconds. How can you get everyone across the bridge within 30 seconds before the battery on the flashlight runs out.

Students were placed into large groups so that they could model the problem with actual flashlights. However, the focal group chose not to model the problem, and instead, used a vertical chalkboard to write their problem-solving strategies. The three cases (Arturo, Gary, and Donesha) were the most influential students in the group.

Arturo was a very eager student who was clearly excited about the opportunity to work collaboratively on a mathematics problem. He was much shorter than the rest of his classmates, and he had a loud voice, often shouting over his peers to be heard. Donesha was similarly excited about problem-solving in a group. She maintained a joyful attitude, often laughing with others while remaining engaged in the task. Gary was a comedian of sorts, resorting to telling jokes to his classmates. He was a tall boy with a large personality to match. Gary moved in and out of productive participation, as will be illustrated shortly.

The broader context of this analysis is important to consider. The lesson analyzed herein was delivered by four preservice teachers who worked collaboratively to design and implement a lesson plan that required sixth-grade students to engage in mathematical problem-solving. The preservice teachers attended a university in Eastern Canada, and the exercise was a requirement of their mathematics methods course. The preservice teachers were given extensive support on their lessons before implementation, and they learned how to use questioning and scaffolding techniques that support student thinking within the methods course. The sixth-graders attended an elementary school in Eastern Canada. The sixth-grade class was not accustomed to working in groups on non-routine problems such as the bridge problem. For this reason, the class was quite active and excited when the preservice teachers introduced a mathematical problem that promoted collaborative problem-solving. There were few norms in place for collaborative work, and the preservice teachers found the experience overwhelming from a classroom management perspective. Nevertheless, the video recording revealed several productive moments of student collaboration and engagement in mathematics. For all such reasons, the present case studies provided ample opportunities to consider social instructional factors that contribute to position shifts while students work in groups on mathematics tasks.

Data Sources & Analytic Methods

The data for this study included a video recording of the focal group working together on the bridge problem. Our analysis included five phases. In the first phase, we watched the video several times to familiarize ourselves with the data (Powell et al., 2003). This process allowed us to gain a general understanding of how the group interacted and who held influence within the group. It was immediately obvious that the three focal cases (Arturo, Donesha, and Gary) were the most influential group members. As discussed above, we chose to examine the processes of the most influential group members because these group members exhibited multiple position shifts.

In the second phase of analysis, we tracked the case students' positions and preliminarily documented the social instructional factors that led to position shifts. Our analysis of student positioning was based on a researcher's perspective. That is, we identified students' positions based on our perspectives rather than taking into consideration students' perspectives on their own positioning. To track student positioning, we watched the video focusing on just one student at a time. Starting at the beginning of the video, we identified the student's position according to Barnes' (2004) classification system. Then, we charted the student's position shifts over time using an excel spreadsheet. We time stamped each position shift and wrote memos to justify our identification of each position. We determined that a student's position shifted if the change in position lasted ten seconds or longer. An example of our memos for Arturo are included in **Table 2**.

Table 2. Arturo position shifts

Position	Definition
Position shift 1	**Expert→outsider** (2:50): Student starts out doing all talking; he talks rather loudly, but his thinking is rather quick & surface-level. At 2:50, instructor enforces collaboration. He says that only one person can hold chalk. This takes chalk out of green shirts hand. At 3:30 he starts to get questioned by another team member. Questioner & another team member who is holding chalk start to physically box green shirt out of problem [go back for image]. At this point, other students start to ignore green shirt even though he is still talking loudly. Instructor asks for group strategy & says "just hold on, I can only hear one person at a time". Then, he allows red shirt girl to speak. Green shirt starts loudly saying "let me talk!" At 6:30 mark, green shirt is actually further away from board than any other group member.
Position shift 2	**Outsider→entertainer** (7:30): "My idea is that 12 stops in middle …" Gives non-purposeful solution proposal just for purpose of making students laugh.
Position shift 3	**Entertainer→collaborator** (8:00): Jessica physically makes room to allow Jadan back into mix. He's physically close to blackboard now. Important here is that Jessica physically made way for Jadan (purposefully). Jadan takes into consideration Donesha's idea that 12 person should go first. He adds to it saying, "and one should go back & forth." Then, he is further questioned by Donesha.
Position shift 4	**Collaborator→audience** (8:40): Gary makes a joke with a non-genuine solution proposal saying that they should be the 8 and 12 on a diet for a couple months. Jadan, along with the rest of the group erups in laughter.
Position shift 5	**Audience→entertainer** (9:10): Class is getting a bit loud, & teacher says, "OK, one voice, one voice … right here". Jadan takes this opportunity to build off of Gary's joke & creates a joke for himself–non-genuine solution proposal … & he says that he can use a "power bag" in his pocket.
Position shift 6	**Entertainer→collaborator** (10:10): Jadan starts collaborating with Jessica in a different language. Two talk about problem together.
Position shift 7	**Collaborator→outsider** (10:45): The two eventually get shut out of their collaboration as the board is erased by Donesha. They are moved away from "where things are happening" at the board. The people with the chalk are drawing at a new location. Jadan and Jessica become outsiders together as talk in their own language without being heard or listened to by others.

In the third phase of analysis, we watched the raw video data again focusing on focal position shifts to construct narratives. Focal position shifts refer to position shifts, wherein students moved in and out of cognitive-oriented positions. As we watched the video, pausing several times, we constructed narratives for each student that described the focal position shifts and the social instructional factors that contributed to each focal position shift. We considered the social (e.g., peer-to-peer interaction) and instructional environment (e.g., teacher intervention) as we constructed narratives. We continuously revised our narratives as we paused, re-winded, and replayed the videos. We continued in this process until we were satisfied[1] with the narratives and their depiction of the social instructional factors that contributed to focal position shifts. Thus, the narratives were created by continuously watching the raw video data.

[1] We purposely avoid the word "saturation" because we acknowledge that our perceptions and biases inform the analyses.

Next, in the fourth phase of analysis, we consolidated the narratives that we created for each student into one seamless narrative. We considered all focal position shifts for each student and created a single narrative that describes each position shift and simultaneously tells a story regarding how the group interacted.

Finally, in the fifth phase of analysis, we examined the narrative and identified/named the specific social instructional factors that contributed to focal position shifts for each student. To do this, we read through the narratives and categorized and named the social instructional factors until we reached a point of satisfaction (e.g., Nowell et al., 2017).

In what follows, we share the narrative that describes students' position shifts and the social instructional factors contributing to position shifts. Then, we explicitly identify the social instructional factors that we identified within the narratives.

FINDINGS

Narrative

The class started with a brief explanation of the task from preservice teachers. After the preservice teachers explained the task, each group was asked to stand at different corners of the classroom near a vertical chalkboard (which groups could use for problem-solving). After being separated into groups, the focal group was still somewhat confused about the task directions. Therefore, one of the preservice teachers huddled the group together and explained the task in more detail. After fielding questions from the group, he placed a hand on Gary's shoulder and stated,

> I think this, uh, fine young person has come up with uh–an interesting solution. So, [Gary] uh if you want to start that discussion, that sounds good.

Gary eagerly asked the preservice teacher,

> Can I use the chalk?

and accordingly grabbed a piece of chalk.

Meanwhile, while the group was organizing itself, Arturo started explaining his strategy to another student (Isa) in a common language that the two shared (different from English). After a short time, Donesha, Gary, and Arturo all walked to the chalkboard, and they stood in a line with Arturo in the middle. The rest of the group was positioned behind Donesha, Gary, and Arturo. Donesha and Gary both held pieces of chalk, and they towered over Arturo, who was much shorter. At this point, the students started vying for positioning.

Donesha started the conversation by pointing to the times that were written on the board (1 sec, 3 sec, 6 sec, 8 sec, and 12 sec) stating,

> But all of this equals to 30 seconds.

Arturo abruptly stole the piece of chalk from Donesha, and he started explaining his strategy to the whole group:

> If you make these two go first, then this is done. And then once this is done, you make this go, make this go across the bridge. Once this is done, this across the bridge. This done, this across the bridge.

As he was talking, Arturo drew lines to represent two people crossing the bridge at the same time (see **Figure 1**). Arturo solidified himself as **expert**, a cognitive-oriented position, with all group members listening to him. However, his strategy was not fully developed or coherent.

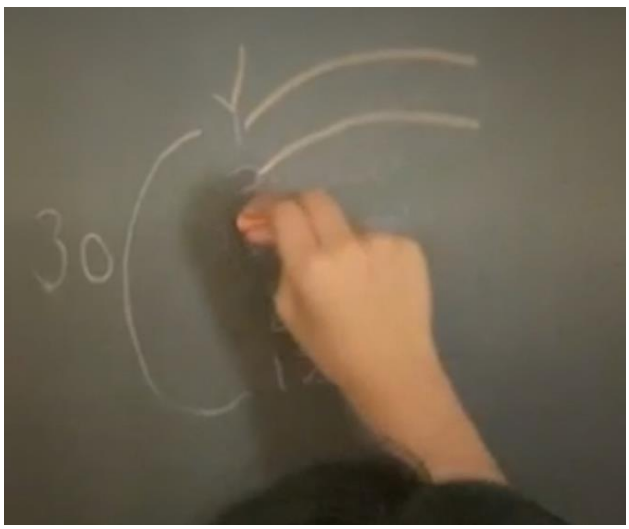

Figure 1. Arturo drawing on board (Source: Field Study)

As Arturo continued his explanation, Donesha exclaimed,

> Excuse me, you took my chalk!

This prompted the preservice teacher to enact a new rule with the group. He explained the new rule to the group:

> OK so guys, here's what I'm going to get you to do. Just so that everyone is working together, we use one piece of chalk, OK? The person who's got the chalk cannot say anything. He or she can only write what the other people are saying. How's that sound?

Donesha nominated Gary to be the person who held the chalk. Therefore, this new rule took the chalk out of Arturo's hands.

Even after relinquishing the chalk, Arturo continued to lead the group. He issued orders to Gary, telling him what he should write on the board. Gary, therefore, took on the position of **helper**, a non-cognitive-oriented position. His role was simply to scribe for Arturo. While Arturo's body was physically angled away from Donesha (toward Gary), Donesha was persistent in her bid to collaborate with the group. She loudly presented a potential issue with Arturo's approach, looking to other group members for comment.

At this point, Donesha's persistence was causing Arturo to lose influence within the group, as she took on the position of **critic**, a cognitive-oriented position. Donesha started to suggest other potential solution approaches to the group:

> What about one and two? One and three?

Arturo, indignant at Donesha's attempt to divert power, loudly exclaimed

> What about the rest?

This left Donesha obviously deflated, as she sighed and angled her body away from the board.

Meanwhile, Donesha and Gary started to communicate over the top of Arturo's head. The two students effectively "tuned out" Arturo. Furthermore, the two students started to physically move closer to one another, which forced Arturo to lose distance from the chalkboard. Donesha and Gary, therefore, were beginning to assume the positions of **collaborator** (a cognitive-oriented position), marking a focal position shift for Gary (**helper→collaborator**). Nevertheless, Arturo continued to shout his solution strategies to Donesha and Gary, and he continually forced his way back to the chalkboard.

The preservice teacher, realizing the rising tension, intervened to ask the group about their strategies. When Donesha and Arturo started to speak at the same time, the preservice teacher said,

> Just hold on, I can only hear one person at a time–Alright, you go [looking at Donesha].

As Donesha shared her idea with the preservice teacher, Arturo had clearly lost influence within the group. He stopped trying to shout his solution approaches and slowly moved away from the chalkboard, marking a position shift to **outsider** (a non-cognitive-oriented position). This concluded Arturo's first focal position shift (**expert→outsider**).

After the preservice teacher intervened, the group became a bit disjointed with several students talking at once. Sandra (another group member) insisted that the group turn their focus to Shelden, who she insisted "has it." Shelden preceded to share a brief solution proposal:

> You send one and three, and then they get across. And you throw the flashlight back.

After sharing his proposal, the preservice teacher intervened, telling the group that "throwing the flashlight back" is not allowed.

Meanwhile, as Shelden was talking, Gary became situated at the back of the group and was disengaged with the task. This concluded another focal position shift for Gary (**collaborator→outsider**). Gary never assumed another cognitive-oriented position after this position shift, though he used his later position of **entertainer** to pull other students away from cognitive-oriented positions (to be explained shortly).

Next, after Shelden's solution proposal, Donesha assumed the role of **expert** for the group. The group angled their bodies towards Donesha so that she was teaching from the board as the others looked at her. Meanwhile, Arturo and Isa started to talk to one another in their common language, standing a few feet away from the chalkboard. Following this short conversation, Isa patted Arturo on the back and opened a path for him to make his way to the chalkboard next to Donesha. At this point, Arturo stood directly between Donesha and Gary at the chalkboard. Donesha and Arturo engaged in the following exchange:

> **Donesha:** Gary thinks the six person should go first. I think the 12 person should go first.
>
> **Arturo:** Yeah, and then the one [second] person just goes back and forth.
>
> **Donesha:** But that would not make sense because we're supposed to get all of them [over].

During this exchange, Arturo was engaging as a **collaborator**, a cognitive-oriented position, by supporting Donesha with her solution strategy, which was another focal position shift for Arturo (**outsider→collaborator**). However, after this short exchange, Donesha and Arturo were interrupted by Gary.

Gary interrupted Donesha and Arturo to give a non-meaningful solution proposal:

> OK, guys, guys, guys … First, we put three, six, eight, and twelve on a diet for a couple of months …

Arturo, Donesha, and the rest of the group erupted in laughter in response to Gary's joke, as he positioned himself as an **entertainer**. Importantly, the preservice teacher also laughed at Gary's non-meaningful solution proposal, which further encouraged Arturo, Donesha, and the rest of the group to continue laughing. Arturo and Donesha both assumed the position of **audience**, as the preservice teacher attempted to bring the group back on task. This resulted in a focal position shift for Arturo (**collaborator→audience**) and Donesha (**expert→audience**).

For the sake of brevity and non-redundancy, we conclude our narrative here. Donesha and Arturo exhibited a few more focal position shifts, but they were caused by factors, which have already been discussed in detail (e.g., Gary's jokes, teacher interventions). Donesha moved in and out of the positions of **expert**, **audience**, and **collaborator,** and Arturo moved in and out of the positions of **collaborator**, **entertainer**, and **outsider** throughout the rest of the group's exchange. Below, we examine the social instructional factors that influenced the students' focal position shifts.

Social Instructional Factors

By examining key elements of each student's focal position shift, our analysis suggests five primary social instructional factors that influenced students' positions to and from cognitive-oriented positions:

(1) Building an ally through common language,

(2) Physical access to the chalkboard and resources,

(3) Tone of voice,

(4) Teacher intervention, and

(5) contestation from peers.

First, "building an ally through common language" was a social instructional factor that was unique to Arturo. Arturo moved into cognitive-oriented positions, in part, because he formed an allied relationship with Isa via communicating in a common language. Arturo became the group's expert at the beginning of the task, but only after he shared his initial ideas with Isa. Furthermore, Isa physically made a path for Arturo and collaborated with Arturo in their common language in subsequent exchanges. In many ways, having an ally who shared a common language was an asset to Arturo's positioning; however, speaking in a language different from English might have also contributed to Arturo's positioning in ways that were not explicit in the narrative. This social instructional factor may have been unique to this classroom context, as the school was culturally diverse.

Second, Arturo, Donesha, and Gary moved in and out of cognitive-oriented positions based on the extent to which they had physical access to the chalk and chalkboard. All three students held cognitive-oriented positions when they were close to the chalkboard. In contrast, the students held non-cognitive-oriented positions when they were far away from the chalkboard. Furthermore, having access to the physical resources (e.g., chalk) influenced students' positioning. For instance, Gary and Donesha both had access to the chalk at the beginning, which supported them in gaining influence. Arturo became an expert, in part, by stealing the chalk from Donesha. When he was forced to give the chalk up, he started to lose influence within the group. The specific classroom context may have contributed significantly to the identification of this social instructional factor. Students were standing up at a vertical surface, and therefore, there was not enough room for all students to stand near the chalkboard. Furthermore, the preservice teacher enacted a rule that only one student could hold the chalk at a time. This meant that there were limited resources available for the group.

Third, the students' tone of voice supported them to occupy cognitive-oriented positions. Donesha and Arturo loudly shared solution proposals and critiques. Speaking loudly seemed to support the students in "winning" positions of influence. It is possible that the students' tone of voice could have also contributed to losing influence. For instance, Donesha became visibly deflated when Arturo loudly argued with her ("What about the rest?"). This may have influenced Donesha to effectively ignore Arturo.

Fourth, teacher interventions influenced many of the position shifts. Arturo, in particular, tended to lose influence within the group when the preservice teacher intervened. When the preservice teacher enacted a new rule about holding the chalk, the rule supported Gary in moving toward a cognitive-oriented position and simultaneously caused Arturo to move out of his Expert position. Furthermore, when the preservice teacher privileged Donesha's response ("Just hold on, I can only hear one person at a time–Alright, you go [looking at Donesha]"), it both supported Donesha's positioning and caused Arturo to lose influence within the group.

Fifth, students' position shifts were largely influenced by the extent to which students' ideas were contested by peers. Arturo's positioning was influenced by contestation from Donesha. Donesha questioned Arturo's solution strategies a few times (e.g., "But that would not make sense because we're supposed to get all of them [over]"), which seemed to contribute to Arturo's shift toward non-cognitive-oriented positions.

DISCUSSION

In this paper, we explored social instructional factors that influenced students' position shifts to and from cognitive-oriented positions. Analyzing a case study of three students (Arturo, Gary, and Donesha), we found five social instructional factors that influenced shifts in position:

(1) building an ally through common language,

(2) physical access to the chalkboard and resources,

(3) tone of voice,

(4) teacher intervention, and

(5) contestation from peers.

In what follows, we situate these findings in the context of prior literature and describe implications for research and practice.

The first contribution of our study is theoretical. Barnes (2004) constructed a comprehensive list of positions that students assume while working in groups in mathematics. To our knowledge, Barnes' (2004) framework has not been used as a framework of analysis. We used the framework as a conceptual lens to understand how meaningful participation was distributed across group members. Our study provides a useful theoretical template for leveraging Barnes' (2004) framework to explore student positioning and its influence on learning.

In addition to theoretical contributions, our study provides empirical insight into factors that influence students' positions in groups and consequently, learning. One of the more interesting findings was that one student (Arturo) was able to "win" cognitive-oriented positions by first sharing strategies in a common language with another student (Isa). It is unclear why "building an ally through common language" supported Arturo's position shifts; however, we hypothesize that Arturo may have felt more comfortable sharing his ideas in English after first sharing them in his first language. This interpretation seems to softly align with Barwell's (2005) study with English as additional language (EAL) students. Barwell (2005) found that nine-10 year old students were able to negotiate social relationships and create mutual understanding by sharing narrative experiences. Thus, "building an ally through common language" might support students to assume cognitive-oriented positions by improving their access and comfortability to share solution approaches.

Engle et al.'s (2014) framework provides a useful lens for interpreting other empirical findings. Engle et al. (2014) suggested that there are four primary factors, which affect who holds influence within groups:

(1) the negotiated merit of each participant's contributions and each participant's,

(2) degree of intellectual authority,

(3) access to the conversational floor, and

(4) degree of spatial privilege.

We found that students tended to move out of cognitive-oriented positions when they moved away from the chalkboard and lost access to physical resources (e.g., chalk). In contrast, we found that students moved into cognitive-oriented positions when they used a loud tone of voice. This aligns with Engle et al.'s (2014) framework, which suggests that students' influence in groups depends on their spatial privilege (i.e., students' body orientations in relation to others) and access to the conversational floor (i.e., ability to share thoughts with others). Furthermore, the extent to which students received contestation from peers influenced their access to cognitive-oriented positions, which aligns with Engle et al.'s (2014) suggestion that the negotiated merit of each participant's contribution affects their influence within groups. Lastly, teacher interventions both supported and limited students' abilities to access cognitive-oriented positions.

The empirical findings complement and add specificity Engle et al.'s (2014) model of influence in persuasive discussions. Whereas Engle et al. (2014) developed a broad theory regarding how students gain influence in groups, our study described specific social instructional factors that support or inhibit students' cognitive access to mathematics while working in groups. In this way, our case study approach contributes to the body of knowledge on how meaningful participation is distributed in groups.

Expanding beyond Engle et al.'s (2014) framework, we hypothesize causal links between the identified social instructional factors and students' abilities to assume cognitive-oriented positions. Each of the social instructional factors either supported or limited students' capacities to be *heard* by their classmates and feel competent in the classroom community. For instance, students who were contested by their peers likely felt less competent (and possibly embarrassed), causing them to move away from cognitive-oriented positions. In contrast, students who dominated with a loud voice had opportunities to be heard, even if forcefully, by their classmates. Future research might tease out how and whether these causal links exist.

Implications

The findings promote several implications for future research and practice. First, in relation to research, it may be beneficial for researchers to expand on this study to explore other social instructional factors that affect students' access to cognitive-oriented positions. As stated in the method section, our purpose was not to pursue generalizability, but to provide a descriptive analysis that paves the way for larger scale analyses in the future. Our study typified several unique factors that support or limit students' learning in groups, but future research should build on this work to identify other specific factors that influence students' access to meaningful positions in group work. Adding specificity to the existing broad frameworks of student positioning (e.g., Engle et al., 2014) can support practitioners and researchers in identifying teaching moves that promote equitable participation during group work in mathematics.

In relation to practice, our study provides implications for how teachers prepare students for collaborative work. It is important that all students feel heard and competent while doing collaborative work. Teachers can support students by manipulating the social instructional factors identified in this study. For instance, it may be productive for teachers to strategically pair EAL students in groups to support their capacity to hold cognitive-oriented positions. Such pairing might give EAL students confidence and comfortability to share their ideas with the whole group (e.g., Arturo). Furthermore, teachers can prepare students for collaborative work by creating classroom norms that promote social instructional factors that lead to equitable and meaningful participation. Classroom norms such as "listening respectfully to others" may support students' equitable participation if these norms are consistently discussed and reinforced.

The findings of our study complement prior research, which reveals that collaborative learning is a highly intricate process (e.g., Dekker & Elshout-Mohr, 2004; Franke et al., 2007). Teachers must simultaneously ask meaningful questions, ensure that groups are on task, provide hints and extensions, and support groups in meeting learning objectives. Teachers need support in navigating these intricacies, and researchers need to identify frameworks, which simplify the processes of facilitating group work. Liljedahl's (2020) framework, "building thinking classrooms," is one such model, which provides specific and detailed direction on how teachers might successfully facilitate group work.

In addition to implications for teachers, this study has implications for teacher education. Teacher educators might consider educating teachers about social instructional factors and their role in influencing students' positioning. Collaborative norms are already a focus of teacher education, but explicitly focusing on social instructional factors might support teachers in developing productive teaching practices.

CONCLUSIONS & LIMITATIONS

While our study provided insight into factors, which affect group positioning and learning, the study did not come without limitations. First, the study was conducted in a specific context (Eastern Canada) with a specific group of students (6^{th} grade students). Some of the findings were unique to our research context. For instance, one of the social instructional factors (building an ally through common language) would not materialize in other settings. It would be prudent for future research to consider a larger sample size. Nevertheless, our study provided an important entry point for future research. Second, the present study did not draw causal links between social instructional factors and position shifts. We hypothesized causal links, but future research should empirically discern why and how certain social instructional factors might lead to position shifts. Third, the thematic findings were necessarily influenced by our personal biases. Other researchers with different positionalities might have drawn different conclusions.

In conclusion, it is vital that all students have opportunities to participate in meaningful ways while working collaboratively on mathematics. To encourage such equitable and meaningful participation, teachers and researchers might focus on supportive social instructional factors. A focus on social instructional factors and their influence on students' positioning can propel the field toward improved collaboration and learning in K-12 mathematics.

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