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Specifying and identifying signs of 'resonance' in teachers' professionalization processes as a condition for teacher change

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ARTICLE INFO	ABSTRACT
	Many studies on professionalization research deal with the question of how teachers actually learn or how teacher change occurs. In this context, Clarke and Hollingsworth (2002) describe a model for capturing teacher change in different domains (external, personal, domain of practice and domain of consequence) and how changes in one domain can affect other domains over time. The present article suggests the notion of resonance for analyzing teachers' professionalization processes in order to better understand the reasons for teacher change on a micro level in their complexity. The idea was to supplement the model by the dimension of teachers' professional conditions, to capture and describe the phenomenon that an active process of engagement with the professional development (PD) content is set in motion. We consider teachers' resonance to be a condition for a successful professionalization process. Insights from two different research projects from Germany and Norway demonstrate the scope of the notion of resonance. Consequences for facilitators and the design of further PD programs[1] are discussed.

INTRODUCTION

The principal goals of professional development (PD) activities are to encourage and support growth by learning. However, there are many different assumptions and conceptualizations, how (mathematics) teachers learn (Goldsmith et al., 2014; Lipowsky & Rzejak, 2015; Wilkie, 2019). In this article we introduce the term *resonance* as a metaphor for the ignition of a durable engagement of teachers in the goals and means of a PD program. Thereby, we try to capture a key aspect of the conditions for teacher change. Many factors can influence the learning processes (Guskey, 2002, p. 387) and it is possible that teachers have different outcomes from the same PD program (Wilkie, 2019, p. 118). One explanation may be that "'opportunity to learn' is not the same as 'being taught'" (Wilkie, 2019, p. 101). This means that PD programs can only provide learning opportunities and that it can never be guaranteed that teachers will learn something in particular, i.e., that the offer will be used as intended. Hiebert and Grouws (2007, p. 379) explain that an "opportunity to learn includes considerations of students' entry knowledge, the nature and purpose of the tasks and activities, the likelihood of engagement, and so on". Wilkie (2019) states that "this quote could also be applied to teachers' opportunities to learn" (p. 101).

In this study we assume, following Putnam and Borko (2000), that the teachers' possible reactions and their learning are situated in the actual PD activity at the micro level and, in the overall PD program and in the teachers' professional life at the macro level. Therefore, the discourse communities and professional contexts in which teachers work and learn have an important impact on the teachers' views and reactions to PD programs. We examine the response of teachers in two projects to specify and identify signs of resonance and, thereby, throw light on the conditions for successful teacher change situated at the micro level. For our examination the notion of resonance was operationalized into a tool in the form of a matrix. The matrix's two dimensions correspond to the model for change within domains (Clarke & Hollingsworth, 2002) and the professional conditions (i.e., profile and context), respectively. The matrix thereby serves to organize data about goals and content of the PD program, and about teachers' learning prospects seen from their perspectives. Interpretation of teachers' navigation in the matrix structure reveals tensions and discrepancies, but also synergies. Thereby, this article aims at contributing to a better understanding of the reasons why teachers adopt certain ingredients of the PD, influencing the overall success of the PD. To this end, the notion of resonance is used as a tool to analyze teachers' professionalization processes at the micro level.

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[1] In this article, PD programs are understood in form of courses, activities, interventions, research & development (R&D) projects, & action research.

First, we describe the rationale behind the construct resonance and operationalize the theoretical background into the matrix. Next, we apply the matrix on empirical data in two cases, to specify and identify signs of resonance. Two cases serve to illustrate the scope of the notion of resonance by setting focus on different phases of the PD. The first case occurred in a German PD program and serves to pinpoint specific conditions for resonance to (potentially) occur, whereas the second case occurred in a Norwegian classroom research project and serves to reveal impact of resonance on actual teaching. Finally, we take one step back and discuss potentials of notion of resonance and the matrix as a tool for inquiry of teachers' kind of navigation through the matrix.

THEORETICAL BACKGROUND: RATIONALE BEHIND THE CONSTRUCT 'RESONANCE'

An Interconnected Model for Capturing Teacher Change

This article's study builds on *the interconnected model* of teacher change (**Figure 1**) (Clarke & Hollingsworth, 2002). Possible domains of teacher change can be distinguished in the external domain, the personal domain, the domain of practice, and the domain of consequence (e.g., Clarke & Hollingsworth, 2002; Guskey, 2002). The external domain includes the external source of information or stimulus, the personal domain summarizes knowledge, beliefs, and attitudes of participants, the domain of practice describes professional experimentation, and the domain of consequence contains salient results.
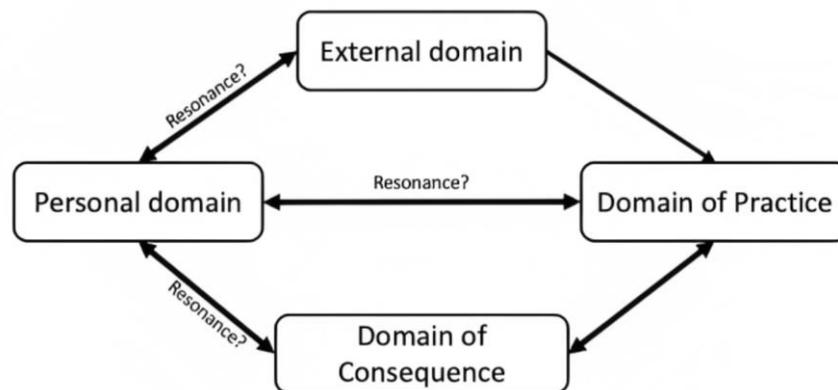
Figure 1. Resonance in the interconnected model of professional growth (Adapted from Clarke & Hollingsworth, 2002, p. 951)

The interconnected model (**Figure 1**) allows several possible directions and learning pathways through the different domains. Changes in one domain may be "translated" into changes in another domain: trying out a new practice and seeing its effects might change the teacher's beliefs for example (Clarke & Hollingsworth, 2002, p. 954). The model thus makes it possible to capture the effects of change in one domain on other domains. In particular, the domains are not regarded as independent of each other, but rather the aim is to better understand the interrelationships. Reflecting on external impulses can, for example, lead to new knowledge, which means a change in the personal domain. This new knowledge in the personal domain can in turn lead to new teaching approaches being tried out in the domain of practice.

Resonance as One Key Factor for Teacher Change

The interconnected model by Clarke and Hollingsworth (2002) serves to identify domains of teacher change and the relations between domains. This article's introduction of the metaphor 'resonance' is an attempt to capture and to operationalize analyses of what is *behind* such change. Following our idea, occurrence of resonance induced by ingredients of PD happens to a teacher either rooted in the teacher's professional context or rooted in the teacher's professional profile (or maybe both). Our study's matrix includes the domains of teacher change in *the interconnected model* as one dimension, and supplies it with a second dimension to include, within each of the domains, potential resonance between PD ingredients and the professional context and/or the professional profile of the participating teacher. The term 'resonance' was borrowed from physics:

> "Resonance, in physics, relatively large selective response of an object or a system that vibrates in step or phase, with an externally applied oscillatory force. Resonance was first investigated in acoustical systems such as musical instruments and the human voice. An example of acoustical resonance is the vibration induced in a violin or piano string of a given pitch when a musical note of the same pitch is sung or played nearby" (Britannica, 2019).

This article's introduction of the metaphor 'resonance' took inspiration from Conle's (1996) investigation of preservice teachers focusing on the processes, where teachers shared narrative inquiries. Conle (1996) was interested in the connection processes, which she labelled "resonance":

> "Resonance is a process of dynamic, complex, metaphorical relations. It is not confined to one single strand of connections. It is a complex relationship among many aspects of a story" (p. 313).

Distinct from the use in Conle's (1996) article, where 'resonance' refers to processes of connecting individual persons' experiences in the form of narratives, we use the term broader, as a metaphor for individual persons' connections drawn when they were introduced to the ingredients of a PD. In this view, the subsequent processes initiated by resonance encompass

connection with, e.g., prior professional experiences as well as processes of reflection and action, which form the basis for durable engagement. In this article, resonance is defined as the phenomenon that an active process of engagement with the PD content is set in motion, usually triggered by a specific impulse (e.g., an input by a facilitator or ideas of other participants in the PD program). Assuming that teachers are active learners, teacher educators can also only ever create learning opportunities and not directly teach anything. For this reason, we refer to facilitators rather than teacher educators in this article.

Teachers' Professionalism as Classifying Background Information

Taxonomy for teachers' professionalism serves to fertilize and enrich the description and conceptualization of teachers' professional development. In Dale's (1998, p. 169) taxonomy, level K1 of professionalism by a teacher is characterized by rationality in and beyond the teaching performance, level K2 includes the teacher's reflections upon and evaluation of curriculum and teamwork with colleagues about the organization of courses, whereas level K3 encompasses the teacher's theory development based on own experiences. Within Dale's (1998) taxonomy for professionalism, it is possible to discern between goals of PD programs within a certain level, and goals implying that the teacher should move from one level to the next. The taxonomy is useful to classify the target group of a PD program in the design phase. In both of the two projects presented in this article, the best assessment would be to rank the participating teachers at level K2 in Dale's (1998) taxonomy. The first project aims at developing teachers' situation specific skills by reflecting their teaching experiences based on the theoretical ingredients of the PD program. The second project concentrates on how the PD program (i.e., the research and development project) can frame a realization of the teachers' shared visions about teaching. Focus is, further, on the teachers' reflections upon and their sharing of experiences from their local classroom experiments.

Tensions and Discrepancies as One Type of Resonance

Rouleau and Liljedal (2018) discuss a case of pre-service teachers shifting from acceptance of an established teaching practice to a determination never to use it. Tension is described as inner turmoil experienced by teachers as the consequence of being pulled in different directions by competing pedagogical demands. Rouleau and Liljedal (2018) advocate those tensions often propel teachers towards professional development. This is another variation of an active engagement process, where the impulse triggers a wish for resonance in the teachers. Therefore, tensions can also be understood as a trigger of resonance.

Teachers might as well experience tensions if they are pulled by a PD activity in a direction in contrast with their own desires about teaching. Therefore, tensions can also be understood as an opportunity to be aware of and to stress a wish for resonance that result in teachers dropping out, so to speak, and not continuing to be willing to follow the PD program. Goodchild (2014) reports a case study with upper secondary teachers and university researchers, taking the premise that sustained and durable transformation of teaching can be achieved when teachers set the goals, make, and implement the plans and reflect on outcomes. However, analysis of the data revealed the teachers' resistance to innovation. A re-analysis revealed tensions between the researchers' and the teachers' view on how to implement the project. The study concluded that "there may be many reasons for their reluctance, but the outcome is that creative innovation that results in goal-directed development of practice does not occur." (ibid., p. 315). In this case there seemed to be a discrepancy between the means of the PD and the teachers' visions.

However, even when the teacher agrees with the goal, the observed teaching will not necessarily target it. Apparent discrepancies and inconsistency between teachers' beliefs or dispositions and their performance of enacted teaching, reveal a complexity of influential factors, urging for further studies (cf. Skott et al., 2018). Other discrepancies between beliefs and performance are found by Liljedahl (2008) in a sub-study, which "clearly show that, (…) there is not always a correlative relationship between teachers' espoused beliefs and their intentions of practice" (p. 52). In line with this, Hundeland (2011) pinpointed individual teachers' dilemma between their wish to support the students' inquiry activities, and the loyalty to the system, which implied a high teaching pace. Such cases stress the importance of the professional context (sociological perspective), and its alignment with the teachers' professional profile, i.e., their visions and personal capabilities (psychological perspective). Professional context is here defined as a generic term for (perceived) external conditions (this can be equipment, the support of the school management, the reaction of children etc.). Professional profile is here defined as a generic term for everything that the teacher (perceived) has in his or her own hands.

Construction of the Matrix

The matrix-model (**Table 1**) was constructed as a means for inquiring signs of resonance by identifying navigation processes between aspects concerning participating teacher's professional conditions (context and profile), and *the interconnected model*'s four domains. The matrix was created by a deductive-inductive process. Both the domains of change in the rows of the matrix and the columns of the matrix through the juxtaposition of normative goals, the professional context and the professional profile, come from theory as outlined above. However, these aspects are not considered in isolation from each other in this article, but in an integrated manner by focusing on navigation processes within the matrix. Combining the aspects as rows and columns of a matrix makes it possible to visualize connections between aspects in order to gain a comprehensive picture of signs of resonance.

Column A in the matrix entails data about the PD's key ingredients. The key ingredients may be in the form of goals and normative descriptions of ideal teaching, application of specific teaching methods or working styles, or in terms of theory-enriched practical knowledge. The key ingredients may relate to any of the four domains; for example, the goal of a project can address local teaching practice within the domain of practice, provide understanding and insight into educational theory in the personal domain, or aim at sustainable change in the domain of consequences.

Column B in the matrix entails data about teachers' view on the key ingredients seen from the perspective of the teachers' professional context. This perspective captures reactions and reflections rooted in, for example, shared norms amongst peers about teachers' role, formal regulations for curriculum, and (prior) experiences with students' reactions in classroom interaction.

Table 1. Matrix for specifying & identifying signs of resonance

	A. Input: Goals & means of PD program	*B. Professional context*: Empirical data about teachers' reactions rooted in professional context for participant's work with his/her PD activity	*C. Professional profile*: Empirical data about teachers' reactions rooted in his or her ideas & values, emotions, beliefs, & visions
External domain	1.A	1.B	1.C
Domain of practice	2.A	2.B	2.C
Domain of consequence	3.A	3.B	3.C
Personal domain	4.A	4.B	4.C

Column C in the matrix entails data about teachers' view from the perspective of their professional profile. This perspective captures reactions and reflections rooted in their background, personal professional visions and situation including, for example, their view on mathematics, beliefs about teaching and learning, and mathematical literacy.

The matrix's distinction between these perspectives goes across the four domains since a teacher's view on, reaction to, or reflections upon potential changes in each of the domains can be rooted in the professional context as well as in the professional profile, in accordance with the interconnected model in Clarke and Hollingsworth (2002) and with the situated perspective in Putnam and Borko (2000).

In addition to the identification of positive type of resonance, search for resonance by identifying navigation processes in the matrix (**Table 1**) may also identify possible tensions and discrepancies as obstacles for success on a micro-level when conducting a PD program. This leads to the following research question:

RQ. Which signs of resonance can be identified on micro level in the teachers' professionalization processes in a PD program?

METHODS

Data Collection

Both projects are about fostering teachers' learning. One project deals with learning processes of teachers in an advanced PD program for language-responsive mathematics teaching at the micro level, i.e., the situational learning processes during advanced PD activities. The other project deals with the long-term learning processes of teachers at the macro level who participated in a project on encouraging students' inquiry and autonomy.

The aim of the first (German) project MuM-Innovation (for an overview of the project see Prediger, 2019) was to improve teachers' performance in language-responsive classrooms by providing new didactic categories and training their situation-specific skills in PD integrated activities (Prediger, 2019) . The teachers participated in the in-service PD program on a voluntary basis (a total of five dates spread over one year from May 2017 until March 2018). In total, data were collected by gathering teachers' (n=65) written products from a PD activity, by observing some teachers' classrooms, and by video-recording teachers' small-group and whole-group discussions during the PD program. This article focuses on the video-recording of one whole-group discussion at the end of the third meeting within the PD program (of five).

The aim of the second (Norwegian) project was to develop and study teaching that encourages students' activity, inquiry, and autonomy (Andresen, 2018; Andresen & Brun, 2015) during small scale problem solving projects. The participating teachers got the opportunity in the research project to realize and reflect on some of their own professional visions about how to organize and enact teaching, distinct from their usual performances. The data consisted of video recordings of teaching in eight Norwegian upper secondary classrooms and two interviews with the participating teachers (30 hours in total) in the autumn of 2013 and spring 2014.

In both projects, resonance proves to be a helpful construct for better understanding the learning processes of teachers. This article presents inquiry by the matrix of two sets of data:

1. The first case was chosen to illustrate the scope of variety of the signs of resonance by the teachers that might foster or hinder teachers' professionalization processes with regard to the new PD content in a discussion of a PD program on language-responsive teaching.

2. The second case was chosen to illustrate, by two examples, teachers' attempts to enact a change of their teaching in accordance with PD goals, which resonated with their professional visions. In this case resonance was revealed in their teaching.

The key difference is that in the German project there are external stimuli from research that teachers might accept or reject (for different reasons). In the Norwegian project, the teachers were involved with setting the project goals themselves, so there would, presumably, be less tensions between the normatively set key ingredients and the professional profile. The Norwegian teachers' challenge was to implement the goals. The study of the Norwegian project can serve to identify and illustrate the role of resonance between the teachers' visions and the PD's goals, for a change of classroom teaching. Contrasting two such very different projects was chosen here to show the scope of variety of possible signs of resonance and, to inspire discussions about the extent to which they are identical or distinct in different projects.

Methods of Data Analysis

The matrix was used as a tool for analysis of signs of resonance in the concrete activities of the PD programs. Resonance was detected in interpretations of data that point to favorable connections between the key ingredients of the PD, and the participants' perceived achievability of their visions through it. Each cell in the matrix was linked to all relevant pieces of data assigned to one or more of the four domains, seen from one or more of the two perspectives. The key ingredients were extracted from data that included project description and background documents, program descriptions, etc., and teachers' statements from interviews, answers from questionnaires or PD activities and acts of teaching (excerpts from videos, etc.). The basic procedure for data analysis is of course very similar for both data sets since the matrix was utilized in both cases. The diversity of the two PD programs that served to demonstrate the scope of the tool, though, implied slight adaptations of the data analysis. Therefore, the analysis procedure for each data sets is explained separately below.

Dataset one

Concerning the German project, in the column "input" the PD ingredients of exactly this PD program are listed, which are currently being discussed in the focused scene. In this case, the PD ingredients were

(1) specifying the mathematically relevant language demands by considering the connections of mathematical content goals, corresponding discourse practices and necessary lexical means and

(2) different discourse-promoting impulses for moderating whole-classroom-discussions (**Figure 2**, cf. Prediger, 2019).

The inquiry aimed at identifying signs of resonance in a group discussion at the end of the third meeting of the PD program, between the PD's goals, the teachers' ideas and visions, and his or her reflection on previous teaching.

The discussion was transcribed and analyzed by a qualitative content analysis (Kuckartz, 2019) with respect to the addressed perspectives and the addressed domains of change and the kind of connections that were drawn. Therefore, signs of resonance were qualitatively analyzed by reconstructing the connections that teachers draw in their utterances in the PD program. So, the focus here is more on the question of the extent to which there is resonance that could explain why change will, or will not, occur within a domain.

Dataset two

In this case the PD program's goal was to encourage students' activity, inquiry, and autonomy (Andresen, 2018; Andresen & Brun, 2015) during small scale problem solving projects. The inquiry aimed to detect and identify resonance in each of the individual cases, between the PD's goals, the teachers' ideas and visions, and his or her actual teaching, i.e., signs of resonance between three groups of data:

1. The PD's goals were present in its project descriptions and working documents, etc.

2. Data on the teachers' ideas and visions came from statements in the interviews, upheld against the researcher's general impression from the meetings and group discussions.

3. The actual teaching was documented by communicational actions in excerpts from his or her classroom sessions, teaching materials and notes, and field notes from classroom observations.

Here again, signs of resonance were qualitatively analyzed by reconstructing the connections that teachers draw in their utterances in the PD program. The matrix structure was used to organize data from part I and part II by non-disjunctive marking, and meaning condensation of the materials from part I followed by distribution into the domains in the matrix's column A on PD's key ingredients. Statements from part II were assigned to all relevant cells in the matrix's columns 'B. Professional context' and 'C. professional profile'. The assignment was based on interpretations of the condensed meanings contextualized by the group meetings. The teacher's statements were weightend up against the PD goals cell by cell in a search for resonance in a horizontal direction. Finally, the results were linked with data from part III with the aim to assess the teacher's compliance with the agreed goals, which is in the domain of consequence. The excerpts of videos etc. from part III were linked with the statements from part II based on our interpretations. Linking might refer, for example, to specific types of interaction in the classroom, special situations, mathematical content or the organization of practical issues. The potential transfer of change from one domain to another was considered by a vertical view of the matrix, to capture resonance between the domains. Thereby, the analysis of data set two contributes to the study of resonance with findings in the form of

(1) agreed goals and

(2) the teacher's compliance with these goals.

In this article excerpts are presented from two of the teachers' statements in the interviews. Each excerpt was linked with an excerpt from the project's classroom sessions with the same teacher. These linked excerpts served to illustrate signs of resonance that could indicate translation of change into the domain of consequences. The two teachers were chosen from the group's eight participants because they expressed concerns in the interview, which were frequently brought up during the group's meetings, whereas they addressed them distinctly. The two incidents relate to different aspects of resonance.

EMPIRICAL INSIGHTS IN TWO CASES

Insights in Teachers' Resonance Processes in a PD Program on Language-Responsive Teaching

The empirical insights show the analysis of teachers' resonance at the end of the third of five sessions of the PD program. At the end of this PD session, the facilitator asks the participants: "What do you take home from today?" The following transcript protocol shows the teachers' answers (summarized in **Figure 2**).

As described in the methods section, each utterance is labeled with the dimension of resonance (columns in the matrix) and the domain of change (rows in the matrix, see **Figure 2**). In this way, the different signs of resonance can be made visible by a navigation process through the matrix.

Figure 2. Horizontal & vertical resonance of the teachers in a PD program on language-responsive teaching (Source: Authors' own elaboration)

#2 Sommer: […] Often it is the math lessons I know, and which are often practiced, you open the book, you look at the book, you look at how I am going to do the lesson, you do some exercises on it and you close the book again. Now let me say provocatively [slip of the tongue]. Now it's no longer about that, but you have to deal with the whole thing, with the whole thing so deeply, in the end, so that you can build up this meaning-related vocabulary […] and I also find it difficult to blow up the meaning-related vocabulary.

Sommer (#2) compares the input from the PD program with her experience from observed lessons from other teachers. Thereby she reflects on the domain of practice with regard to her professional context (2.B.c). Referring to one of the didactic categories we talked about in the meeting before, she draws the conclusion that one has to deal with the content more deeply (in the future) in order to be able to build up meaning-related vocabulary. So, there is resonance with her professional profile in the external domain (1.A.a & 1.C.a). However, she also mentions that she is struggling with identifying the relevant meaning-related lexical means. Thereby she discusses the domain of consequence by doubting whether she is able to implement the innovation addressing the domain of consequence (4.C.e). The requirement to identify meaningful language resources requires a high degree of autonomy for teachers. Although the PD activity has raised awareness of the need, Sommer's statement also shows that she sees difficulties in implementation.

#6 Lippott: So, I find for me personally it is incredibly difficult to determine these … yes, these linguistic means and uh, exactly in advance. Where exactly do I really want to go linguistically? We were just discussing that. So, what is it now actually and then to consider, yes with the language actions I come there now and yes, the changes we have in the language and that I do not stay with the description but that I then go to the other levels. I find it incredibly difficult for me to determine THAT at all for a series of lessons.

Lippott (#6) follows this statement and explains that she has great difficulty in determining the meaning-related linguistic means in advance. Thereby she addresses the domain of consequence by doubting whether she is able to implement the innovation (4.C.e). So, she seems to acknowledge the necessity of implementing this didactic category from the external domain (1.A.a & 1.C.a) but she has problems with really implementing it in her practice.

#7 Facilitator: Can you imagine that there is still something happening IN the process with the search movements that you make before, that it changes something in the classroom, … or is it so unwieldy that it can no longer be relevant for action?

#8 Lippott: I think that just because we exchange ideas here, you just think of things from time to time and then you react to them spontaneously in class.

#9 Leopold: So especially for the conversation in class erm it is very important for me to give the students the time to formulate it. You are often too rushed in class, and you want to have your answer somehow yes and also your precise answer. And then, when the students already come with vague ehm descriptions, then you say yes now, say it now more concretely, what do you mean now actually? And instead of just letting them finish, often something comes along and then you realize, where the problems actually are. And what words are actually missing.

#10 Facilitator: Mhm (approvingly).

#11 Gerster: Yes, and also to give the opportunity to speak more. So, I found that in the video as well, I liked it a lot, that you just say that we discuss first and can exchange wrong things with each other and then, … you bundle it or move it in any direction, but that this discussion comes about at all, that is often after two or three student comments, is that then clarified and then it continues. So that is–. Well, that you simply give it more room. I believe that that's what I have resolved to do in any case, to somehow pay attention to it.

#12 Facilitator: Mhm (approvingly).

#13 Lippott: Yeah, and that you try to take yourself back a lot more. Just to give that back to the students, that they should talk and not always talk about us, which is so classical.

Lippott (#8, 13), Leopold (#9), and Gerster (#11) pick up another point here concerning demanding language. They explain that they want to let their learners speak more in the future and give them more space in the classroom conversation. Thereby, they address the domain of consequence (4.C.b), which is adapted from the PD programs' input in the external domain (1.A.b). Leopold (#9) thereby reflects their previous teaching. So, she refers to the domain of practice concerning her professional profile (2.C.c). Gerster (#11) makes explicit that she liked the way of teacher moderation in the video before. There is resonance between the domain of practice (here: the design heuristics of showing classroom videos) (2.A.a) and her professional profile (2.C.a). They all indicate here a change in the perception of their role as teachers. They plan to withdraw themselves in the future and let learners talk more.

#14 Facilitator: But this is a longer-term process, ne? You said it would not work the first time (…).

#15 Niehaus: So, I have now with this class 7, which I did not know at all, I had to somehow introduce negative numbers. And then we just took different settings, the account, the thermometer, the elevator, the

surface of the earth. I wrote that on the board and then I first looked at what was coming. And there's actually a lot that comes up and then always bundle it up like that […] I have now resolved to pay more attention to this, this explanatory context. Explain the meaning of–and I would introduce this as an operator and I would always say this, so that they know when I say explain the meaning of, then they know what to do. I want to practice that with them. What this explaining means.

Niehaus (#15) reflects her own teaching experience in the domain of practice (2.A.c) and describes here that she wants to introduce the explanation of meaning as an operator in her teaching, so that learners always know exactly what is meant when she gives this work order. This is also a part of "demanding language" in the domain of consequence (4.C.b). In doing so, she apparently recognizes the necessity of this discourse practice, which has been talked about for a long time in the PD program before. In contrast to the other teachers at the beginning, she has developed a strategy for herself to make this action feasible for the classroom. This addresses the personal domain (3.A.c). To what extent this really succeeds in practice cannot be clarified at this point, of course.

Tremnitz (#16) also takes up a didactic category by building upon his experience in the domain of practice with one of the PD activities (2.A.a). He explains how he has thought about how he could call for diverse language actions and, in particular, how he could diagnose them. By connecting this to his last own teaching experiences (domain of practice 2.A.c), he concludes that writing assignments are well suited to this. He also relates this to the institutional framework (#18). He states that written products are always assessed in examinations, which also belongs to the domain of practice but with regard to the professional context (2.B.c). There should also be corresponding learning opportunities in the classroom, which addresses the domain of consequence (4.C.c).

#16 Tremnitz: It helped me a lot that I was able to read the lesson plans again and I also found written language in class tests and exams. So, I thought about what I could do to extend the language action to a variety of language actions. So not about the class discussion, but when students really write things down. Where can I perhaps read them in again [promised]. Because in what is actually written I have a second to look at it again and do not have to think about it in moderation, but actually have to do justice to other students. So … language now not only on the spoken language, but also on the written language even more … to direct.

#17 Facilitator: This is a tremendous relief strategy. If you can just think for a moment how I'm going to react to this, right?

#18 Tremnitz: The last homework was very useful, because I understood the plurality of thinking of my students better, but if it is now also about language, … that is the means of language, which we also use in school, next to the talk -, next to the class discussion is in the foreground and in the exams, in the performance reviews, it is actually the means that we write something in writing. When do we ever have an oral exam in math, except at the end or when a few of us have failed and then have to take the final exam.

Figure 2 summarizes the identified signs of resonance (horizontal arrows indicate acknowledging the PD ingredients, vertical arrows indicate resonance between the change domains). A dark solid line stands for positive resonance, a dark solid line without color inside means here something does not fit. Dashed line means that a difficulty is seen, i.e., one cannot say, for example, whether something will become a consequence for example. The signs of resonance are diverse and come from different domains. When talking about the personal domain, it tends to be about personal difficulties. The domain of practice is either about reflecting on teaching actions or about planned teaching actions. The domain of consequence tends to be more about consequences for lesson preparation than about the consequences that actions might have.

It is noticeable that most teachers only pick up individual concrete tools. Only two teachers take up the didactic categories, which are rather comprehensive. However, both take up the categories to the extent that they find that implementation is difficult. Experiences from PD activities can seemingly be just as important as personal experiences from one's own teaching.

Insights in Resonance Processes in a Project on Students' Strategies on Problem-Solving

The following examples serve to illustrate how the matrix was used for pinpointing various aspects of the teachers' experiences from the project. Next, this helped to provide a more nuanced interpretation of the teachers' choices and enactment of the project's visions. In the examples, the two teachers asked distinct types of questions in the classrooms in line with those different aspects.

Alice's and Maj's statements must be understood in the context of the project, where Maj and Alice taught two classes partly together. They brought up in the group discussions how their students were reluctant, sometimes opposed, to participating in activities, which they thought would not directly pay out in the examination. The teachers expected support in the project to introduce the more open-ended and creative group-work by the students. They also expected support when struggling with reluctant or unwilling students. Hence, Maj and Alice were aware of the students' reactions to the project's teaching style. They wanted to justify the project by pinpointing outcomes relevant for the examination.

Alice and Maj had experienced students unwilling, also, to solve tasks without having a well-known solution strategy and, thereby, to give up their "secure position". Alice had experienced the students' demands for immediate help but in her view the students had to be forced to work on their own before getting help. Otherwise, she believed, they would not get a profound learning outcome. The following (2, 3, and 4) is one statement from Alice in the second interview. Alice emphasized the relevance for the oral examination of the students' presentations and, implicitly, defended her decisions against students' potential critique.

Table 2. Excerpt from Alice's teaching

No	Act (L: Alice & E: Student)	Statement
1	L writes example of series' elements on blackboard while standing & talking: 1, 5, 9, 13; 1+5+9+13+ …	L: If we take these numbers & try to sum up.
2	L turns around facing class.	(Tacit)
3	L turns back towards blackboard continues to talk & write: 1+5+9+13+ … & $a_1 = 1, a_2 = 5 = a_1 + 4, a_3 = a_2 + 4, etc.$	L: And we find.
4	L turns around facing class.	L: What is it that we find?
5	Facing class, shoulder to blackboard, points to text while talking.	L: This series is called arithmetic series. We call it so because it follows a certain pattern.
6		L: Do you see the pattern?
7	L points to terms on blackboard.	L: We look at first term, look at the second term.
8		E: It jumps by four.
9	L writes: $a_n = a_{n-1} + 4$.	L: It jumps by four. I add four each time I take one step.
10	L turns around facing class.	L: Thus, increase each term by four & that is called the series' … the difference

She implicitly referred to this discrepancy between mutual demands between herself and the students and addressed a lack of immediate result of the teaching experiment.

> #1 Maj: The students spent a long time on the series project, and we collaborated [with Alice]. The students liked it but they did not bring it with them. They felt that they had spent much time on something different from what they should do. But they engaged in the discussions and most of them succeeded. It was funny but I do not know how they will benefit from it in the future. Maybe they will see later that it was useful.

> #2 Alice: I totally agree in what you tell.

This (#2) refers to the students' reactions to the content ('unusual tasks' are manifest of inquiry and creativity, i.e., 1.A.a in the matrix). Alice considers it in the classroom context, i.e., she applies a sociological perspective on the project's idea of inquiry. The sociological perspective relates to her professional context. Statement #2, hence, is assigned to cell 1.B.a. Alice agrees (#2) with Maj's interpretation (#1) that the students did not really accept the tasks. Therefore, in our interpretation, there is a tension between 1.A.a and 1.B.a here.

In #2 Alice also refers to her personal observations and experiences, i.e., to her view from a psychological perspective (column C) on the teachers' choice of theme and plan for the teaching sequence ('teachers' choice of class, theme and plan' is a key ingredient in the domain of practice, i.e., 2.A.a in the matrix), by evaluating and commenting on the atmosphere in the classroom and on the feedback from students. The psychological perspective relates to her professional profile. Therefore, Alice's statement #2 is also assigned to cell 2.C.a in the matrix. Regarding the personal observations and experiences, Alice agrees with Maj's interpretation that the atmosphere and feedback was positive. In our interpretation that would mean a sign of positive resonance between 2.A.a and 2.C.a in the statement.

> #3 Alice: I found it particularly useful for them to make the presentation. They will benefit from that at the oral examination.

Here (#3), Alice evaluates one of her choices ('teachers' choice of class, theme and plan' is assigned to 2.A.a) in the professional context (column B) by talking about the students' prospective benefit from her organizing. Therefore, the statement is assigned to cell 2.B.a in the matrix. The statement is positive, hence, in our interpretation there is a sign of positive resonance between 2.A.a and 2.B.a in this statement.

> #4 Alice: But when doing series–tasks, they do not look more for patterns. They still want help immediately.

In #4, Alice evaluates her students' reactions (classroom context; sociological perspective, column B) to the content ('unusual tasks' is assigned to cell 1.A.a), which means that #4 is assigned to cell 1.B.a. According to Alice, the students' reactions show lack of success with regard to 1.A.a, which means, in our interpretation, a tension in statement #4 between 1.A.a and 1.B.a. In #4, Alice also evaluates the students' intellectual independence (which is assigned to cell 1.A.b). Therefore, #4 is also assigned to cell 1.B.b. Alice notices that the students' attitudes have remained unchanged. In our interpretation, hence, there is a tension between 1.A.b and 1.B.b in statement #4.

In the following excerpt (**Table 2**), Alice offers a paradigmatic example of 'learner's inquiry' to the students: Alice demonstrates to the students how they are supposed to deduce the expression of a series from the series' first numbers. She asks questions and gives the answer herself, to show a reasoning procedure, which may lead to the result. By doing that, Alice adapts the project's inquiry approach to the teaching style, which she and the class by negotiation have established as the norm for classroom teaching. The excerpt was cut from a teaching sequence on series, which is described in detail by Andresen and Brun (2015).

Alice agreed with the key ingredients (column A), in accordance with the resonance between 1.A.a and 1.C.a. The contextualized statement from the interview pointed to the students' reactions ('B. Professional context' in the matrix). Hence, we did not see any signs of resonance neither between 1.A.a and 1.B.a nor between 1.A.b and 1.B.b. The excerpt demonstrates how Alice adapted an inquiry teaching style to the demands from her students. She gave a paradigmatic example of problem solving – a 'safe way' for the students to meet an unfamiliar challenge. In this case there was a strong resonance by the teacher with the goals and content. The resonance that was revealed in the teachers' utterance was in the personal domain. Finally, her envision of a changed practice in the future, revealed in the interview and in the group discussions during the project, should be assigned

to the domain of consequences. To Alice, hence, her agreement with the goals (apparent in the resonance between 1.A.a and 1.C.a) translates from external domain to domain of consequence despite the tensions between the cells in column A and B.

Niels shows a different kind of resonance. Niels's statements below must, as well, be understood in the context established by discussions and talk in the group during the project. In the project Niels and one of his colleagues sometimes joined their classes together and split them in two parts after the students' capabilities. In the group discussions the two teachers had explained their concerns about the variety in the students' activities and reactions.

Niels and his colleague were also concerned about the students' willingness, and capability, to think creatively, and to develop mathematical ideas on their own. According to statements in the group's discussions Niels expected that the project would point out a direction for him to handle the variety in his students' capabilities and motivation. The project's set up with students' group work on problem solving could be a way to differentiate between the different groups of students and maybe manage some of those issues. On this background Niels was eager to set new standards for encouraging discussions in the classroom and in the groups of students.

In his statement below, Niels points to his own, new role as more of a coach or supervisor, and to the new demands this role puts on him. Niels reflects on the new teaching style he must develop, and he expresses a positive attitude towards it although it will be complicated. Niels is positively engaged in the project's form of classroom interaction. The last part of the statement addresses the observation that one part of the students never started the group work. He does not suggest any explanations for that. The following (#101, #102, #103, and #104) is one statement from Niels in the second interview.

#101 Niels: My role is turned upside down; they must be asked questions that make them think further.

Here Niels points to his role as a teacher, which is in the domain of practice (assigned to cell 2.A.b in the matrix). By talking about how he felt a need for changing it and by giving personal reflections on the development of his own teaching style, Niels' statement (#101) was aligned with his professional profile (column C) and, hence, assigned to cell 2.C.b in the matrix. Niels seems to accept and engage in the need for change. Therefore, in our interpretation, there is a positive sign of resonance between cell 2.A.b and 2.C.b in the matrix, expressed in statement #101.

#102 Niels: Give examples with tools they can use.

This is Niels' reflection on how to show accountability to the subject (which was assigned to cell 3.A.b in the matrix). Reflections upon one's own teaching are aligned in the matrix with column C. Hence, statement #102 was assigned to cell 3.C.b in the matrix. Like in the previous example, Niels does not question the need for a change so, in our interpretation, there is a sign of resonance between cell 3.A.b and 3.C.b in statement #102.

#103 Niels: That is complicated.

Niels gives personal reflections (aligned with psychological perspective related with professional profile; column C) on the project's inquiry approach (cell 3.A.a), therefore, statement #103 was assigned to cell 3.C.a in the matrix. When Niels finds it complicated, it is not easy to say whether there may be a sign of resonance between cell 3.A.a and 3.C.a in statement #103.

#104 Niels: Some of the groups did not really start.

Niels talks about the students' reaction (which is column B: professional context) to the unusual working style, ('unusual working style' is manifest of 1.A.c working mathematically in a problem setting). Statement #104, hence, was assigned to cell 1.B.c in the matrix. The statement reveals some dissatisfaction, i.e., it is not positive and therefore, we do not interpret it as a positive sign of resonance between cell 1.A.c and 1.B.c.

In the following excerpt (**Table 3**) Niels asks the students the kind of questions they are supposed to ask themselves. By asking those guiding questions, he shows how the students can start a reasoning procedure, which may lead to a result. Niels is careful about not giving the answers himself and insists on asking guiding questions until he gets an answer from somebody. The excerpt was cut from a teaching sequence on descriptive statistics, which is described in detail by Andresen and Brun (2015).

Niels, as the rest of the teachers, agreed with all the PD's goals, according to his interactions in the project. Our interpretation of the contextualized statement had pinpointed his development of a new teaching style, suitable for his new role, as the issue at stake. This issue was organized under 'C. Professional profile' in the matrix. There was good resonance between the teacher and the project's means, which he saw as a potential tool for overcoming actual difficulties in his classroom. This is in accordance with the signs of resonance between 2.A.b and 2.C.b, and between 3.A.b and 3.C.b, in our findings.

The excerpt from the video demonstrates Niels' attempts to ask more, and more insistent, questions to the students in accordance with his interpretation of the new role, in the statement from the interview. This was aligned with the domain of practice. Niels' try out of this more coaching role and his reflections upon the students' reactions in the interview can, in addition, be interpreted as steps of translation of change towards the new role for the teacher in the project, which was aligned with resonance between cell 4.C.b and cell 4.A.b in the matrix, in the domain of consequence. Like in the case of Alice, all four domains are involved for Niels. Apparently, Niels put most emphasis on change in the personal domain by focusing on his own role. Besides, Niels' wish to solve the problem with inactive students stresses his need for translation of the changed practice into the domain of consequences.

Both Alice and Niels agreed with the goals of the PD, that is, we saw signs of resonance between cells in column A and cells in column C (see **Figure 3**). They both revealed signs of resonance in their involvement with the group, in the interviews, and in their teaching in the experiment.

Table 3. Excerpt from Niels' teaching

No	Act (L: Niels & E: Student)	Statement
1	L is facing class while talking. Slide says arithmetic mean & spread.	L: We have new concepts of arithmetic mean & spread. It is already clear from names what this is about. Mean is in center, so this is about center of our data, & then we measure how they are spread out.
2	L goes to next slide showing graphs of two discrete sets of data with distinct distributions.	L: Take a look at this example with two sets of possible ratings in two different classes. All red dots mark students' individual rates.
3	L points to data on slide while explaining how to read graph.	L: Two students got a 1, (…), but now, we take a look at other class, here.
4	L points to slide while taking, facing class.	L: And when you have such sets of raw data, such type of an inquiry, (…), then will those two concepts be useful. How can we compare & analyze two sets?
5	L steps aside, facing class.	L: Anyone who can imagine? Yes?
6	E raises a hand, mumbling.	E: Another examination?
7	L steps aside, facing class.	L: Yes, but that is about method. What if we only want to look at results?
8	E raises a hand, mumbling.	E: Look at rates.
9	L stands at one side of blackboard, facing class.	L: We can look at that. And how can we manage this?
10	E raises a hand, mumbling.	E: Look at rates one by one.
11	L stands at one side of blackboard, facing class.	L: Look at rates one by one.
12	L stands at one side of blackboard, facing class.	L: We can look at average rate. How can we calculate average?
13	E sits mumbling, only moving his hands while explaining.	E: (Explains how to calculate average value of rates).
14	L stands at one side of blackboard, facing class.	L: We just talked about mean value & spread. Which one of these are linked with average?
15	E sits without moving.	E: (Small sound, not understandable).
16	L stands at one side of blackboard, facing class.	L repeats question.
17	L moves to screen & starts drawing on slide.	L starts to explain how mean value appears from graphical representations in slide.

For Alice, resonance in the example concerned the professional context, i.e., students' reactions to the unusual tasks and working style in the experiment. In Niels' example, the resonance concerned his professional profile, i.e., development of his new role as a teacher, which he saw more like a supervisor or coach in the experiment. Alice adapted her teaching to the demands of the students by providing a question–and–answer sequence, whereas Niels realized his ideas about coaching students by asking insistent questions, answered by a handful of students. During their reflections, Alice addressed the lack of immediate results or change. Niels, as well, mentioned that one part of the class never entered the group work. Both teachers, in agreement with the rest of the group, had concluded that more time and longer teaching experiments would be needed for a real change.

The two examples of Alice and Niels from a project, where resonance had occurred illustrate changes in the domain of practice translated into the domain of consequences. The changes were interpretations based on our identification of signs of resonance between data assigned to cells in the matrix. The first example of resonance concerned more the teacher's professional context; the second example of resonance concerned more the teacher's professional profile.

RESULTS AND DISCUSSION

Our research question (**RQ**) was "which signs of resonance can be identified on the micro level in the teachers' professionalization processes in a PD program?" For answering this question, the empirical examples provide insights in two different possible uses of the resonance matrix:

(1) for analyzing the resonance within a PD program and

(2) for analyzing the resonance as impact of PD on teachers' performance in classrooms.

Accordingly, the signs of resonance in the first, German example are in the form of utterances of agreement with the PD ingredients whereas the signs of resonance in the second, Norwegian example are in the form of teachers' performance in agreement with single ingredients of the PD.

Data Set One

The resonance matrix makes it possible to visualize the PD contents taken up and the different signs of resonance (**Table 2**). In the preview perspective on teaching, this shows that it is mainly the perspective of the professional profile that is addressed. In the professional context, observed classroom action is only addressed once, as well as examination requirements. Where concrete options for action for the domain of practice are concerned, there seems to be more resonance at first glance than, where the more fundamental acquisition of knowledge in the personal domain is concerned. Of course, the fact that teachers see difficulties here does not necessarily mean that there is no resonance. It could also be that dealing with this content leads to implementation in the medium term (and that not dealing with it at all stands for a lack of readiness).

A limitation of this analysis is that the teachers were of course not forced to express themselves and therefore some activated signs of resonance may not have been made explicit. Further analyses could also reconstruct whether certain types of teachers can be identified who also show similar signs of resonance in different situations, or whether the response and their kind of resonance differ in individual situations. In any case, the different signs of resonance indicate that not all teachers are "picked up" with the same argument or the same continuing PD design. It was possible to identify different signs of resonance and thus possible reasons for the acceptance or rejection of new PD contents. This is important because these kinds of resonance must be

approached differently. Those who do not consider it relevant presumably need to be further sensitized. Those who think it is relevant but difficult to implement need further examples of what it might look like in concrete terms. Those who believe that they cannot implement it in spite of everything because of reservations on the part of their school management or the like need arguments to convince the school management etc. By focusing on the teachers' resonance, these starting points for further professionalization can be made visible. This is the basis for the adaptive design of PD programs to the needs of the teachers.

Data Set Two

Although Alice and Niels in line with the rest of the group pointed to the absence of convincing results in the form of fundamental changes of students' attitudes, none of them expressed doubts about the project's basic ideas, goals and means (**Figure 3**). On the contrary, based on their critical reflections there was a common agreement in the group that the experiences from the project would serve as a platform for their future professional work. The lack of immediate results was explained by the group by the short period of time spent on the experiment.

These facts point to support our claim of resonance in the project. Resonance was not only between the project's goals and means and the teachers' visions about teaching. The project's design, which gave room for the individual teacher's choice of class, organization and content of the experiment, was tailored for the level of professionalism in the group of teachers. The participating teachers would be ranked at level K2 in E. Dale's (1998, p. 169) taxonomy. This level necessitates reflections by the teachers upon, and evaluation of, curriculum and teamwork with colleagues about the organization of teaching. Likewise, the organization of the project group's work with studies, discussions and exchange of experiences was appreciated by the teachers for supporting their professional development.

Figure 3. Horizontal & vertical resonance of teachers in a PD program on problem-solving (Source: Authors' own elaboration)

The Norwegian cases illustrate the effect of the resonance on the implementation of change in teaching: Both teachers choose tasks and a working style distinct from what was usual in the classes. They encouraged the students' independent thinking; one teacher changed her way of reviewing the task, the other changed his questioning technique.

This presentation of the analysis demonstrates details of resonance, where the units of analysis are single statements and related teaching segments by one teacher. The cases concerned resonance in the professional context and professional profile, resp., chosen to show various incidents of resonance. Importantly, the intention here was not to classify or characterize teachers after, *where* resonance occurred. Alice and Niels might, or might not, react to other perspectives on the project's goals and means as well–that would not interfere with the cases in data set two. The fine details of resonance are important for understanding the teachers' learning processes. In each of the two cases, use of the matrix structure supported the analysis by laying out a red thread for interpretation of the teacher's statements and teaching segments.

Another limitation of this study is that in both data sets, some teachers also partly pick up exactly what was said before. Possibly in face-to-face situations other things would have been mentioned.

CONCLUSIONS

The study contributes to elaborate on the idea of analysis and re-analysis of teachers' learning processes in PD programs. We introduced the term "resonance" to analyze teachers' learning processes by describing the phenomenon that an active process of engagement is set in motion, usually triggered by a specific impulse. Introduction of this term helped us to capture possible mechanisms behind changes that occurred at the micro level during the PD program, within and between the domains introduced by Clarke and Hollingsworth (2002).

Taking as our starting point that teachers are active learners who actively control their learning process, we inquired what ingredients of the PD program the teachers picked up and took to their heart, seen from the perspective of their professional context or professional profile. Our underlying assumption was that teachers in general address exactly those ingredients in the PD program that are important to them. Thus, analysis of their statements in terms of resonance could provide more detailed information about how the professional context or the professional profile might trigger conditions for success in achieving the goals of the PD program.

The analyses of data revealed explicit signs of resonance in the form of direct statements by the teachers, i.e., in data set one, Tremnitz (#18). They also revealed implicit signs of resonance based on our interpretations of actions, i.e., the discussion of the excerpt in **Table 3** in data set two.

We observed that various ingredients inspired and pushed the teachers at the micro level. For example, some referred more often to the professional context and others to their professional profile. In addition, some teachers were very concerned that the goals should align with their personal beliefs, while others addressed the need for the goals to match their classroom experiences. One explanation could be that the situation in the German project involved a concrete thematic focus, which was met with different previous experiences and attitudes. In the Norwegian project, many ingredients were discussed and settled in advance and not mentioned in the teachers' reactions in data. Furthermore, it became apparent that the response did not need to be immediately positive in order to initiate a productive teacher change. It seemed to be more relevant whether the teachers were touched or not, for dealing with the PD content. This is in line with the presumption that an intensive discussion is a prerequisite for teacher change (Gregoire, 2003).

The matrix made it possible to identify navigation processes between the addressed domains of change and the addressed professional context or professional profile. We found that not all domains were of equal importance to teachers, neither were the professional context and the professional profile. However, by comparing several teachers in a matrix, tendencies for the group as a whole can still be derived. In the German project, it is noticeable that the teachers mainly talked about their professional profile and, in particular, about the question to what extent the ideas from the PD program will influence their teaching.

In the Norwegian project, the teachers were much more concerned with the question of reactions in the professional context. We point to the major differences between the projects and cannot make a general statement about the teachers in the two countries. Nevertheless, it is interesting to notice these differences in what is important to the teachers during the PD program or, at least, in what they address. The two sets of data illustrate a wider range of possible aspects and domains of change to be addressed. A facilitator should be able to respond to this flexibly.

As a consequence, for the design of PD programs, we conclude that a range of appropriately diverse ingredients should be addressed in order to create (potential) connecting points, conducive to resonance. The guidelines for participatory design (Könings et al., 2014; Spinuzzi, 2005) will then offer good opportunities for reflections in advance, leading to the creation of contact points supporting resonance in the PD. In addition, we assume it helpful that facilitators are sensitized to different forms, triggers and conditions for potential resonance in order to be able to respond adequately to the participants' reactions.

This study focused on teachers' resonance at the micro level, that means in a short-term perspective. It would be fruitful to further investigate long-term effects of resonance within the project, or to analyze how teachers' resonance develops over time.

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