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Social and socio-mathematical norms constructed by teachers in classes through the development of noticing skills

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ABSTRACT

This paper examined how teachers' noticing skill affects the social and socio-mathematical norms they construct in the classroom. The study was carried out with four mathematics teachers working at a secondary school in the eastern regions of Turkey. A case study among qualitative research methods was used. The data of the research were collected via videotaped lessons. The data obtained from teachers' classroom practices were evaluated performing descriptive and content analysis. Research findings show that noticing skill positively improves secondary school mathematics teachers' ability to construct social and socio-mathematical norms. In addition, it has been noticed that social norms and socio-mathematical norms are not independent of each other.

Keywords: social norm, socio-mathematical norm, teacher noticing, video club

INTRODUCTION

Culture can be defined as a whole of actions and discourses created by a society or community in an interactive process. Culture constructs the norms that are defined as implicit rules that are not written in the system of values it contains and determines the limits of how individuals who live with these norms should behave. Classes, micro-cultural areas, are settings in which norms that determine and characterize the roles of teachers and students in the classroom are constructed. According to Sekiguchi (2005), a classroom is a community of practice, where teachers and students gather, share common goals, discuss, and shape their roles. As a result of mutual expectations in the classroom, this community generates, maintains, modifies, or completely eliminates various patterns such as norms, obligations, rules, and routines.

Accordingly, in order to understand the classroom process, which includes all the variables (e.g., teacher behavior, student behavior, and other situations) that may occur in the classroom and explain how this process developed, Yackel et al. (1991) introduced the concept of classroom social norms in their study. The researchers defined this concept as implicit rules that emerge as a result of interaction between teachers and students in the classroom and are located in the micro cultural area of that class. These norms are valid for all subject areas and each classroom can develop them according to its own micro cultural structure. Although social norms form a basis for determining the roles of teachers and students in the classroom, it is necessary to establish mathematical norms for the execution of mathematical activities (Roy et al., 2014). In this respect, socio-mathematical norms highlight the normative aspects of discussions specific to mathematical activities (Yackel & Cobb, 1996). These norms are social norms or social interactions depending on mathematical content, materials, and learning (Widodo et al., 2019). For instance, it is a social norm for both students and teachers to be aware of justifying mathematical explanations while basing the justifications on acceptable mathematical explanations is a socio-mathematical norm. In this sense, while social norms regulate students' participation in discussions, socio-mathematical norms are related to the quality of students' mathematical participation (Partanen & Kaasıla, 2014).

The implementation of socio-mathematical norms in the classroom supports the mathematical development of both teacher and students (McClain & Cobb, 2001). However, mathematical development also requires a questioning perspective and a questioning classroom environment in which explanations are presented with reasons and mathematical differences are discussed should be created for the development of socio-mathematical norms. This fact helps teachers take a central place in the development of norms by promoting the quality of classroom discourses. Obviously, teachers have an important role in creating norms, and understanding the importance and effects of norms in the learning environment is considered the first step in constructing norms (Van Zoest et al., 2012). Therefore, the role of teachers in the formation and development of norms has been the focus of many studies (e.g., Dixon et al., 2009; Jankvist et al., 2018; Kazemi & Stipek, 2001; McClain & Cobb, 2001; Yackel, 2002).

Teachers need to make sense of what and how students think in order to develop an inquiry-based learning environment and construct a classroom culture in this direction. Therefore, the most important point for teachers is to notice what is important in the classroom environment and to develop effective reasoning strategies on these situations (van Es & Sherin, 2006). The more the teacher is aware of the student, the more effective instructional decisions can be made, and the more sensitive teaching practices can be developed in this direction. The relationship between students' thinking and instructional decisions requires teachers to critically evaluate their teaching practices. This can be possible with learning environments that will direct teachers' attention to the important aspects of classroom interactions and help them define their teaching practices (Sherin & van Es, 2009). Video-based professional development environments, which have become especially popular in recent years, provide opportunities for teachers to develop their ability to attend and interpret meaningful situations in the classroom (van Es & Sherin, 2008). Video clubs, which are widely used for this purpose, are environments, where teachers come together at regular intervals to watch their or their colleagues' video-recorded lessons and have reflective discussions on the situations they notice. In this respect, it is regarded as a professional development process used to stimulate teachers' sensitive teaching practices (Sherin & van Es, 2009; Tekkumru-Kisa & Stein, 2017).

When the related studies were examined, some researchers focused on the situations that teachers observed through video clubs and were interested in characterizing these situations (e.g., Star & Strickland, 2008; van Es, 2011). Some researchers have focused on students' mathematical thinking by addressing a more specific aspect of noticing (e.g., Goldsmith & Seago, 2011; Jacobs et al., 2010; Luna et al., 2018; Sherin & van Es, 2009; van Es & Sherin, 2008; Walkoe, 2015). Others examined the changes in classroom practices of teachers who participated in the video club (e.g., Borko et al., 2014; Ozdemir Baki & Kilicoglu, 2020; Seago, 2004; Seago et al., 2018; Sun & van Es, 2015). Although the studies have expanded in different subject contexts and focal points, these studies clearly reveal that video clubs are effective in the development of teachers' noticing skills. Undoubtedly, these results bring to mind the idea that the development of noticing skills of teachers participating in the video club will have a healing effect on the social and socio-mathematical norms they construct in the classroom. From this point of view, the study focused on the social and socio-mathematical norms that teachers who employ their noticing skills construct in their classrooms.

THEORETICAL FRAMEWORK

Teacher Noticing

We can define noticing as focusing on something that draws our attention in our environment or understanding a situation that creates complexity in our minds. The teacher's noticing as a structure includes considering classroom events and making sense of these events. In the related literature, there are different conceptualizations of teacher noticing based on Goodwin's (1994) concept of professional vision and Mason's (2002) studies on noticing discipline. For example, van Es and Sherin (2002) stated that teacher noticing consists of three processes:

(a) identifying important events in the classroom,

(b) reasoning about these events using knowledge of the context, and

(c) making connections between specific events and broader teaching and learning principles.

Jacobs et al. (2010) more specifically defined noticing students' mathematical thinking as involving three interrelated skills. These are attending in student strategies, interpreting students' mathematical understanding, and deciding how to respond based on students' understandings.

In many studies made upon the teacher's noticing, the recorded lesson videos were used as a learning tool. In some studies, video clips containing certain moments were used for teachers to learn to notice (e.g., Jilk, 2016; Sherin & van Es, 2009), while in others, videos recorded by the whole lesson were used (e.g., Santagata & Angelici, 2010). Guler and Celik (2022), on the other hand, included both video-recorded lessons and video clips in their work process on the e-mentoring experience, considering the positive effects of both practices in terms of professional development. Recent studies have turned to the use of 360-degree videos (videos with omnidirectional recording) and representations such as extended reality (XR) instead of standard videos (Kosko et al., 2020, 2022). In this direction, Kosko (2022) used holographic representations in his study to support prospective teachers to notice students' mathematical reasoning. In the research, it was revealed that it is more beneficial for pre-service teachers to view holograms first than viewing standard videos. According to the researcher, the use of representations such as 360-degree videos and expanded reality makes it easier for teachers to notice, as they better approach the feeling of being in the classroom. As a result, different types of video applications opened to the classrooms allowed teachers to see what was going on in the classroom.

In the studies conducted in this direction, special frameworks were used to analyse the teachers' noticing skills and interpret classroom interactions. For example, Walkoe (2015) developed an "algebraic thinking framework" to develop pre-service teachers' noticing based on students' algebraic thinking and used it to structure the discussions in the video club. Similarly, Guler et al. (2020) investigated the effect of an alternative teaching sequence enriched with videos to improve pre-service teachers' noticing skills. In order to evaluate the noticing skills of the participating teachers, they asked the participants to take note of the critical events they observed in the classroom and explain why they found these events remarkable. Researchers used Santagata and Guarino's (2011) lesson analysis framework. On the other hand, van es and Sherin (2021) expanded their previous conceptualization of teacher noticing and revised the learning framework they developed in 2002 for learning to notice. In the expanded framework, the researchers included the "shaping" dimension in addition to the attention and interpretation dimensions. The shaping dimension involves teachers constructing interactions to access additional information to support their greater noticing. According to the researchers, this dimension requires the teacher to interact actively with the students in the process of noticing. The current study is based on the definition of noticing revised by van Es and Sherin (2021). In the study, the social and socio-mathematical norms that the teachers who participated in the video club process created in the classrooms as a result of the development of their attention and interpretation skills were examined. Therefore, this study argues that shaping skill, which is considered as the third dimension of noticing, is actually related to the social and socio-mathematical norms that teachers construct in the classroom.

Teacher Noticing in Constructing Social and Socio-Mathematical Norms

The classroom social norm is a concept put forward by Yackel et al. (1991) as a result of the experimental study they conducted in 2nd grade mathematics lessons of a primary school. Yackel et al. (1991) described this concept as the implicit rules that each student adopts even if they are not written and emerges as a result of the individuals' interactions in the classroom and settles in the micro cultural structure of that classroom. Classroom social norms are formed in accordance with the mutual expectations and beliefs of teachers and students (Yackel, 2000). In addition, these norms are very important in determining which behavior is important in a classroom or which behaviors are more valuable (Ozmantar et al., 2009). Furthermore, they help teachers develop a common understanding of students' positive and negative behavior patterns (Loh et al., 2009). Therefore, the norms created by each classroom in accordance with its micro cultural structure are what distinguish one classroom from another. For instance, in a classroom, expectations such as students' expressing situations they do not understand, explaining their solutions, justifying their explanations, and trying to understand the thoughts shared in the classroom constitute the social norms of that classroom. However, Yackel and Cobb (1996) who argued that such norms are valid for every subject field and are general norms that sustain classroom micro cultures, put forward the concept of socio-mathematical norm by expanding their studies with the normative aspects of in-class mathematical activities.

Socio-mathematical norms are different from social norms since they form structure specific to mathematical practices (Kazemi & Stipek, 2001). Sekiguchi (2005) used the concept of mathematical norm instead of socio-mathematical norm and described this concept as important cultural knowledge of mathematical activities. In this sense, Sfard (2008) pointed out that a behavior should be adopted by the majority of the class in order to identify it as a social or socio-mathematical norm. Researcher also suggested that this behavior should be prominently manifested in classroom interactions by students' and teachers' internalizing. Although social norms are the basis for identifying the classroom roles of teachers and students, socio-mathematical norms must be applied for mathematical activities (Roy et al., 2014). For example, it is a social norm to expect the student to offer a different solution to the problem while expecting him/her to offer a solution with a mathematical difference is considered as a socio-mathematical norm (Yackel & Cobb, 1996). Similarly, the fact that students in a classroom should be aware how to explain their reasons while answering the question is a social norm.

It is a socio-mathematical norm that students' answers should include persuasive and acceptable mathematical explanations (Iannone & Cockburn, 2008). Therefore, while social norms refer to the way students participate in classroom activities and methods (Kazemi, 2008), socio-mathematical norms refer to regulations that affect the development of participation and practices in mathematical activities (Gorgorio & Planas, 2005, p. 94). These types of norms are shaped by mutual expectations and common discussions of both the teacher, and the students. The creation of a learning environment based on research and inquiry for having these norms formed needs to be emphasized here because the norms that may occur in a learning environment have aspects specific to the approach of questioning (Yackel, 2001).

These results indicate that teachers who adopt an inquiring analytical approach by participating in the video club can also be effective in using norms based on this approach in the learning environment. Group analysis of video-recorded lessons allows for expanded perspectives on the same event. Therefore, since teachers can change various perspectives in this process, they develop an inquiring perspective to make sense of student thinking (Tsai, 2007). For example, Sun and van Es (2015) revealed that pre-service teachers who made video analyses used more practices that focused on student thinking over time. Similarly, Ozdemir Baki and Kilicoglu (2020) argued that teachers participating in video clubs use different approaches to reveal students' thinking in their classroom practices. Researchers have revealed that teachers tend to use approaches such as questioning students' prior knowledge, prompting students for further explanations, and asking students to explain their answers and reasoning. These research results provide evidence that video clubs improve teachers' classroom practices. These evidence indicate that the approaches adopted by the teachers participating in the video club in their classroom practices will be internalized by the teacher and the student over time and direct the construction of norms.

In the current study, teachers' development of rich discourses about teaching practices in video club meetings may also provide an opportunity to differentiate their classroom interactions with their students. In this study, teachers' development of rich discourses about their instructional practices based on student thinking in video club meetings will also provide an opportunity to differentiate classroom interactions. Therefore, it is foreseen that teachers will shape the skills they have gained by experiencing them with their colleagues in the classroom environment by interacting with their students.

The current study aims to extend previous research on teacher noticing by making important claims about the impact of video club participation on teachers' instruction. In particular, the study suggests that the result of teachers developing their noticing skills in a video club will be an increase in the teacher's use of social and socio-mathematical norms in the classroom. This is an important topic for several reasons. First, a great deal of research on professional development has used self-report data from teachers to make claims about the impact of the program. Looking inside teachers' classrooms before and after the program is therefore an important step. Second, there are many aspects of teacher expertise, one of which is teacher noticing. Research that is able to link teachers' noticing skills with other classroom practices will help us to better understand how teacher noticing impacts opportunities for student learning. Finally, we provide a strong rationale for linking the development of a teacher's noticing skills and the ability to construct social and socio-mathematical norms. We claim that the interaction dimension (shaping) of the noticing skills can only be explained by observing the norms established in the classrooms. Undoubtedly, investigating this relationship will provide valuable information for the field. The findings of the study will give clues to explain which norms come to the forefront in the classrooms with the use of noticing skills. Under this point of view, an answer to the following question was sought in the study.

1. How does teacher noticing affect secondary mathematics teachers' skills to construct social and socio-mathematical norms?

METHOD

Research Design

In our previous study (Ozdemir Baki & Kilicoglu, 2020), we revealed what kind of changes occurred in the classroom practices of teachers who participated in the video club. In this study, we examined the social and socio-mathematical norms constructed by the teachers who developed the noticing skills as a result of participating in the video club. The current study was a case study based on qualitative research. Case study, in general, is an approach based on the thorough definition and analysis of a limited system (Merriam, 2009). In addition, it provides an important advantage in cases, where "how" and "why" questions are focused (Yin, 2008, p. 13). The analysis unit of this research was the teachers' behaviors reflecting the norms they created in their classrooms. The researchers tried to exemplify in-class interactions related to social and socio-mathematical norms by concentrating on these norms having occurred in the learning environment. For this purpose, the rich conversations involving classroom interactions were frequently included in the study.

Participants

This study was conducted with four mathematics teachers working at a state secondary school located in the east of Turkey. In fact, five teachers were involved in the video club. However, one of the teachers left the study due to out-of-province assignment. For this reason, four teachers' lessons were video-recorded after the video club process. Teachers' professional experience was over five years, as the study focused on the social and socio-mathematical norms that experienced teachers construct in classrooms (Berliner, 1994). All of the teachers participating in the study were women and their teaching experience ranged from six to eleven years. The teachers' having at least six years of professional experience was taken into consideration with the idea that it would facilitate their focus on important aspects of classroom interactions. The teachers' working at the same school made it easier for them to gather and allowed them to keep in touch. Due to the ethics of the research, code names were preferred instead of the teachers' real names.

Data Collection

In this study, video recordings of teachers' lessons and field notes taken by the researchers were used as the data collection tools. Before and after the video club process of the teachers, two lesson hours were recorded every week for six weeks. These records were used to examine the norms that teachers constructed in their classrooms. The lessons were recorded by the first author. During the video recording, the necessary situations were tried to be clarified with close-ups. The video camera was first placed at the back of the classroom. However, it was determined that the individual work of the students could not be observed sufficiently in the fixed shooting. For this reason, in order to clarify individual studies with close-up shots, shots were taken in which the researcher was active in the classroom. Thus, it was tried to reduce the negligence cases in the observation of classroom situations. Sherin et al. (2021) stated that camera footage can distract students and suggested that discussing why lessons are recorded is a good way to overcome this. Therefore, the researcher helped the students to adapt by discussing with the students why she was shooting an active video rather than a fixed video. In addition, since the first author worked at the same school with the teachers participating in the research between 2012 and 2018, he established a deep-rooted relationship with these teachers and implemented video club applications with them to support their professional development. For this purpose, with the support of the school administration, a video club group was organized with the participation of five volunteer mathematics teachers.

Video clubs are designed to support teachers in realizing student mathematical thinking. The main purpose of video club meetings is to support teachers to pay attention to students' mathematical thinking and to make productive interpretations on these thinking. For this purpose, video club meetings were held once a week for twelve weeks. Before each meeting, a teacher's lesson was video-recorded and viewed at club meetings. Therefore, the weekly contents differed, as a different teacher's lesson was video-recorded each week. After recording, the videos, lasting approximately 30 minutes, were viewed at the video club meeting. Unlike most studies in the literature, videos containing the whole lesson were viewed instead of video clips in order not to neglect the integrity of the lesson and the different situations in the classroom. Therefore, the video club meetings took about an hour. Video club meetings were held once a week at the school, where the teachers work. The meeting time was arranged in accordance with the teachers' school departure time. Before the teachers watched the video lesson, the facilitator explained at which grade level and which topic the video lesson was about.

Teachers took written notes for the situations they observed while watching the video. The facilitator paused the video when there were classroom discussions, teacher-student interactions, student solutions to a mathematical problem, and when the student explained their idea or solution. That is, the facilitator tried to direct teachers' attention to important aspects of classroom interactions. For example, at the video club meeting in the 7th week, T3's video-recorded lesson was viewed. The video includes solutions to a percentage problem asked by T3 and students' explanations. The facilitator paused the video to get the teachers' views on the students' solutions and asked, "What did you notice in the video?"

Table 1. Coding categories of social and socio-mathematical norms

	Categories	Teacher behavior	PVC	AVC
Classroom social norms	Justification	Presenting the rationale for solutions/statements		
	Elaboration	Generating alternative solutions/statements Generating ideas for the solutions made or adding new ones to the ideas put forward		
	Explanation	Explaining solutions/statements Expressing incomprehensible solutions or clarifying complex situations		
	Questioning	Questioning the accuracy of solutions/statements		
	Sharing	Share ideas freely		
	Listening	Hearing what others say for the purpose of understanding		
	Proving	Demonstrate that something is true		
Socio-mathematical norms	Different explanation	Explaining to the subject with different approaches such as using interdisciplinary approach Offering an unusual solution		
	High-level explanation	Explaining using high-level skills for the statement		
	Convincing explanation	Making an effort to persuade others		
	Discussion	Discussing mathematical concepts/Knowing more about the concepts		

Note. PVC: Pre-video club & AVC: After-video club

T1: Students generally understood the problem and were able to express the ways of solving the problem. However, they had difficulty in making the solution.

T4: It caught my attention as well. The student said, "I can tell, but I do not know how to do it" and said that the problem can be solve in the direct proportion, but he could not show it operationally.

T2: Yes, the student who went to the blackboard had difficulty in making a solution and needed the support of the teacher. Despite the teacher's hints, the student could not perform the operation.

T1: I think that the student's inability to continue with the process shows that he has difficulty in converting fractional expression to decimal expression.

Facilitator: Why do you think so?

T1: Because the student first multiplied the expression 50 by 2.5/100, first simplified the zeros and found the result of 12.5/10, but she had difficulty in adding up with 50 because she could not write the result she found as a decimal.

Discussions continued in line with the opinions of the teachers. As can be understood from the dialogues, the teachers mostly expressed their opinions about student thinking. In the continuation of the video, the teachers expressed their views on the instructional practices of T3 to make sense of student thinking. Therefore, although the main focus of video clubs is the student's mathematical thinking, teachers made evaluations about their own and their colleagues' classroom practices in order to make sense of these thinking. At the end of the video club meeting, the teachers wrote reports containing their specific pedagogical recommendations for the video they watched. Data from the video club process were analyzed according to the theoretical framework developed by van Es and Sherin (2008, 2010). The analysis results showed that the teachers participating in the video club paid attention to student thinking over time and displayed an interpretative analytical stance based on these thinking. In the current study, the focus is on the norms constructed by the teachers who improve their noticing skills by participating in the video club. For this reason, the lessons of the teachers recorded before and after the video club and the field notes of the researchers were used as data collection tools.

Data Analysis

In this study, the theoretical framework developed by the researchers for determining the norms was used for the data analysis. This framework consisted of two parts, the social norm, and the socio-mathematical norm. While determining the norms, relevant studies were taken into consideration (e.g., Buyumez, 2011; Edwards, 2007; McClain & Cobb, 2001; Pang, 2001; Yackel, 2000, 2001; Yackel & Cobb, 1996), and an extensive theoretical structure was tried to be created. As a result of the literature review, the behaviors of students and teachers were clustered according to their similarity, and each phrase was expressed as a category. For instance, if an in-depth explanation about the concept was desired in the classroom, or if alternative ideas were required to be understood for the comprehension of the concept, these behaviors were regarded under the social norm category of elaboration. If the teacher wants students to produce unusual solutions, this behavior has been included in a different category of explanation as a socio-mathematical norm. In addition, the theoretical framework was finalized considering there would be changes if new situations to be encountered in the analysis process of the videotaped lessons (**Table 1**).

As a matter of fact, the behaviors of expressing or explaining solutions that were not understood in classroom social norms during the analysis process were evaluated under the category of explanation contrary to the literature. In the literature, such situations were included under the title of elaboration. However, in the data analysis process, it was determined that elaboration and explanation were different from each other, so the 'explanation' category was created.

As seen in **Table 1**, social norms were explained in seven categories: justification, elaboration, explanation, questioning, sharing, listening, and proving. Socio-mathematical norms were explained in four categories as different explanation, high-level

Table 2. Data of the teachers regarding the norms

	Category	T1		T2		T3		T4	
		Pre	After	Pre	After	Pre	After	Pre	After
Social norms	Justification	√	√	√	√		√		
	Elaboration	√	√	√	√	√	√		√
	Explanation	√	√		√	√	√	√	√
	Questioning	√	√	√	√	√	√	√	√
	Sharing	√	√	√	√	√	√		√
	Listening	√	√				√	√	√
	Proving	√	√		√		√		√
Sociomathematical norms	Different explanation				√				
	High-level explanation						√		√
	Convincing explanation		√	√			√		
	Discussion	√	√		√		√		

explanation, convincing explanation, and discussion. Videotaped lessons of each teacher were carefully monitored, and the identified behaviors were specified under the relevant norm. This was performed separately preceding and following the video club. For the coding example, T2's in-class dialog is, as follows:

> T2: Do you think this is the height?
>
> Ata: No (The teacher allows time for the student to correct, and the student cannot answer. He then gives someone else a say and asks the student not to delete the previous drawing for comparison. The student draws a diagonal instead of a height).
>
> T2: Do you think this is height?
>
> Kemal: Yes.
>
> T2: Why yes, why not? (The teacher gives someone else a say because Kemal remains silent, but also includes Kemal in the process).
>
> …

According to this dialogue that took place in the classroom, it was noticed that the teacher (T2), while forming mathematical ideas, opened the concepts for discussion throughout the class, listened to the ideas of the students and created an environment for them to listen to each other's thoughts, and questioned the thoughts of the students in order to have comprehensive knowledge about the concept in question. According to this section obtained from T2's videotaped lesson, it was deemed appropriate to be evaluated under the category of justification and questioning from social norms. Analyses continued in this way. The other researcher also made the analyses simultaneously. The agreement between the two researchers was found to be 86% using Miles and Huberman's (1994) formula. For situations of incompatibility, the researchers gathered and discussed until a consensus was reached. The researcher repeated the analyses every two weeks until becoming completely sure of the analyses, and then they were finalized.

RESULTS

In this section, the findings about teachers' classroom social and socio-mathematical norms were included. The emergence of the behaviors regarding the norms observed in each teacher's lesson before and after the video clubs was analyzed. The results of this analysis were presented descriptively in **Table 2**.

According to **Table 2**, four important situations attract attention. The first is that both classroom social norms and socio-mathematical norms were observed more frequently after the video clubs. Secondly, the norms having existed before the video clubs were generally observed after the video clubs as well. Thirdly, the classroom social norms were observed more frequently than the socio-mathematical norms. Fourthly, this is a very important issue, it has been determined that both social and socio-mathematical norms are used more frequently by the teacher after the video club. For example, T1 used "the questioning" before and after the video club. However, it was observed that T1 used "the questioning" more dominantly in her lessons after the video club. Detailed analyses of these four situations were presented under the headings of the classroom social and socio-mathematical norms.

The Findings Regarding the Classroom Social Norms

The behaviors reflecting the classroom social norms the teachers created were evaluated within the framework of the relevant norms. The data obtained were summarized in **Table 3**.

When **Table 3** was analyzed, it was observed that each teacher exhibited behavior reflecting at least one social norm. However, this was different for each teacher. For instance, while all the classroom social norms were observed preceding and following the

Table 3. The teachers' data regarding the classroom social norms

	Category	Pre-video club	After-video club
Social norms	Justification	T1 & T2	T1, T2, & T3
	Elaboration	T1, T2, & T3	T1, T2, T3, & T4
	Explanation	T1, T3, & T4	T1, T2, T3, & T4
	Questioning	T1, T2, T3, & T4	T1, T2, T3, & T4
	Sharing	T1, T2, & T3	T1, T2, T3, & T4
	Listening	T1 & T4	T1, T3, & T4
	Proving	T1	T1, T2, T3, & T4

video clubs during T1's lesson, four norms before the clubs and all of the norms after the clubs were observed during T3's lesson. On the other hand, while all of the norms were observed in the lesson of only one teacher (T1) before the video club; after the video club, all norms were observed in the lessons of two teachers (T1 and T3), and six norms were observed in the lessons of two teachers (T2 and T4).

According to **Table 3**, if the classroom social norms of each class are considered separately, it can be stated that the teachers paid more attention to make explanations in their lessons before the video clubs, they mostly questioned accuracy of the students' thoughts and encouraged the students to share their thoughts. For instance, before T3's lesson started, she told the students that "I want you to discuss the concepts during the lesson, please do not be afraid of giving wrong answers and do not feel embarrassed, because it is important to share your thoughts." In this way, she stated that sharing thoughts was important and, after the lesson, she tried to construct the subject by taking the students' thoughts. T1 encouraged the students with expressions such as "yes, you are moving right, you can say without hesitation", "what can we say more, let's continue", "yes, definitely good", and made them active in the lesson by establishing constant eye contact with students. On the other hand, T4 seemed to encourage the students with discourses such as "I want you to share your ideas without thinking whether they are right or wrong". However, it was observed that teachers were not very successful in considering students' thinking each other's thoughts as in sharing category. For instance, in the pre-club lesson of T2, it was seen that the teacher gave other students the right to speak until the correct solution was found, that the students made solutions independently from the previous solution and the teacher did not care about this situation.

> T2: … You must draw from corner D. You seem to have gone through DC. Is your way out, correct? Be careful (The student quietly erases his solution and takes reference to corner D. He tries again). Do you think this is height?
>
> Ata: No.
>
> T2: If you want to correct, do it, otherwise I will ask someone else ... (Then, T2 asks someone else and tells the student not to erase the previous drawing for comparison. The student draws a diagonal instead of height). Do you think this is height?
>
> Kemal: Yes.
>
> T2: Why yes? ... (The student waits silently, and the teacher reiterates that if he is unable to explain, she will ask someone else).

In the conversation above, T2 repeated that if the student who had the right to speak could not solve the problem, she would give another person the right to speak. It was observed that the students who had the right to speak acted independently from their friends' responses, yet T2 asked the students in this problem not to delete their drawings and to make a comparison. However, in practice, it was seen that this situation was not realized and that the answers given were slurred over. It can be said that the teacher did not make the students listen to their friends' thoughts in the classroom and make them feel that this behavior was necessary and important. It was observed that the teachers wanted to make explanations especially on the subjects that could be misunderstood, or they needed to explain more when teaching was newly started. It was also observed that the teachers questioned the students about what they knew or what the meanings of things they knew were. This was a particularly prominent situation after the video clubs. Before the video clubs, it was revealed that the questions asked by the students were passed by expressions such as 'this is not our subject at the moment' and 'what you say is different, this is different'. However, after the video clubs, it was observed that the teachers did not leave any questions unanswered regardless of whether they were related to the lesson or not, and the confusions were clarified. T3 wanted the whole class to notice that the student solving the problem on the board had used the equals sign incorrectly. The student statement of this situation is shown in **Figure 1**. Then a dialogue about this situation was given.

> T3: Do you see anything wrong here? (The students think). Nisa, Sena, Esra, and Sibel. Come on, I want to say something. These are the things I have specifically mentioned. What is the mistake? You can say whatever comes to your mind. Feel comfortable (The students are eager to speak).
>
> Taha: Teacher! Equals sign was misspelled.
>
> T3: Yes. What can we understand from here? The value of *x* is understood as (*five lemons x nine and 6/10 Turkish Liras*), right?

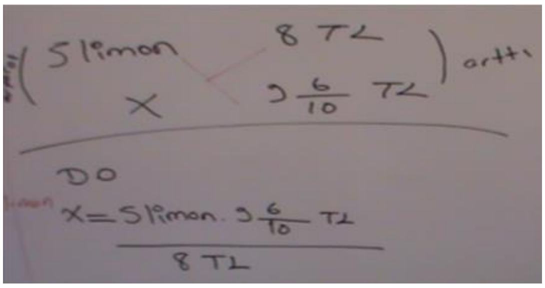

Figure 1. Student's problem-solving status in T3's post-club lesson (Source: Student's own illustration, reprinted with permission of the student)

Class: Yes.

T3: If the equals sign is here, was this expression (pointing to what is written on the board) written as an equation?

Class: No.

T3: Of course not. So, where should we write the equals sign?

Class: In the middle.

T3: In the middle of the fraction line. Thus, we cannot accept it this way. Can you delete and edit it, Sema? If we write in this way, we can understand what is in the numerator and denominator of the fraction. For example, 2/3/5. Do you remember which number to be divided here?

Class: Yes.

T3: What did we say before? Who wants to say?

Ezgi: Where the great line is.

T3: Did I say great line? Please, think carefully again.

Ezgi: Fraction line.

T3: I think there is something more?

Sibel: I mean the point, where fraction line and equals sign meet.

T3: Yes. So, if the equals sign is here (she writes in front of the upper fraction line) we divide 2 by 3/5; If it is here (she writes in front of the bottom fraction line) we divide 2/3 by 5. The two situations are different. Then, it is important, where to write the equals sign.

Figure 1 shows the student's problem solving and misuse of the equal symbol is seen. As can be seen from the dialogue, T3 also gave information about the use of symbols while discussing mathematical concepts. She asked the students to explain their thoughts about misuse of the equals sign. She also occasionally questioned the students. For instance, she asked the students if the result would change when denominators are equalized in multiplication. However, it was seen that the teacher usually explained the proofs of the statements herself and did not ask any justification or proof from the students. It was observed that after the video clubs, T3 paid attention to justification and proving.

While only one teacher (T1) exhibited the behavior of proving before the video clubs, it was observed that other teachers adopted this behavior in their lessons following the video clubs. The teachers promoted their students' learning by directing them questions such as *'Why did you think so? How can you be sure that the answer is correct? So, how can we know that this solution is correct?'* in order to help them think through questioning. The facts that the teachers asked the students to crosscheck the incorrect operations, to show with an exemplary case that the hypothesis could be refuted and to show the accuracy of an expression with different solution methods were regarded. For instance, T1 created a discussion atmosphere after the video clubs about what the measure of the narrow angle of other two angles in a right triangle should be:

T1: What should we pay attention to when creating a triangle? What angles ca not we use ... Let's examine the first example.

Cemre: No.

T1: Why not?

Cemre: Because … (She remains silent).

Table 4. The teachers' data regarding the socio-mathematical norms

	Category	Pre-video club	After-video club
Socio-mathematical norms	Different explanation		T2
	High-level explanation		T3 & T4
	Convincing explanation	T2	T1 & T3
	Discussion	T1	T1, T2, & T3

T1: How was the isosceles triangle supposed to be? (She remains silent) Can you think of the edges?

Cemre: Its two edges must be equal.

T1: What about the angles?

Cemre: They also have to be equal.

T1: What else should we check in order to draw such a triangle?

Cemre: We have to see if the sum of the interior angles is 180°.

Here, it was seen that T1 asked guiding questions to the students to show the correctness of their answers for the questions. It was found that the students justified their responses in most of the teacher's lessons. In addition, it was observed that in T1's lesson, the students questioned the accuracy of mathematical expressions by making simple checks.

Moreover, it was revealed that before the video clubs, the teachers had often carried out lessons in which they were active in the process. For instance, while T3 tried to construct mathematical concepts before the video clubs, she justified the accuracy of a mathematical expression and paid attention to prove it. However, this continued unilaterally, and the students were not encouraged to demonstrate such behavior. Therefore, such situations were not evaluated under the relevant norm category.

The Findings Regarding the Socio-Mathematical Norms

The data related to the socio-mathematical norms created in the classroom by the teachers were analyzed before and after the video clubs, and the data obtained were presented in **Table 4** descriptively. Then, qualitative explanations and examples of these descriptive data were presented.

According to the data in **Table 4**, it can be stated that socio-mathematical norms were observed less than the classroom social norms. Unlike preceding the video clubs, more teachers included the socio-mathematical norms in their classrooms after the video clubs. It can be mentioned that particularly the norm of discussion was observed after the video clubs, and the researchers encountered the behaviors regarding each norm during the analysis process. However, it was found that teachers did not exhibit a behavior that would be evaluated in the high-level explanation category before the video clubs. In addition, it was seen that the teachers formed the same norm before and after the clubs. For instance, T1 was concerned with mathematical discussion in both cases, but it was observed that T1 reminded the prior knowledge and immediately switched to the new subject and created a discussion environment about the concepts towards the middle of the lesson in her first lesson. In her final lesson, however, T1 created a mathematical discussion environment and questioned the prior knowledge during this process. Similarly, for the norm of discussion, while T2, T3 did not create a mathematical discussion environment before the video clubs, they created a mathematical discussion environment after the video clubs. For instance, T3 encouraged the students to discuss quadrangle, parallelogram, square and rectangle. She seemed to encourage students to present various ideas and to get new ideas based on each other's ideas by asking them questions such as *'Do you think the quadrangle and the square are drawn in the same way? What are the differences? Would you like to try? What about the parallelogram and the rectangle? Is there a difference between the parallelogram and the quadrangle?* Thus, T3 showed that it was important for the students to share their ideas, that is, to have an in-class discussion, to construct the concepts in their minds.

While nothing related to the high-level explanation was observed before the video clubs, two teachers (T3 and T4) guided students to make high-level explanations. For instance, a section of T4's lesson was, as follows:

T4: How do you make inferences in this question (by pointing to **Figure 2**)?

Fatma: There may be differences between the short and long edges of rectangular regions with the same area.

T4: Find the smallest and greatest perimeters (The student shows 18 and 42 br, respectively). So, there is a correlation between the edges of these greatest and smallest perimeters. What can we infer? Please, be careful! (Silence prevails in the classroom). Look at the short edges, please!

Fatma: Aah! OK. As values get closer to each other, the perimeter becomes smaller, the perimeter gets bigger as they move away from each other.

T4: Can you show this? (The student makes markings on the **Figure 2**) ...

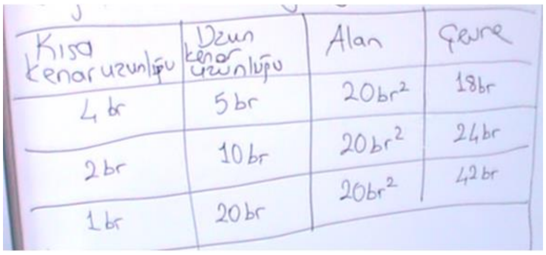

Figure 2. Student's problem-solving status in T3's post-club lesson (first line short edge length, second line long edge length, third line area, and fourth line perimeter) (Source: T3's own illustration, reprinted with permission of T3)

In **Figure 2**, the change in the circumference and area of the rectangles with different lengths has been the subject of discussion. T4 asked the students to reach mathematical generalizations and encouraged them to generalize. This fact indicated that T4 was trying to use high-level skills in her classroom.

In the convincing explanation category, it was seen that T2 was in an effort to convince the students before the video clubs. However, since the teacher's persuasion efforts were prolonged redundantly, it caused confusion rather than comprehension in the classroom. A conversation related to this matter is below.

T2: 2.8:0.07=? Does anybody have an idea?

Kerim: We need to multiply both of them with 100?

T2: Why?

Kerim: Because 0.07 is bigger.

T2: Is it bigger or …

Kerim: It has more digits.

T2: More digits after the comma, right? Any other ideas?

Selma: Since 0.07 has more digits after the comma, we have to multiply with 100.

T2: Both of them need to get rid of the comma at the same time, right?

Selma: Yes.

T2: So, based on what should I decide? Who wants to tell?

Beren: On the number of digits after the comma.

T2: Which number? For example, there are two numbers. One has five digits after the comma and the other has four digits, according to which shall I decide?

Beren: The one with five digits.

T2: Why?

Beren: Because it has more digits…

The discussion continued for about 15 minutes in the same way. While trying to persuade her students, T2 included different examples (for example, there are two numbers, one has five digits after the comma and the other has four digits, according to which shall I decide?) and caused confusion. After the video clubs, it was seen that T1 and T3 tried to persuade the students by giving consecutive examples on the topic and even encouraged the students to persuade their friends.

After the video club, it is seen that only T2 cares about presenting unusual solutions for mathematical concepts in the category of explanation different from socio-mathematical norms. The fact that T2 gave examples from other branches of science, such as art, in her explanations about what the concept of height was in his class, shows that she made different explanations rather than the classical definition. In addition, it was determined that she created discussion environments in order to mentally construct the important concepts of the subject such as orthogonality and parallelism and collected the ideas of many students. Before starting to explain the concept of height, T2 starts the lesson by asking students to guess how tall the tallest tree in the world will be. Thus, by sharing a scientific knowledge that will attract attention, she makes a transition to the concept of height. It reinforces the sense of curiosity in students by taking the opinion of the whole class.

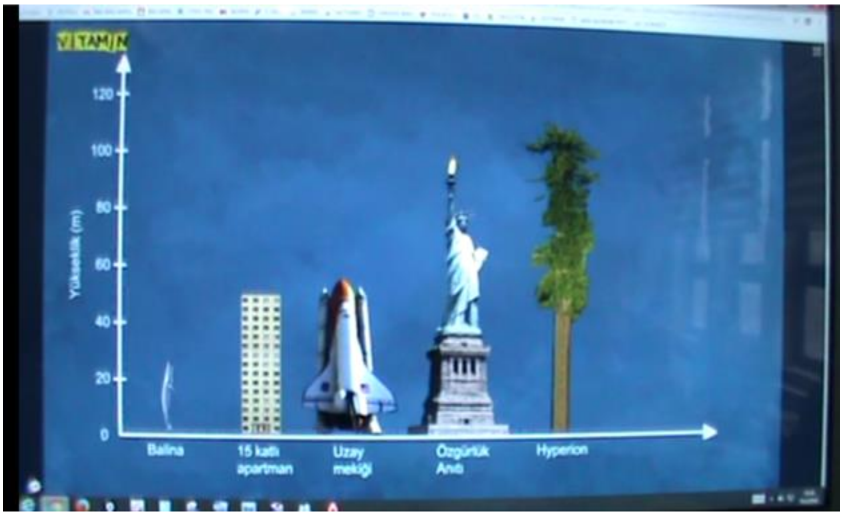

Figure 3. Representation of lecture in T2’s post-video club (Source: Turkey's Vitamin Education, one of Turkey's online education platforms, reprinted with permission from owners)

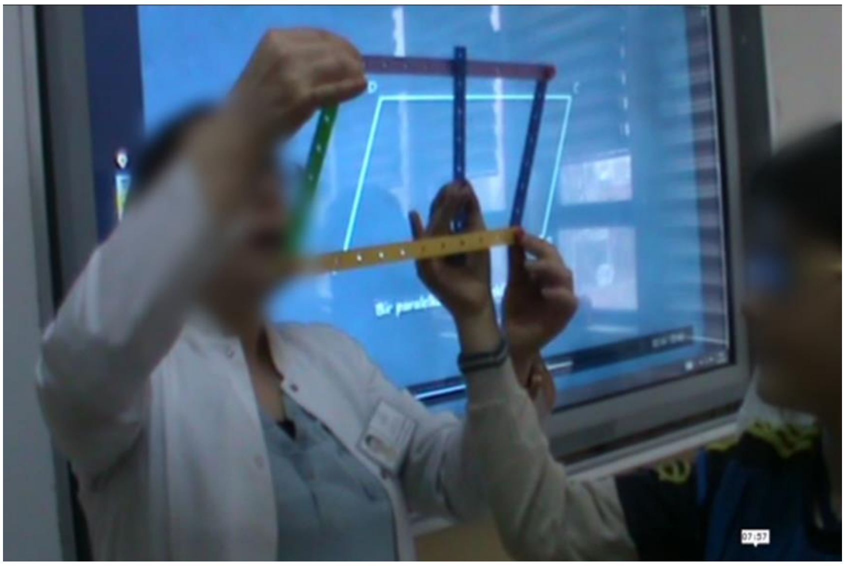

Figure 4. Representation of lecture in T2’s post-video club (Source: T2's own representation, reprinted with permission of T2)

As seen in **Figure 3**, T2 shows the world’s tallest tree by comparing it with other examples from daily life in the coordinate system. In the meantime, she gives examples of other concepts and revitalizes the lesson with different subjects. Here is a section of the interaction that takes place in the classroom related to height:

T2: (She asks Ali to stand up). Ali, can you show me your ground clearance?

Ali: (Pointing to his head with his foot) That’s it.

T2: What do you think about the angle you made with the ground?

Ali: Upright.

T2: Then what can we say about height?

Ali: The measure of height is 90°.

…

After T2 tries to create the concept of height on the student, she continues her lesson with a material (**Figure 4**). Then, she explains about height on the board and creates a discussion environment on how to determine the heights of different shapes such as rectangles and parallelograms. In the meantime, it was observed that she deepened the class discussion by questioning how the heights of geometric shapes would change when they changed direction.

DISCUSSION AND CONCLUSION

This paper examined how teachers' noticing skill affects the social and socio-mathematical norms they construct in the classroom. The video clubs were organized to reveal the teachers' noticing skills, and for this purpose, the classroom social and socio-mathematical norms created by the teachers were identified. Four basic conclusions were reached with the findings obtained. The first was that teachers' noticing skills and the norms in their classrooms were related. It was observed that the video clubs, which were made to support their noticing skills, had an impact on the norms that the teachers created in their classrooms. Therefore, teachers adopted the norms based on the students' thinking in their classrooms as a result of their noticing the students' mathematical thinking. In this sense, it can be claimed that there is a relationship between the development of noticing skills and norm formation. van Es and Sherin (2008) mentioned the effect of noticing what is important in the classroom environment and overcoming such situations on the classroom atmosphere. In this study, which was conducted to follow the development of teachers' noticing skills, it was revealed that the rules created in the classroom showed progress in parallel with the noticing skills. In the study carried out by the same researchers in 2021, the social and socio-mathematical norms created by the teachers who participated in the video club process as a result of the development of their skills of consideration and interpretation were examined, and it was suggested that teacher noticing skills are related to teachers' ability to use social and socio-mathematical norms in the classroom (van Es & Sherin, 2021). Similarly, according to Borko et al. (2014) the teachers' being aware of their students' knowledge is closely related to the behaviors that would arise during the teaching process, and this positively affects the understanding process of the students. In addition, Van Zoest et al. (2012) states that the creation of social and socio-mathematical norms in teacher education will increase the awareness of teacher candidates about the processes of creating norms through their knowledge and continuous application and experience about the norms. Ozdemir Baki and Kilicoglu (2020) presented evidence that teachers participating in video clubs improved their classroom practices. These evidence are related to the more frequent use of approaches such as supporting students' explanation, reasoning, and questioning skills in the classroom. Based on this evidence, the researchers argue that the approaches adopted by the teachers participating in the video club in the classroom will be internalized by the teacher and the student over time and affect the formation of norms.

The second result of the research was that the classroom social norms were created more widely than the socio-mathematical norms. It was observed that a teacher who cared about the formation of a socio-mathematical norm in her classroom also created various social norms to create this norm. For instance, it was observed that a teacher paying attention to creating a mathematical discussion environment in her classroom also created the social norms such as justification, elaboration, explanation, and questioning. Therefore, it is not possible to consider the classroom social norms independent from socio-mathematical norms. Levenson et al. (2009) stated that the socio-mathematical norm is in a nested structure with the classroom social norm. There have been several studies in the literature revealing the relationship between the two norms (Gorgorio & Planas, 2005; Iannone & Cockburn, 2008; Kazemi, 2008; Yackel & Cobb, 1996). It was understood that these studies concurringly explained the concept of socio-mathematical norm with the definition of the social norm. While making these claims, it has been considered that the formation of norms may vary according to the education systems and values of the countries. As a matter of fact, even if the norms vary, the approaches such as justification, elaboration and questioning used in putting forward ideas are objective. Therefore, the claims made regarding the association of the two concepts have been presented with the support of international studies.

The third result of the research was that for a behavior in the classroom to be regarded as a norm, it should be exhibited by both the student and the teacher. For instance, merely the teacher's justification about a solution or a mathematical judgment and her proving the solution undermines the feature of this behavior's being in the classroom because the members of a classroom are teachers and students. If a behavior is performed by both the teacher and the students, then it becomes a norm. Otherwise, the lesson remains as a design that cannot go beyond teaching through presentation. According to Tsai (2007), an effective way to change students' thinking or approach styles is the teacher himself. Dixon et al. (2009) stated that there are multivariate factors in the formation of the norms in the classroom, and the teaching style of the teacher is an important factor. Indeed, Elliott et al. (2009) state that socio-mathematical norms, specialized content knowledge and practices for teacher sharing are three important frameworks for teachers' learning opportunities. According to the researchers, these frameworks are thought to support the professional development of the teacher and also have a positive effect on the learning of the students. Similarly, according to McClain and Cobb (2001), the teacher's implementation of socio-mathematical norms in the classroom is a tool used to promote the student's mathematical thinking and such tools contribute to the improvement of the classroom atmosphere. Therefore, it can be expressed that the norms cannot be the output of a stable lesson process.

The fourth result of the research was that the video clubs positively affected the frequency of occurrence of both classroom social and socio-mathematical norms. After the video clubs, the lesson process, in which alternative ideas were generated, new details were added to the ideas put forward and the accuracy of these ideas were proved were encountered. In the lessons before the video clubs, listening to the students' thoughts and trying to understand them were ignored in some cases, and proving the accuracy of the ideas put forward was rather weak. In this case, it is possible to talk about the positive effects of the video clubs on the norms. Guler et al. (2020) revealed that an alternative teaching sequence enriched with videos has positive contributions to pre-service mathematics teachers' noticing skills. Similarly, some researchers claimed that video clubs help teachers make sense of class interactions and improve their skills to focus on students' mathematical thinking (Borko et al., 2008; Jacobs et al., 2010; Sherin & Han, 2004; Sherin & van Es, 2009). In addition, the creation of the classroom social and socio-mathematical norms is important for student learning (Yackel et al., 2000) and conceptual teaching of mathematics (Kazemi & Stipek, 2001). For example, Stenmark (1989) and Yackel and Cobb (1996) reveal that promoting an alternative solution, which is a social norm, has positive effects on students' learning, the development of mathematical thinking skills, and their existing knowledge being able to connect with the information they have just learned.

Implications

In this study, the video clubs are designed to support teachers' noticing skills. In this way, the classroom social and socio-mathematical norms constructed by the teachers were examined implicitly. However, if the video clubs had been planned for the creation of norms, different results could have been obtained. In future studies, the content of video clubs can be designed with a focus on social and socio-mathematical norms. Thus, in club meetings, teachers can discuss the social and socio-mathematical norms they use in their classrooms. In fact, it can be observed in more detail which norms are more prominent in the classroom. In fact, Van Zoest et al. (2012) state that creating norms that make meaningful teaching and learning possible for individual and collective mathematics learning in the mathematics classroom is the key point of teacher education. Therefore, it is suggested that further studies consider this situation. In this direction, the research results can be enriched by the use of 360-degree video and XR representations that increase the feeling of being in the classroom instead of standard videos. Finally, in the light of the data obtained in this research, it is thought that social norms and socio-mathematical norms formed in the classroom are not independent. It is important to carry out studies aiming to reveal the relationship between these two concepts in order to clarify this claim.

Limitations

In this study, since the teachers followed the curriculum of the Ministry of Turkish National Education, the content of the courses before and after the video clubs coincided with different learning areas. Comparing the norms constructed in the classroom for the same grade level and the same subject is supposed to be a facilitating feature for data analysis. In addition, longer classroom observations should be made to examine the norms that teachers construct in classrooms.

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Declaration of interest: No conflict of interest is declared by authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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