International Electronic Journal of Mathematics Education
2022, 17(4), em0717
e-ISSN: 1306-3030
https://www.iejme.com

MODESTUM

Research Article **OPEN ACCESS**

The influence of practical illustrations on the meaning and operation of fractions in sixth grade students, Kosovo-curricula

Saranda Kamberi [1], Ismet Latifi [1], Shpetim Rexhepi [1*], Egzona Iseni [1]

[1] Department of Mathematics, Mother Teresa University, Skopje, NORTH MACEDONIA
*Corresponding Author: shpetim.rexhepi@unt.edu.mk

Citation: Kamberi, S., Latifi, I., Rexhepi, S., & Iseni, E. (2022). The influence of practical illustrations on the meaning and operation of fractions in sixth grade students, Kosovo-curricula. *International Electronic Journal of Mathematics Education, 17*(4), em0717. https://doi.org/10.29333/iejme/12517

ARTICLE INFO

Received: 22 Jul. 2022
Accepted: 04 Oct. 2022

ABSTRACT

In this paper, we have tried to clearly explain the meaning of fractions and operations with fractions. Also, we have tried to illustrate with some examples how to apply the rules of operations with fractional numbers, giving different practical techniques through different figures, visualization methods, and concretizing problems. The paper presents with examples of the most common mistakes made by students, giving suggestions for their avoidance, as well as through a questionnaire, which considered the survey of 60 students of grade 6 of school, "Elena Gjika" City Prishtina, Kosovo, with the help of the statistical test we have concluded that practical techniques such as figures, visualizations, and video lessons have a close dependence on the meaning and operation of fractions.

Keywords: fractions, learning fractions, problems, misunderstandings

INTRODUCTION

Fractions are one of the basic concepts in mathematics and as such they are valued not only for their importance in this field but also for other human fields and activities. Although we find them widely included in curricula and textbooks, again everywhere in the world their understanding and learning is one of the key problems for students, teachers, and parents, therefore we have found it necessary to research more about the meaning of fractions straight as well as operations with them. This problem is also encountered in countries with a tradition in learning mathematics. There are many different opinions on when it is time for students to learn fractions, during the research we came across various recommendations that are said in some countries are taught earlier while in the country where we are, in Kosovo, are taught in the 5th grade, and in the 6th grade, where the rules and operations with fractions are elaborated in a more extensive way. Fractions are important for students because they can be applied in everyday life throughout life. While working with students we encountered difficulties in learning and understanding fractions, so we found it appropriate to research more about this issue. One of the topic areas that learners display numerous errors in in Mathematics is fractions (Lemonidis & Pilianidis, 2020; Makhubele, 2021; Reinholda et al., 2020). Research indicates that learners have difficulties with operations involving the addition, subtraction, division, and multiplication fractions (Baidoo, 2019; Ubah & Bansilal, 2018).

Many studies have revealed that fraction is a complicated mathematics topic for students. Students struggle to solve problems including comparison and addition of fractions correctly. There are three types of errors in solving mathematics problems, which are factual error, procedural error, and computational error (Lestiana et al., 2017). The causes of student mistakes are the following:

1. Implementation of laws and strategies that are not relevant, one example of the application of the law and the evidence irrelevant strategy that students use the concept of the sum fractions in solving division operations on fractions.
2. Be short of understanding of the basic concepts of multiplication and division of whole numbers with fractions.
3. Be short of mastery of prerequisite skills on fractions (Maelasari & Jupri, 2017).

To date, a large number of studies have engaged paper-and-pencil test to measure students' instrumental understanding about fractions, but few have sought to learn about children's fraction sense. (Kor et al., 2018). Fractions are numbers and like common sense, fraction sense is an abstract concept. The definition of fraction sense comes in several versions. McNamara and Shaughnessy (2015) contended that fraction sense is an essential component to students' success with fraction operations. Prediger (2013) notes that although it is important to connect different models for fractions as a way of developing conceptual understanding of a fraction, establishing this connection is not the only means of effective instruction. Among other things,

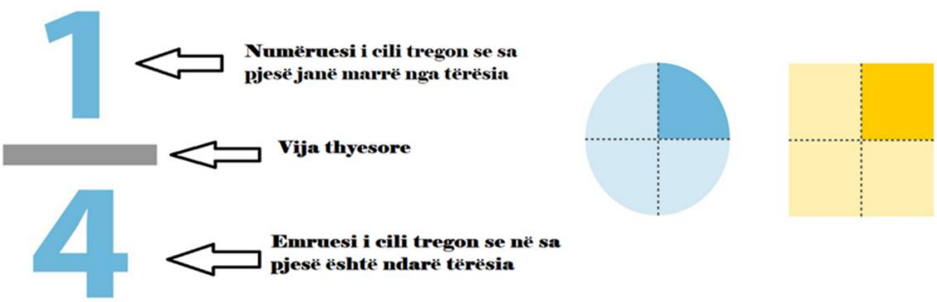

Figure 1. Illustration of the fraction $\frac{1}{4}$

Prediger (2013) examined different strategies of connecting different models for fractions when the learning process is aimed at generating visual representations of fractions. This paper aims to contribute to research that investigates in-depth pupils' fraction sense in solving questions about fractions in particular questions about fraction "concept", "representation", and "operations" using different teaching interactive methods, as practical and illustrations methods. There are many methods to explain and illustrate by concretizing techniques using different figures and visualization methods, in order students can more easily understand fractions and operations with fractions and make the lesson and explanation more attractive.

The technology and the time we are living make this easier because we can also use the resources from the internet for additional lessons and to see various illustrations and videos that explain the meaning of fractions, operations with them in more understandable and attractive way for the students and help them a lot to obeying the rules and avoiding the mistakes with operations with fractions. Except illustrations important role in understanding the fractions plays homework, classroom commitment, extra hours, and extra exercises (Aliu et al., 2021).

The illustrations have greater impact compared with using of the program GeoGebra and Mathematica for solving exercises by students then solving exercises on the whiteboard, since students of lower classes hardly find themselves when working with GeoGebra, compared to higher classes (Mollakuqe et al., 2021).

METHODOLOGY

In this study, it was aimed to determine the common mistakes and misunderstandings of 6th grade elementary school students in fractions and operations with fractions. For this purpose, the case study model, which is a qualitative research design, was used in the research. The case study is defined as the in-depth exploration of a specific system (e.g., an activity, event, process, or individuals) with the help of different data collection tools. Thus, to be able to deeply examine the misconceptions of primary school students in fractions, the case study model was chosen for this research.

Participant

In the paper, we use the table of specifics, where the main purpose was to discover the problems and difficulties that students have. For this reason, this study was conducted in a state level with some 6th grade students.

Data Collection

To correct the mistakes and misunderstandings of the students we have given the students some exercises for the 6th grade. Before these types of exercises were formed only the existing misunderstandings were defined. We have also created a questionnaire to see how much students understand fractions and how important it is to use concretizing tools to understand them.

Data Analysis

In the analysis of the research data, a contingency table was used. We answered the answers to the questionnaire questions as variables *X* and *Y*, where the first variable has to do with using the figures and concretizing tools in understanding the concept of fractions and the second variable has to do with that if the students have understood the fractions and operations with them. According to the answers made by the students, we conclude that these variables are dependent to each other.

UNDERSTANDING THE FRACTIONS

The notion of fraction for someone can be a new term for him and it is always suggested that once we get acquainted with that term and know what we call fraction in our case.

We denote the quotient of two natural numbers a and b by $\frac{a}{b}$ and call it a fraction. The number a is called the fraction nominator, while b is called the fraction denominator and it is denoted as: $\{\frac{a}{b}: a, b \in Z; b \neq 0\}$.

Figure 1 depicts the fraction $\frac{1}{4}$, where 1 notes the numerator that shows how many parts were taken from the whole, – denotes fraction line, and 4 notes the denominator that shows how many parts are separated from the whole.

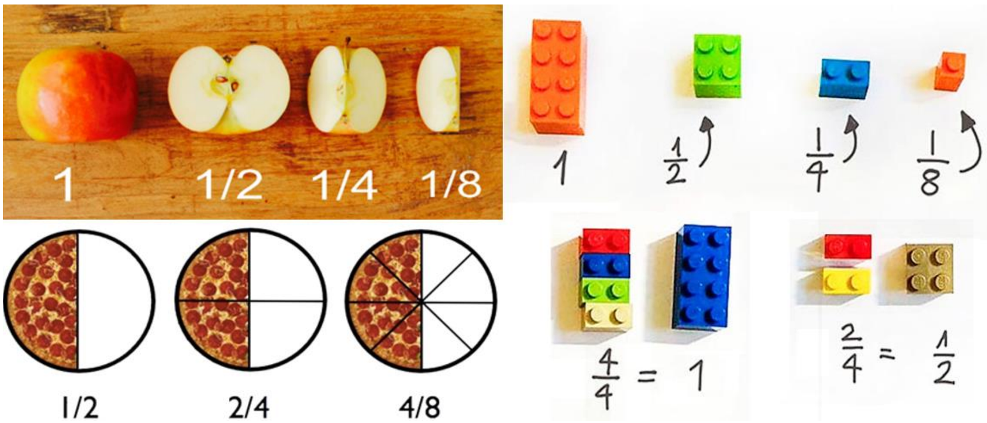

Figure 2. Representation and illustration of some fractions through different pictures

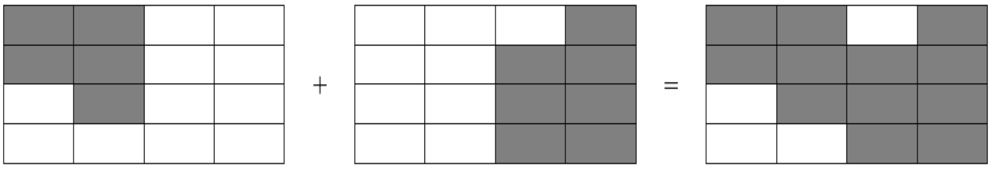

Figure 3. Addition of two fractions by illustration

A common fraction like $\frac{1}{2}, \frac{8}{5}$, or $\frac{3}{4}$ consists of the numerator (above the line) and the denominator (below the line). The numerator represents a number of equal parts while the denominator shows how many such parts make up the whole. For example, in the fraction $\frac{3}{4}$, the numerator 3 tells us that the fraction represents equal parts, while the denominator 4 tells us that 4 parts make a whole. **Figure 2** shows some fractions through different pictures.

Whenever we make the connection between theoretical and practical problems based on real life, they become more understandable. So, we can also explain the fractions using different things or illustrations in order to make them easier to understand and in our case as an example we have taken food like apples, pizza, and cake. It is worth mentioning that any natural number is a fractional number because it can be written like this $3 = \frac{3}{1}$. So, any natural number can be written as a fraction with denominator 1 (Zejnullahu et al., 2004).

ADDITION AND SUBTRACTION OF FRACTIONS

In the following, to make the addition and the subtraction of fractions clearer, we will take some examples where we will illustrate how two fractions should be added and subtracted regularly because at first most students encounter problems and misunderstandings while performing fractional operations.

Example 1

Two sisters Arta and Blerta buy a chocolate together. Arta ate $\frac{5}{16}$ of the chocolate, while Blerta ate $\frac{7}{16}$ of it.

1. How many pieces of chocolate did the two sisters eat?
2. Which of the sisters ate the most and how much?

To make the following actions clearer, we are representing **Figure 3**.

1. $\frac{5}{16} + \frac{7}{16} =$? After the chocolate was divided into 16 equal parts, the two sisters ate 5+7=12 parts of chocolate. So, they ate $\frac{5}{16} + \frac{7}{16} = \frac{5+7}{16} = \frac{12}{16}$ of the chocolate, as illustrated in **Figure 3**.
   There are several cases when students misunderstand the rule of addition of two fractions. In our example the most common mistake by the students is: $\frac{5}{16} + \frac{7}{16} = \frac{5+7}{16+16} = \frac{12}{32}$ (?)
   This way of solving the exercise happens very often in students at the beginning of learning the lessons with fractions and operations with them.
2. $\frac{7}{16} - \frac{5}{16} =$? Blerta ate seven pieces, while Arta ate five pieces of the chocolate. Then it is clear that Blerta ate two parts more than Arta (7-5=2). So, Blerta ate more: $\frac{7}{16} - \frac{5}{16} = \frac{7-5}{16} = \frac{2}{16}$ of the chocolate.
   Misunderstanding as in addition also occurs with subtraction of fractions:
   $\frac{7}{16} - \frac{5}{16} = \frac{7-5}{16-16} = \frac{2}{0}$ (?), which is impossible.

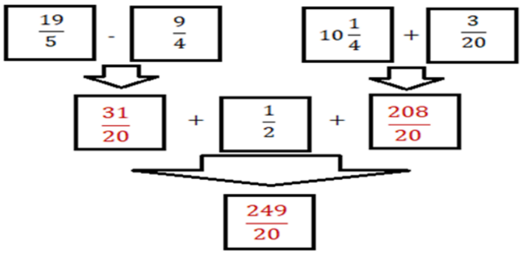

Figure 4. Illustration of example 3

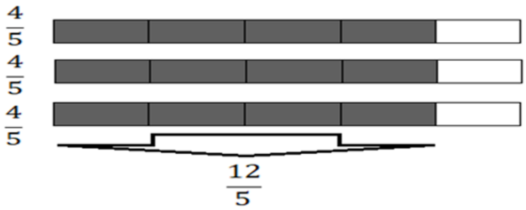

Figure 5. Multiplication of two fractions by illustration

1° fractions with equal denominators are added by adding the numerators, while the common denominator is described. Symbolically: $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$.
2° fractions with the same denominator are subtracted, when we subtract the numerator of the denominator from the numerator of the denominator, that difference is taken as the numerator of the fraction with the same denominator. Symbolically: $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$ $(a>b)$.
To add (subtract) fractions with different denominators, we first bring them into equal denominators, then add (subtract) them as fractions with equal denominators.

Example 2

A pool is filled by two pipes. The first pipe in one hour fills $\frac{1}{6}$ of the pool, while the second pipe in one hour fills $\frac{2}{9}$ of the pool. Which part of the pool is filled by both pipes within an hour?

To solve the exercise, we need to perform the addition $\frac{1}{6}+\frac{2}{9}$. To add the fractions $\frac{1}{6}$ and $\frac{2}{9}$, we must first convert these fractions into fractions with common denominators. Since the least common multiple (6, 9)=18, then return both fractions with denominator 18 by extending them by the appropriate number: $\frac{1\cdot 3}{6\cdot 3}=\frac{3}{18}$ and $\frac{2\cdot 2}{9\cdot 2}=\frac{4}{18}$. From where: $\frac{1}{6}+\frac{2}{9}=\frac{3}{18}+\frac{4}{18}=\frac{7}{18}$.

In practice this will be done, as follows: $\frac{1}{6}+\frac{2}{9}=\frac{1\cdot 3}{6\cdot 3}+\frac{2\cdot 2}{9\cdot 2}=\frac{3}{18}+\frac{4}{18}=\frac{3+4}{18}=\frac{7}{18}$.

Even when we have different denominators, there is often a misunderstanding by students about the adition: $\frac{1}{6}+\frac{2}{9}=\frac{1+2}{6+9}=\frac{3}{15}$ $(?)$

Example 3

Complete the missing values in **Figure 4**.
$\frac{19}{5}-\frac{9}{4}=\frac{19\cdot 4-9\cdot 5}{20}=\frac{76-45}{20}=\frac{31}{20}$.
$10\frac{1}{4}+\frac{3}{20}=\frac{4\cdot 10+1}{4}+\frac{3}{20}=\frac{41}{4}+\frac{3}{20}=\frac{41\cdot 5+3\cdot 1}{20}=\frac{205+3}{20}=\frac{208}{20}$.
$\frac{31}{20}+\frac{208}{20}=\frac{31+208}{20}=\frac{239}{20}, \frac{239}{20}+\frac{1}{2}=\frac{239\cdot 1+1\cdot 10}{20}=\frac{239+10}{20}=\frac{249}{20}$ (Zejnullahu & Shabani, 2004).

MULTIPLICATION AND DIVISION OF FRACTIONS

Multiplying Fractions by A Natural Number

Multiplying a natural number *x* by a natural number *n* means taking *n*–the multiple of the number *x*, which is the sum of *n* adders equal to *x*. This also applies to multiplying a fraction by a natural number. Let's start with an example.

Example 4

The mother brought from the village three jars of honey, each weighing $\frac{4}{5}$ kg. How many kg of honey was there in total?

The task revolves around finding the amount $\frac{4}{5}+\frac{4}{5}+\frac{4}{5}$.

To calculate this output, we use **Figure 5**.

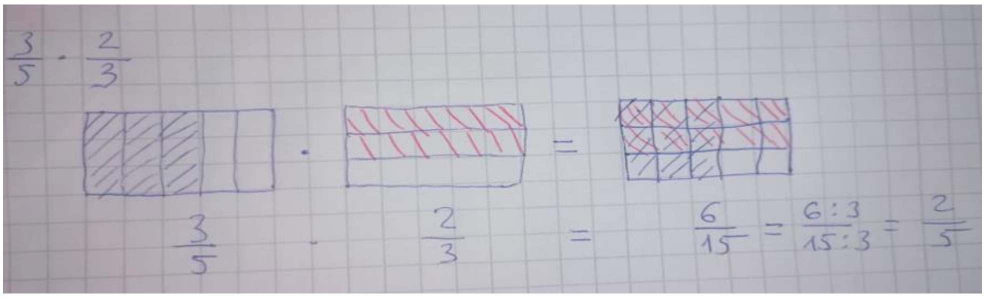

Figure 6. Multiplication of fractions using visual method

In each of the wholes divide by $\frac{4}{5}$ and add all those pieces. It can be seen that a total of 12 fifth parts are divided, so: $3 \cdot \frac{4}{5} = \frac{12}{5}$ or $3 \cdot \frac{4}{5} = \frac{3 \cdot 4}{5} = \frac{12}{5}$. So, the fraction $\frac{4}{5}$ is multiplied by the number 3 when we multiply its numerator by the number 3.

Multiplying a fraction by a fraction: Let $\frac{a}{b}$ and $\frac{c}{d}$ be two fractions. Then the formula is true: $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$.

In words: The product of fractions is equal to a fraction, which has the numerator the output of the numerators, while the denominator the output of the fraction denominators factors.

Example 5

If 1 m of fabric costs $3\frac{2}{5}$€, how much do $6\frac{1}{2}m$ of fabric cost?

$$6\frac{1}{2}m = 6\frac{1}{2} \cdot 1m = 6\frac{1}{2} \cdot 3\frac{2}{5}€ = \frac{2 \cdot 6 + 1}{2} \cdot \frac{5 \cdot 3 + 2}{5}€ = \frac{13}{2} \cdot \frac{17}{2}€ = \frac{13 \cdot 17}{2 \cdot 5}€ = \frac{221}{10}€ = 22\frac{1}{10}€$$

Example 6

In **Figure 6**, we present the multiplication of fractions visually ($\frac{3}{5} x \frac{2}{3}$), according to the method of rectangles, worked by students as homework.

Division of Fractions

Dividing the fraction by a natural number: If $\frac{a}{b}$ is a fraction and k is a natural number, it is really representing an equation: $\frac{a}{b} : k = \frac{a:k}{b}$.

In words: The fraction is divisible by a natural number by dividing the fraction numerator by that natural number. Fraction division by fraction: The division of the fraction $\frac{a}{b}$ with the fraction $\frac{c}{d}$, is expressed by the equation: $\frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$.

In words: The quotient of two fractions is equal to the product of the first fraction with the reciprocal value of the second fraction.

Example 7

Calculate $3\frac{4}{5} : 2\frac{6}{7}$.

We convert the mixed numbers into irregular fractions and then apply the above rule. We have: $3\frac{4}{5} : 2\frac{6}{7} = \frac{5 \cdot 3 + 4}{5} : \frac{7 \cdot 2 + 6}{7} = \frac{19}{5} : \frac{20}{7} = \frac{19}{5} \cdot \frac{7}{20} = \frac{19 \cdot 7}{5 \cdot 20} = \frac{133}{100} = 1\frac{33}{100}$.

COMPARISON OF FRACTIONS

Comparison of Two Fractions with Equal Numerator: The Largest Is the One with the Smallest Denominator

1. We always refer to the same whole.
2. Students are introduced to models for fractions that describe the relationship between part and full size
3. The more pieces, the smaller the size of each piece.

Students know (understand) the size of the pieces: $\frac{4}{5} > \frac{4}{8}$.

Comparison of Two Fractions with Equal Denominators: The Largest Is the One that Has the Largest Numerator

1. Students use concrete materials to compare fractions and use this knowledge in abstract thinking in later grades.
2. Concrete models should first focus on area models.
3. The parts are the same size (equal).
4. The largest numerator represents (shows) the largest fraction (Wu, 2011).

Figure 7 shows comparison of fractions.

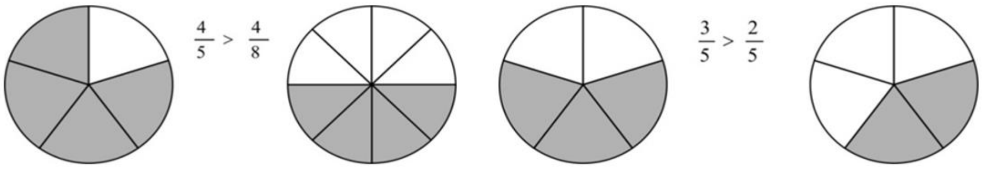

Figure 7. Comparison of fractions by illustration

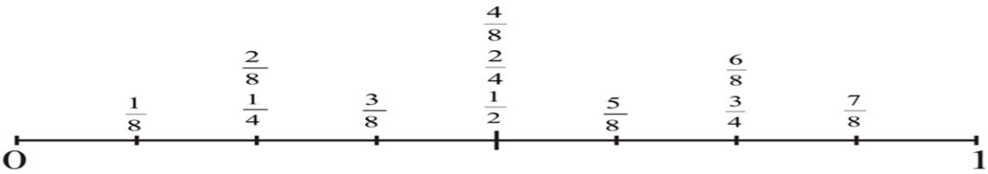

Figure 8. Comparison of fractions in the numerical line

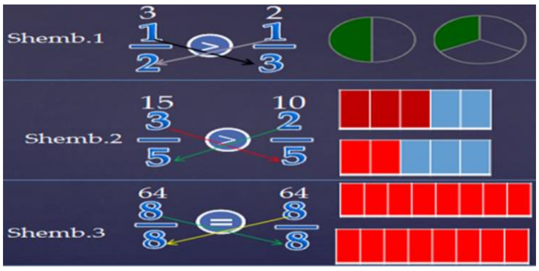

Figure 9. Comparison of fractions using the rules

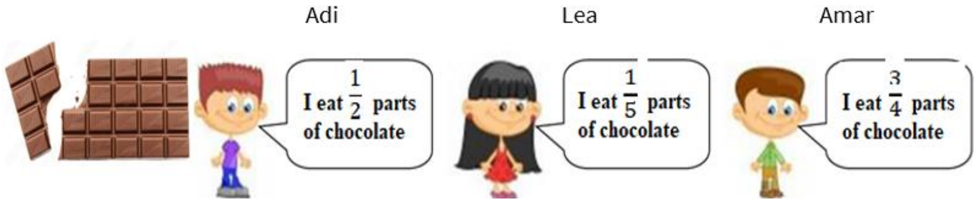

Figure 10. The problem relating comparison of fractions

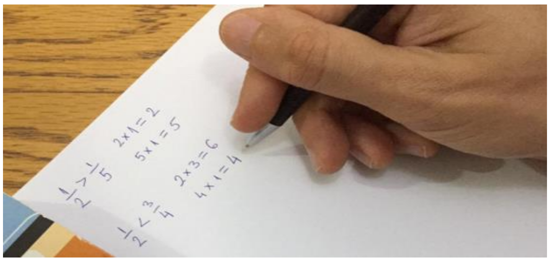

Figure 11. Students' strategy in comparing fractions

Comparison of Fractions with the Same Numerator and Denominator

Figure 8 shows comparison of fractions in the numerical line (Lippens, n. d.).

Let the fractions $\frac{a}{b}$ and $\frac{c}{d}$ be given, where $a, b, c, d \in N$.

1° $\frac{a}{b} < \frac{c}{d}$, then and only then when $ad < bc$.

2° $\frac{a}{b} = \frac{c}{d}$, then and only then when $ad = bc$.

3° $\frac{a}{b} > \frac{c}{d}$, then and only then when $ad > bc$.

Figure 9 shows comparison of fractions using the rules.

The problem related to the comparison of fractions in the study was, as follows: Adi, Lea, and Amar have a piece of chocolate. They then said how much they have eaten (**Figure 10**).

List the students from the one who eats the smallest chunks of chocolate to the one who eats the largest chunks of chocolate!

In this problem, students' solution was correct. Most of them used the cross-multiplication rule, as shown in **Figure 11**.

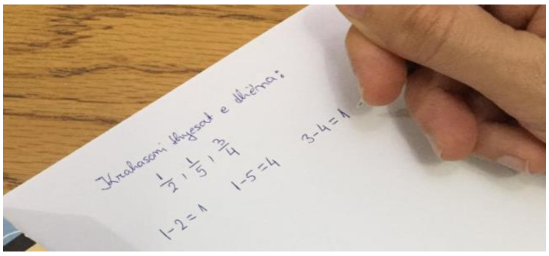

Figure 12. Students' mistakes in comparing fractions

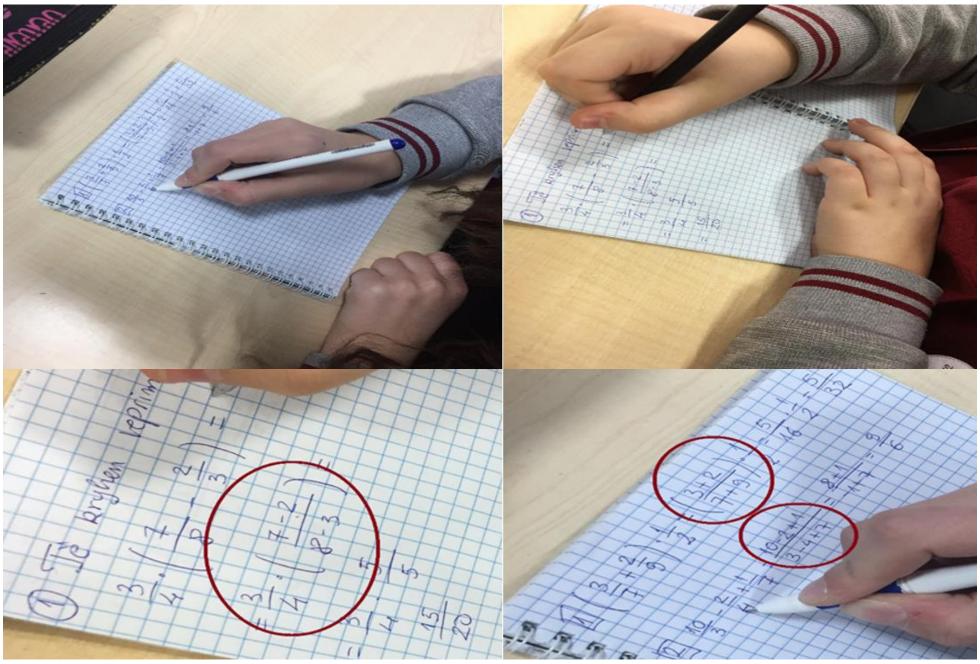

Figure 13. Students' mistakes when performing operations with fractional numbers

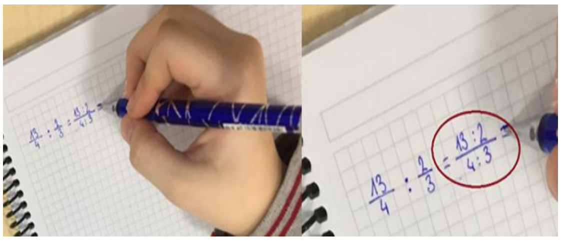

Figure 14. Students making mistakes when dividing fractions

In comparing two fractions using the cross-multiplication strategy, students multiplied the numerator of one fraction and the denominator of another fractions. The bigger multiplication result is, the bigger the value of the fractions.

However, some students made some mistakes strategies and most common was subtracting the denominator from the numerator, as presented in **Figure 12**.

It can be inferred from **Figure 12** that students did both procedural and factual mistake. Firstly, they applied incorrect procedure to compare fractions. Secondly, when they subtract 2 from 1, the answer was wrong, 1 instead of -1. In this case, the student might not be aware of concept of number fact.

SOME OF THE MOST COMMON MISTAKES THAT STUDENTS MAKE IN OPERATION WITH FRACTIONS

In **Figure 13**, we can notice some of the mistakes of the students that were encountered during the exercises of fractional actions. Over time and with the maximum commitment of the teacher in collaboration with the parents these mistakes will be avoided and will be more rarely from students in the coming years.

Figure 14 and **Figure 15** show the mistakes students make when performing operations with fractional numbers by not obeying the rules and the properties.

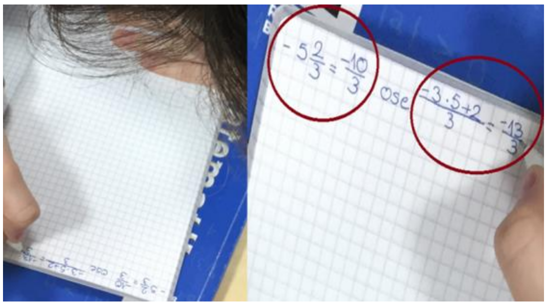

Figure 15. Student mistakes converting mixed numbers to fractions

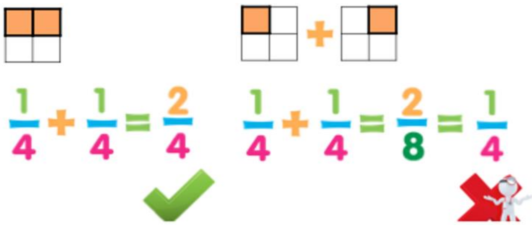

Figure 16. The right and wrong way of addition of fractions

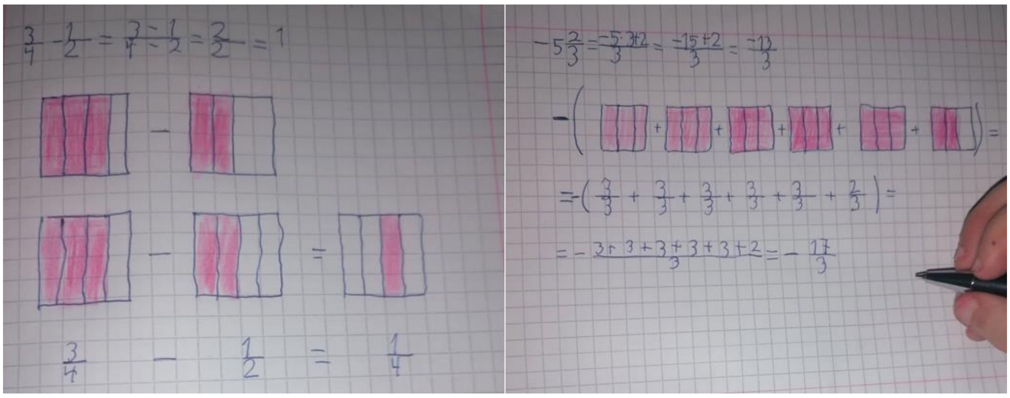

Figure 17. Error investigation using the visualization method to subtract fractions and convert mixed numbers to normal fractions

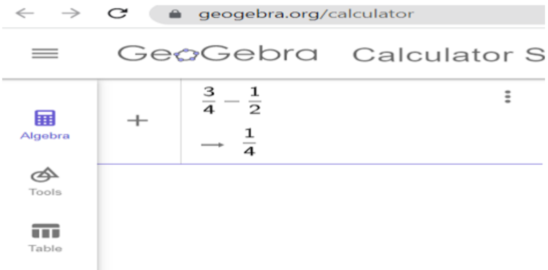

Figure 18. Using GeoGebra during class to compare results

To avoid the most frequent mistakes with fractional operations, in addition to the additional exercise that students have to solve, during the experience with students we witness that visual method of solving such exercises also plays an important role.

We give some concrete examples, through photos while working with students in the classroom (**Figure 16** and **Figure 17**).

The left part of **Figure 17** shows one of the most common mistakes when subtracting fractions, not following the rule, subtract the denominator with the denominator and the numerator with the numerator. In the right part of **Figure 17** displays the mistake that students make in most cases when converting mixed numbers into fractions. These are common mistakes that happen to students especially 6th graders. It really should be like this: $-5\frac{2}{3} = -\frac{5\cdot 3+2}{3} = -\frac{15+2}{3} = -\frac{17}{3}$. In both cases, the student noticed the error by not matching the result with the classical method as well as with the visualization method, where both of these methods were examined in class during the lessons.

Despite the significant impact of tablets, GeoGebra is a valuable pedagogical tool for visualizing a wide variety of mathematical formulae in both algebraic and geometric representations (Mollakuqe et al., 2021). Geogebra calculator for fractional operations is easily used considering the age of students and arouses discussion and interest by comparing the results obtained in students (**Figure 18**).

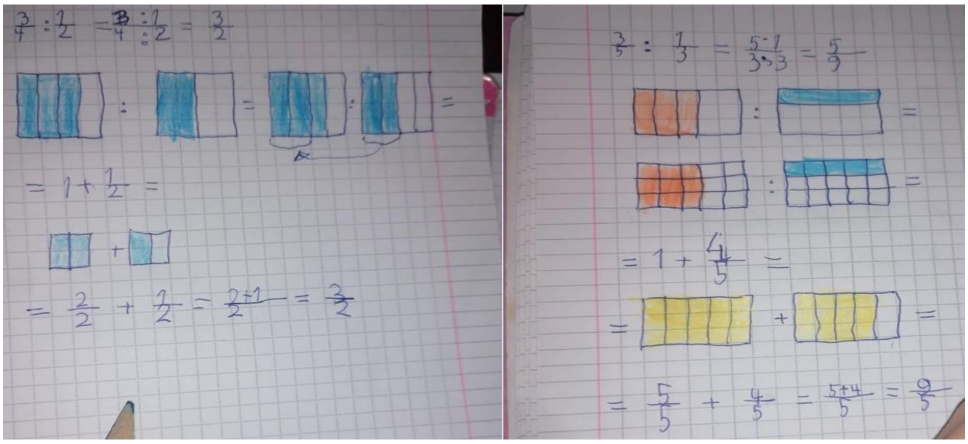

Figure 19. Error investigation using the visualization method to divide fractions

Figure 20. Photographs of students answering questionnaire willingly

Similar to subtracting fractions, as well as dividing fractions, a considerable number of students make the mistake of dividing denominators by denominators and numerators by numerators or intuitively make cross multiplication. In general, from the classroom experience of visual methods, 6th graders are clearer, and this method is always more attractive to them and as a method helps them rarely make mistakes in fractional operations.

Let us discuss for example the right part of **Figure 19**. With $\frac{3}{5} \div \frac{1}{3}$, our common denominator is 15, so we can create a grid of 15 squares. $\frac{3}{5}$ takes up nine of these squares (colored in orange) and $\frac{1}{3}$ takes up five (colored in blue) of these squares. So, all of $\frac{1}{3}$ can fit into $\frac{3}{5}$, plus an additional 4. We can then see that $\frac{4}{5} \div \frac{2}{3} = 1$ and $\frac{4}{5}$, respectively $\frac{9}{5}$.

In these cases, when different results are achieved by the students using different methods, usually the students become even more curious and ask the question with which method they made a mistake and how to correct that mistake.

THE IMPACT OF PRACTICAL ILLUSTRATIONS, VISUALIZATION, INSTRUCTIONAL VIDEOS ON UNDERSTANDING FRACTIONS AND OPERATIONS WITH THEM

We did a questionnaire with the students of the 6th grade where we aimed to see how much they understand the fractions and how much they have a problem with the fractions. It was a short but timely questionnaire because it is in this period that they are learning about fractions and operations with them. A total of 60 6th grade students were surveyed. In the following, we will elaborate on their answers.

Questionnaire

1. Do you think that using figures and concretizing tools help you understand the concept of fractions?
   a. Yes
   b. No
2. Do you understand fractions and operations with them?
   a. Yes
   b. No

Photographs in **Figure 20** prove that the students answered the questionnaire very willingly.

Table 1. Observed values

	Answer to the first question			
Answer to the second question	*X/Y*	Po	Jo	Total
	Yes	36	12	48
	No	4	8	12
	Total	40	20	60

Table 2. Expected values

	Answer to the first question			
Answer to the second question	*X/Y*	Po	Jo	Total
	Yes	32	16	48
	No	8	4	12
	Total	40	20	60

Table 3. Calculation of Chi-square value

O-observed	E-expected	O-E	$(O-E)^2$	$(O-E)^2/E$
36	32	4	16	0.5
12	16	-4	16	1
4	8	-4	16	2
8	4	4	16	4
				Total: 7.5

The results of the answers are given with the contingency table in **Table 1** (Rexhepi et al., 2021).

Random variables *X* and *Y* accept set values {yes, no}. With difficulty level we will prove that the answers to the second question by the students do not depend on their opinion regarding the use of practical techniques in relation to the fractions.

We first calculate the absolute expected frequencies for each of the quadrants and the contingent table. Based on the preliminary table, the expected values are shown in **Table 2**. **Table 3** depicts the estimation of Chi-square value.

Next we calculate the Chi-square $\chi^2 = \frac{(36-32)^2}{32} + \frac{(12-16)^2}{16} + \frac{(4-8)^2}{8} + \frac{(8-4)^2}{4} = 7.5$.

With significance level $\alpha=0.01$, we set $\chi^2_{1,0.01} = 6.6349$, so, the critical domain is $C=(6.6349, \infty)$ and since $7.5 \in C$, we conclude that *X* and *Y* are dependent (**Table 3**).

CONCLUSIONS AND RECOMMENDATIONS

During our work as teachers, we have encountered difficulties with students in understanding and implementing operations with fractions, especially those in the 6th grade.

This prompted us to research more about this topic, where after all this work we have seen that the use of various figures, illustrations, video-lectures facilitate our work as teachers but at the same time helps students to understand the fractions and make operations with them. This is clearly seen from both the classroom experience with the students where some of the experiences were presented above in the paper, as well as from the statistical testing.

After all the research done and the results we have achieved, we recommend all teachers to use Internet resources as well as other illustrative and demonstration methods to make the teaching of fractions and operations with them as attractive as possible.

Author contributions: All authors have sufficiently contributed to the study and agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Declaration of interest: No conflict of interest is declared by authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

REFERENCES

Aliu, A., Rexhepi, S., & Iseni, E. (2021). Analysis and comparison of commitment, homework, extra hours, preliminary grades, and testing of students in mathematics using linear regression model. *Mathematics Teaching Research Journal, 13*(3), 21-52.

Baidoo, J. (2019). Dealing with grade 10 learners' misconceptions and errors when simplifying algebraic fractions. *Journal of Emerging Trends in Educational Research and Policy Studies, 10*(1), 47-55.

Kor, L. K., Teoh, S. H., Mohamed, S. S. E. B., & Singh, P. (2018). Learning to make sense of fractions: Some insights from the Malaysian primary 4 pupils. *International Electronic Journal of Mathematics Education, 14*(1), 169-182. https://doi.org/10.29333/iejme/3985

Lemonidis, C., & Pilianidis, N. (2020). The 8th grade students' competencies in alternating different symbolic representations of rational numbers. *International Electronic Journal of Mathematics Education, 15*(3), em587. https://doi.org/10.29333/iejme/7865

Lestiana, H. T., Rejeki, S., & Setyawan, F. (2017). Identifying students' errors on fractions. *Journal of Research and Advances in Mathematics Education, 1*(2), 131-139. https://doi.org/10.23917/jramathedu.v1i2.3396

Lippens, M. (n. d.). *Strategies and resources for teaching fractions through the common core*. http://mrslaug.weebly.com/uploads/1/4/7/4/14749530/math_institute_teaching_fractions.pdf

Maelasari, E., & Jupri, A. (2017). Analysis of student errors on division of fractions. *Journal of Physics: Conference Series, 812*(1), 012033. https://doi.org/10.1088/1742-6596/812/1/012033

Makhubele, Y. E. (2021). The analysis of grade 8 fractions errors displayed by learners due to deficient mastery of prerequisite concepts. *International Electronic Journal of Mathematics Education, 16*(3), em0645. https://doi.org/10.29333/iejme/11004

McNamara, J., & Shaughnessy, M. M. (2015). *Beyond pizzas & pies, grades 3-5: 10 essential strategies for supporting fraction sense*. Math Solutions.

Mollakuqe, V., Rexhepi, S., & Iseni, E. (2021). Incorporating Geogebra into teaching circle properties at high school level and it's comparison with the classical method of teaching. *International Electronic Journal of Mathematics Education, 16*(1), em0616. https://doi.org/10.29333/iejme/9283

Prediger, S. (2013). Focussing structural relations in the bar board–A design research study for fostering all students' conceptual understanding of fractions. In B. Ubuz, C. Haser, & M. A. Mariotti (Eds.), *Proceedings of the 8th Congress of the European Society for Research in Mathematics Education* (pp. 343-352).

Reinhold, F., Hoch, S., Werner, B., Richter-Gebert, J., & Reissa, K. (2020). Learning fractions with and without educational technology: What matters for high-achieving and low-achieving students? *Learning and Instruction, 65*, 101264. https://doi.org/10.1016/j.learninstruc.2019.101264

Rexhepi, S., Iseni, E., Kera, S. (2021). *Introduction to mathematical statistics with exercises*. Gostivar

Ubah, I. J. A., & Bansilal, S. (2018). Pre-service primary Mathematics teachers' understanding of fractions: An action–process–object–schema perspective. *South African Journal of Childhood Education, 8*(2), a539. https://doi.org/10.4102/sajce.v8i2.539

Wu, H. (2011). *Teaching fractions according to the common core standards*. https://math.berkeley.edu/~wu/CCSS-Fractions_1.pdf

Zejnullahu, R., & Shabani, A. (2004). *Matematika VI permbledhje detyrash te zgjidhura* [*Mathematics VI summary of solved exercises*]. Dukagjini.

Zejnullahu, R., Gjergji, R., Vula, E., & Bilalli, S. (2004). *Matematika per klasen e VI* [*Mathematics for VI grade*]. Dukagjini.