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Metacognition and Its Role in Mathematics Learning: an Exploration of the Perceptions of a Teacher and Students in a Secondary School

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ABSTRACT

The study aims to explore teachers' and students' perspectives regarding metacognition and its role in mathematics learning. The use of case study was a methodical means to achieve elaborate data and to shed light on issues facing the study. The participants consisted of a case study class from a secondary school in Saudi Arabia. The instruments used for data collection were semi-structured interviews and classroom observation. The data produced essential finding based on thematic analysis techniques, regarding study's aim. Firstly, the traditional method can hinder mathematics teaching and learning through metacognition. Secondly, although metacognitive mathematics instruction should be planned, the strategy that is introduced should be directly targeted at improving the monitoring and regulation of students' thought when dealing with mathematics problems.

KEYWORDS
Metacognition, Mathematics and IMPROVE Programme

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Introduction

Flavell (1979) indicated that young children have thinking limitations of cognitive enterprises. Therefore, researching of cognitive monitoring and cognitive regulation is important in developing these kinds of activities for children and adults alike. The term 'metacognition', coined by Flavell, emerged from this research area (Flavell, 1979). Use of the term 'metacognition', according to Brown (1987), began in psychological literature within two different research areas: knowledge about cognition, and regulation of cognition. The

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former refers to one's knowledge concerning thinking processes, whereas the latter refers to the regulation and monitoring of one's course of thinking. Similarly, Kluwe (1982) claimed that research relating to metacognition is based on distinguishing between one's own knowledge about cognition and the executive processes of cognition. The former refers to one's own knowledge about features of one's cognition and that of others, whereas the latter refers to the monitoring of cognitive activity, its application, and its effects on problem solving strategies, in addition to the regulation of the course of cognition.

Despite these premises, to present a certain definition of the metacognition concept is still difficult because of its cross-disciplinary nature. Adding to this difficulty in definition, differentiating between cognition and metacognition has proven to be another issue. Efklides and Misailidi (2010) underscored this by explaining that the differentiation between cognition and metacognition is challenging and that the wide range of metacognitive phenomena would indicate that no one mechanism suffices in their description. Taking a more literal approach, Peña-Ayala and Cárdenas (2015) explained that cognition means to know, and elaborated on this by suggesting cognition involves an individual's perception and comprehension of the world, and how he/she behaves in that context. According to them the process of cognition covers acquisition, development, and exploitation of a range of knowledge-based and cognitive functions. Regarding knowledge itself, it consists of memories which have been shaped by the manipulation and integration of 'raw input' – or rather information processed through one of the five senses or resulting from cognitive functions such as thought, reasoning, recall, learning and experiences.

According to Peña-Ayala and Cárdenas (2015), the simplest of human actions rely entirely on cognitive activity. This activity is manifested in different abilities so as to organize, control and exploit it effectively to achieve cognitive tasks. The challenge of definition appears, as cognitive abilities cannot necessarily be distinguished from one another, they can overlap. Hence, cognition has been divided into wider cognitive abilities, for example, perception, attention, reasoning, speaking, planning, learning… a difference between the metacognitive and cognitive processes, as pointed out by Kuhn (2000). He further explained that cognitive processes are involved in doing, while the metacognitive processes are involved in choosing and planning what is required and monitoring what is being done.

Taking all these arguments into account, the presentation of a definition for metacognition does not mean that there is unanimous agreement about the borders of the concept. This is due to the fact that, over time, the scope of definition has grown in tandem with metacognition becoming a multifaceted concept (Buratti & Allwood, 2015). Despite this, a need for theoretical clarity is certainly present. This would include improved definitions and descriptions of the numerous components of the concept (Azevedo & Aleven, 2013). Hence it can be concluded that metacognition from an educational standpoint refers to one's knowledge and the monitoring and control of one's own systematic cognitive activity which requires certain metacognitive skills such as planning and evaluation. Noteworthy in the context of discussing the concept of metacognition, the important issue remains determining the basic subject of the concept of metacognition. Particularly since Brown (1987) mentioned that the concept of self-monitoring and control method is essential in the growing field of metacognition and Kluwe's view (1982, p. 220) being that "the subject of metacognition is regulation of one's own information processing".

Metacognition and Mathematics

There are several essential dimensions regarding the nature of the relationship between metacognition and mathematics, which in turn provide this study with important points of discussion.

Firstly, a key finding in the literature was that students perceive difficulties in mathematics and problem solving tasks because they are neglecting a wide range of cognitive or metacognitive processes (Cardelle-Elawar, 1992; Grizzle-Martin, 2014; Tok, 2013; Wolf, Brush, & Saye, 2003). However, this could lead to the assumption that struggling students are lacking in crucial metacognition (Coles, 2013). Secondly, many studies asserted that mathematical performance is significantly and positively affected by applying metacognitive strategies (Bernard & Bachu, 2015; Desoete, 2007; Gillies & Richard Bailey, 1995; Goos, 1993; Grant, 2014; Sahin & Kendir, 2013; Schoenfeld, 1987). Hence, metacognition plays a central role in the learning process, which ultimately affects the student's academic performance at school generally and their mathematical performance specifically (Almeqdad, 2008; Grizzle-Martin, 2014; Panaoura & Philippou, 2005; Schoenfeld, 1992). Thirdly, and more specifically, the student's inability to perform the required monitoring and controlling process in their learning is the factor behind low performance in mathematics, rather than a lack of mathematical knowledge (Grant, 2014; Tok, 2013; Yimer, 2004). Hence, the effectiveness of a problem solving process will increase when a student becomes capable of monitoring and controlling his/her own learning processes (Grant, 2014; Sahin & Kendir, 2013; Schoenfeld, 1987). Fourthly, students can be trained to improve mathematical performance through metacognitive skills such as monitoring or regulation (Grant, 2014; la Barra et al., 1998; Sahin & Kendir, 2013). Fifth, teachers need to explicitly instruct their students to monitor and subsequently control their learning processes in order to become more self-directed in their mathematical performance (Desoete, 2007, 2009; Grizzle-Martin, 2014; Raoofi, Chan, Mukundan, & Rashid, 2013; Schoenfeld, 1987). Sixthly, it is important that teachers themselves begin to reflect metacognitively on the means to improve metacognition in students. This should come in tandem with professional training, but the teacher should have a stake in the theory itself, as a genuine belief in its importance for learning will assist in effecting change in others (Larkin, 2000). Teachers will not be able to perform this as long as they are not provided with sufficient training in this field, as stressed by the study of Sahin and Kendir (2013). Thus, teachers first and foremost should be educated about instructing students on this so they can fully engage their students in gaining such strategies. Coles (2013) pointed out an absence of studies investigating metacognitive requirements placed on the teacher seeking to enhance this type of skill among their students. This absence is concerning, considering the numerous studies demonstrating that an instructor's conceptualization of mathematics and student learning has an impact on classroom discourse. In terms of subject-specific metacognition, Larkin (2010) asserted that the process pertains to the nature of the task at hand along with specialized skills needed for specific subjects. Therefore, the use of metacognition, particularly in mathematics teaching, will remain a wide area of inquiry, requiring further research.

Based on these several essential dimensions regarding the nature of the relationship between metacognition and mathematics, in light of the reality of mathematics learning and teaching in Saudi Arabia, this study - which are notably absent in the educational context of this country - sought to identify teachers' and students' perspectives regarding the use of metacognition in the mathematics classroom. This study sought to identify inadequacies in mathematics learning and teaching in the classroom regarding metacognition in that country. How does the use of metacognition (if used at all) play a central role in mathematics learning and teaching, and why? What are the main encouraging signals and difficulties perceived by students and teachers wishing to improve their mathematical performance through metacognition? What are the characteristics that seemed to enhance the positive effects of employing metacognitive processes by analysing the beneficial effects of metacognitive training with students?

IMPROVE programme

The IMPROVE programme was presented by Mevarech and Kramarski (1997). It encompasses three interrelated components (Mevarech & Kramarski, 1997, p. 369):

> (a) Facilitating both strategy acquisition and metacognitive processes; (b) Learning in cooperative team[s] so four students with different prior knowledge: one high, two middle, and one low-achieving student; and (c) Provision of feedback-corrective-enrichment that focuses on lower and higher cognitive processes. (p. 369)

IMPROVE is an acronym for the instruction steps that comprise the method: Introducing new concepts, Metacognitive questioning, Practising, Reviewing and reducing difficulties, Obtaining mastery, Verification, and Enrichment. This is designed for implementation in smaller groups which include four students of diverse capabilities, particularly after a concept has been introduced to a class. Students pose three forms of metacognitive questions, these being categorized as comprehension, strategic and connection questions.

The IMPROVE method has proven to have a sizeable positive impact on mathematical performance in problem solving for seventh-grade students (Cetin, Sendurur, & Sendurur, 2014; Grizzle-Martin, 2014; Kramarski & Mevarech, 2003; Kramarski & Michalsky, 2013; Mevarech & Kramarski, 1997; Mevarech & Amrany, 2008).

There are some reasons that the IMPROVE programme was chosen to be implemented in this study. Firstly, the IMPROVE programme uses the metacognitive perspective and how it can be activated in mathematics teaching and learning. Secondly, the programme is centred on the belief that learning is not a rote process but rather one of interpretation, as many constructivists would argue. In doing this, students build meaningful relationships between new and previous knowledge, thus leading to the assertion that this is a process of construction rather than recording and memorization. This conforms to the current study which was engaged in the socio-cultural perspective. Thirdly, the programme includes cooperative learning which in turn helps in understanding metacognition and mathematics within the socio-cultural context as it was presented in relevant section of the current study. Fourthly, the IMPROVE method has proven to have a sizeable positive impact on mathematical performance in problem solving across several age groups. (Cetin et al., 2014; Grizzle-Martin, 2014; Kramarski and Mevarech 2003; Kramarski, Mevarech, and Arami, 2002; Kramarski and Michalsky, 2013; Mevarech and Amrany, 2008; Mevarech and Kramarski, 1997). Despite all these reasons, it is important to assert that the IMPROVE programme was carried out in order to enable the formulation of a clearer and more complete picture of mathematics teaching and learning through metacognition in Saudi Arabia, rather than seeking to improve a specific strategy or to measure students' achievement.

Study aim and questions

Based on these several essential dimensions regarding the nature of the relationship between metacognition and mathematics, in light of the reality of mathematics learning and teaching in Saudi Arabia, this study - which are notably absent in the educational context of this country - sought to identify teachers' and students' perspectives regarding metacognition and its role in mathematics learning in secondary schools in Saudi Arabia. Consequently, this study sought to respond to two questions:

1) How do secondary students and their teacher perceive metacognition in mathematics teaching and learning?

2) What are the experiences of secondary students and their teachers in Saudi Arabia of metacognition in relation to mathematics after the implementation of the IMPROVE programme, regardless of improvements in specific strategy or the aim to boost students' achievement?

Methodology of the study

The current study focussed on a phenomenon present in certain contexts in which the relevant behaviours cannot be regulated. The researcher has little or no influence over behaviours and external influences. In addition, when direct observation of events and interviews with those involved in such events are introduced, a case study design is favoured. According to Yin (2014: 12), 'The case study is preferred when examining contemporary events, but when the relevant behaviours cannot be manipulated. The case study relies on many of the same techniques as a history, but it adds two sources of evidence not usually available as part of the historian's repertoire: direct observation of the events being studied and interviews of the persons involved in the events.'

In this study, given the research aims, objectives and questions, an explanatory approach was adopted although elements of the evaluative approach were also incorporated. According to Robson (2002), a case study can be defined as an approach where the concentration is on a phenomenon in context and multiple methods of data collection, such as interview and observation, are typically utilised in this situation. In collecting the qualitative data for this research, the methods used are individual semi-structured interviews and participant observation. The study was carried out in order to enable the formulation of a clearer and more complete picture of the nature of the relationship between metacognition and mathematics, in light of the reality of mathematics learning and teaching in Saudi Arabia rather than improving specific strategies or seeking to boost students' achievement.

Participants

Since this study does not seek to generalize its results but to understand 'what is happening' and 'the relations linking the events, purposive sampling was used as the method of selecting the sample (Merriam, 1998). The participants were chosen based on a purposive sampling technique. I chose a small city which might be a more suitable environment to fulfil the following requirement criteria: the number of students in the class should not exceed 30 students, and teachers found who were cooperative and enthusiastic to implement the idea of metacognitive teaching. In addition, there should be a pre-existing practice of cooperative mathematics learning among students and teachers. Since in the Saudi Arabian education system the concept of metacognition in mathematics teaching and learning is unfamiliar, considering these criteria to find a suitable environment might help me to focus on the main subject of the study, particularly the IMPROVE programme based on cooperative learning. There was one class at the secondary school; this class is considered a case study which contains 30 students and their teacher. They were observed with each observation session lasting 45 minutes. Through the particular period I bore in mind that it was planned for the teacher to present eight lessons over a period of seven weeks in which the theory would be applied. The timing of this study fulfils Schraw and Gutierrez's suggestion (2015) which explains that programmes ranging from six weeks to several months tend to be more effective.

Seven students and their teacher gave individual interviews. These students were chosen through co-ordination with the teacher in order to determine which students were best able to express themselves on their opinions and feelings, with these students being of various educational achievement levels. The teacher's interview lasted 45 minutes, with the students' interviews lasting approximately 30 minutes. As for the teaching staff involved in this research was called Mr Fallatah as a pseudonym. Subsequent to gaining his undergraduate degree in mathematics at King Abdulaziz University in 1998. All the participating students in Mr Fallatah's class were 17 years old. All participating students lived in the same area of city.

Main Study

Before I began the main study I met the teacher twice; each time the meetings lasted one hour. These meetings were scheduled in order to discuss the IMPROVE programme and how the teachers could implement it in the maths classroom context. The aim of doing the entire stage was to enable me to formulate a clearer and more complete picture of mathematics teaching and learning through metacognition rather than evaluating the IMPROVE programme, or improving a specific strategy, or even seeking to measure students' achievement. I gave the teacher the freedom to choose appropriate situations in which to apply the IMPROVE programme, based on the content of the lesson and the preparedness of the students. Through this particular period I bore in mind that it was planned for each teacher to present eight lessons over a period of seven weeks in which the theory would be applied. The timing of this study fulfils Schraw and Gutierrez's suggestion (2015) which explains that programmes ranging from six weeks to several months tend to be more effective. This is because longer-term programmes enable students to model, practice and automate strategies, while also enhancing conditional knowledge. Furthermore, another benefit is that instructors himself improve his teaching and modelling of strategies over a lengthier period of time.

As a result of the discussion surrounding the IMPROVE programme, it was underlined that this programme encompasses three interrelated components (Mevarech & Kramarski, 1997). Based on these three elements, the teacher prepared the following:

1) Work groups consisting of four students of differing academic attainment, based on previous reports of the teachers. The number of students in the class was 30, so there were five groups containing four students, and two groups containing five students each. It is noteworthy that the nature of work in this school is originally based on cooperative learning, which facilitated the implementation of this study.

2) Mathematical problems suitable for learning according to the metacognitive questions as stated in the IMPROVE programme. These were questions relating to understanding the question, the solving strategy, and linking previously and newly learned information.

3) Worksheets for the student groups to solve the problems chosen in (2) above.

4) The steps which should be considered by the teacher during instruction, as noted in IMPROVE, which were: introducing new concepts, metacognitive questioning, practising, reviewing and reducing difficulties, obtaining mastery, verification, and enrichment. IMPROVE is an acronym for the instruction steps that comprise the method.

All of these applications were observed. Throughout each observation I sat with one of the work groups in the class solving the activity presented by the teacher. After gaining student consent I audio recorded their conversations as they solved the problem. I also observed their way of discussing and examined the steps of their work according to the metacognitive questions found in IMPROVE. These were questions relating to understanding the question, the solving strategy, and linking previously and newly learn information. They were asked when necessary about the items of the observation schedule prepared previously. When the teacher presented the next activity I moved to another group, and undertook the same work, in addition to noting the observations of the teacher's method of instruction. This was based on the items of the observation schedule.

At the conclusion of this period I conducted semi-structured interviews with the teacher and the previously mentioned seven students. Interview sought to respond to two questions: how do secondary students and their teachers perceive metacognition in mathematics teaching and learning? And what are the experiences of secondary students and their teachers in Saudi Arabia of metacognition in relation to mathematics after implementation of the IMPROVE programme?

Data analysis

Thematic data analysis has been discussed by Braun and Clarke (2006) who explain that data themes can be categorised as inductive ('bottom up'), or theoretical ('top down'). The method employed in this study involved elements of both approaches; inductive then theoretical thematic analysis. Overall, coding reflected information that had been expected to be found before the study, but also surprising and unforeseen data were collected in the field, along with other significant and pertinent information relating to the study.

In terms of data analysis procedures, first, I immersed myself in the data through intensive reading of the interview transcripts which involved searching for meanings, patterns and themes, while making initial notes for coding that could be reviewed later. The individual interviews, along with the observations were conducted in Arabic, and transcribed and analysed in that language to preserve the meanings. After acquainting myself with the data and having formulated some general ideas about the notable features within it, I then began to generate preliminary coding by assigning a 'code' to specific content using a software called MAXQDA (MAXQDA is professional software for qualitative data analysis that organises and categorises data, retrieves results and creates illustrations and reports). In order to do this I uploaded the transcript to the software and assigned a code to a highlighted segment of text. After the entire transcript had been coded, I had a long list of codes that were assigned to extracts. I then examined each coded extract and organised these codes into groupings that I called 'categories'. These categories were checked by a colleague (who holds a doctoral degree in Education) who agreed with the logical aspect of these groupings after extensive discussion. This phase involved sorting these different codes into potential categories, and collating all the relevant coded data extracts within these categories using the software. I then read through the 'code system' (as it is called in the software) and pondered how much each code agreed with the category. Then I created themes that were inferred based on the link between the different categories.

Finding

As shown by interview data Mr. Fallatah was questioned on his perception about the concept of metacognition, in which he explained the presence of logical thinking, and the importance of practising planning, management, monitoring and evaluation. Mr. Fallatah also discussed the function of metacognition in learning. He thought that metacognition would help him to discuss students' thinking rather than simply discussing solving methods. Metacognitive teaching encourages students to participate in a constructive learning process. Overall, Mr. Fallatah considered that metacognitive instruction was a positive experience for teachers.

The findings showed that in the initial stages of IMPROVE implementation, lessons were delivered ineffectively. Through practice this improved, as the seven steps of IMPROVE were better adhered to. The teacher's choice of activities was more appropriate for metacognitive teaching. This made students enthusiastic to solve problems, and after correction, they fully understood the problem.

As shown by the interviews, student perspectives of metacognition can be summarized as follows: the concept of metacognition was perceived as an awareness of thought and being able to judge its course in a positive way. In discussing its function in learning, several points were discussed, including evaluation and adjusting thought, helping students in understanding mathematics problems and concepts, and thinking logically when dealing with mathematics problems. Metacognitive skills were also emphasized as significant, these being planning, management of planning, and evaluation. Observation demonstrated that the students' learning method at the beginning of IMPROVE's implementation centred on direct solutions and this was done regardless of any thought method. However, with practice, the students took on a thought method for the mathematics problem in accordance with the metacognitive questions. As outlined in IMPROVE, these were: questions to understand the problem, the solving strategy and questions of linkage. The students' learning method in mathematics transformed from a complete reliance on the explanation and solving of the teacher to them making efforts to search for knowledge and building upon it.

Mr Fallatah expressed requirements for the successful implementation of metacognitive learning. The activities should involve indirect solutions, previous experience, hold new ideas, and should be challenging. Sensitive handling of student weakness as well as practice were also identified as key requirements. Students outlined a set of characteristics to be embodied in the teacher: he or she should hold knowledge of various styles of thinking in dealing with mathematics problems. Readiness, evaluation skills and the setting of suitable activities were also identified as important factors. Another of the major requirements was for the role of the student to be in searching for and building knowledge, rather than simply receiving knowledge by the method of memorization from the teacher. The creation of work maps for dealing with mathematics problems would enable them to monitor their thinking and help in its adjustment and its improvement. Students also felt the need to be well prepared and trained for full benefit. Another need expressed was for students to have a role in evaluating their method of thinking, with this being done with a mental work map for dealing with mathematical problems.

Obstacles identified in interviews included the domination of the traditional method over mathematics learning, lack of teacher readiness, and students being limited to a single way of thinking. Syllabus and textbook content was highlighted as being too large. It was noted that learning through metacognition required more lesson time. As for the challenges that might confront teaching mathematics metacognitively, Mr. Fallatah saw these as revolving around five issues. Firstly, the teacher being long-accustomed to teaching mathematics in a particular way, requiring traditional courses and investment of resources. Secondly, the absence of preparation and training for teaching through metacognition, be it at university or during a teacher's service in education. Thirdly, the general lack of previous adoption of metacognition in education is an obstacle and the lack of pursuit of how we can implement metacognition in reality. Fourthly, the school administration remains unconvinced because its focus is on the direct academic attainment of students and the completion of curricula. Finally, an absence of teacher evaluation criteria for using metacognition and a focus on superficial issues when evaluating teachers were both identified and explained as obstacles to the implementation of the method.

Discussion

The subject of teaching and learning mathematics metacognitively grounded in the implementation of the IMPROVE programme (Mevarech & Kramarski, 1997) is discussed. As a prelude to this discussion, two important issues will be pointed out. Firstly, domination of the traditional method that pervaded the reality of teaching methods in mathematics. Secondly, metacognitive mathematics teaching as a planned procedure.

Dominance of the traditional method in mathematics teaching

As for the Traditional Method, the term in this context refers to the presentation of mathematical concepts in a direct manner, i.e. without linking to other concepts or explaining how such concepts really work, so the students are aware of how to imitate but they do not know why they are doing things. This is a process of rote learning. This method is not conducive to shaping mathematical thinking to deal with varying problems using differing methods in changing contexts.

The findings indicated a tendency to depend on linking a presented solution to ones which had been previously encountered, a point of great interest to students which was often asked about. They also seemed to lack focus when tasked with thought monitoring or adjustment, and rather preferred to link current to previous solutions. This link was not of the reflective, metacognitive sort, but instead was a form of rote learning and imitation. This was consistent with Larkin's (2006) study, which identified a lack of sufficient opportunities for students to cooperate on a higher cognitive level as a key obstacle. This was evidenced by the findings of this study, in which steps to solve problems directly were focused on, rather than the thought methods involved in that process. Hence, the teacher-student relationship was neither participatory nor constructive but rather one in which monitoring errors was the norm. When the teacher serves as the central point of the learning process and his role does not extend beyond the delivery of information, it results in a hindered manifestation of metacognition. In a similar vein, Hurme, Järvelä, Merenluoto, and Salonen (2015) concluded that, in regards to problem solving in mathematics, student groups were neglectful of the analysis aspect. In addition, they failed to monitor and regulate workflow, which is a key component of metacognition. Therefore, neglecting analysis and verification weakened the use of metacognition and the full realization of its potential.

Metacognitive mathematics teaching as a planned procedure

When the IMPROVE programme had been implemented, the manifestation of indicators of metacognition and their extent in mathematics learning were observed. Based on the study's findings, there were indeed many signs of metacognitive mathematics learning. This highlights that the process of teaching mathematics metacognitively is one that must be planned and intentional, which is consistent with the assertions of literature. For example, the study of Naglieri and Johnson (2000) indicated that the provision of explicit metacognitive strategies can further enhance students' performance in mathematics – displaying the importance of planning to ensure effectiveness. Adding to this, Grizzle-Martin (2014) recommended the use of clear teaching that concentrates on cognitive and metacognitive strategies.

The findings of this study demonstrated the importance of metacognitive mathematics teaching as a planned and intentional process. To this end, the provision of a model to assist both teachers and students in achieving this type of learning is essential. This is consistent with Hartman (2001) who outlined studies surrounding the development of metacognitive practice. She summarized these as containing four main approaches, targeted at: raising general awareness through teacher-presented models, enhancing metacognitive knowledge, improving metacognitive skills, and developing learning environments. A wide body of research has suggested that teaching the use of metacognitive strategies assists students to regulate and direct themselves along with improving their performance overall (Raoofi et al., 2013). Hence, if learners are capable of discerning how they understand concepts, they are enabled to think introspectively and furthermore analyse how knowledge and its meanings are built through metacognition (Grizzle-Martin, 2014). Moreover, Grizzle-Martin (2014) expressed the view that IMPROVE is an explicit form of teaching. This is because teachers direct and guide learners during problem solving, but eventually seek to enhance their abilities as independent learners.

Facilitating both strategy acquisition and metacognitive processes

The choice of an appropriate strategy for learning through metacognition plays an important role in mathematics learning. This was displayed by the study's findings and the strategy can be considered as a mind map. This has a role in learning as the presence of a strategy being represented as a mental map for the student in dealing with mathematics problems would assist them in monitoring and adjusting their thinking for its enhancement. The findings point to the strategy's systematic nature in pointing out its utility as a method to monitor and adjust thought. It was also highlighted that this method helped in identifying and locating errors, before remedying them.

The IMPROVE programme is based upon the processing of new information based on pre-existing information. This is done through metacognitive questions, the formulation and answering of which are targeted at processing such information, according to Mevarech and Kramarski (1997). This is because a key feature of control and regulation is "the decisions one makes concerning when, why, and how one should explore a problem, plan a course of action, monitor one's actions, and evaluate one's progress." (Lester, 1989). According to Mevarech and Kramarski (1997), there is a clear case for instructing students to create questions that may result in rich and elaborate explanations. Such questions are targeted at (a) the structure of the problem, (b) connections between the new and existing knowledge, and (c) specific strategies and principles that are appropriate for solving the new problem.

The IMPROVE programme presents a useful vision in the field of metacognitive mathematics teaching in this area. However, it is essential in this context that the use of a strategy is targeted at assisting students in monitoring and adjusting their thought when dealing with mathematics problems, which was underlined by the findings of this study. Despite the importance of a clear strategy for learning mathematics - be it in problem solving or understanding new mathematical concepts - it is also essential not to limit students to a single strategy. Doing so would not be consistent with conscious reflection on the efficiency or learning for the development of metacognition (Thomas, 2012). Hence, limiting students to dealing with a single strategy in mathematics learning was seen not to help the students in creating and innovating with new strategies, which would enable students to develop an ability to choose the most appropriate strategies for learning concepts and solving numerous mathematics problems - the absence of which means an absence of learning through metacognition. This is consistent with Thomas's (2012) assertion that if students are not consciously reflecting on the newer tasks introduced to the classroom and the impact on learning then the development of metacognition can be questioned. However, the problem of confining students to certain strategies can be alleviated through distinguishing between a general and limited suggested strategy such as a mind map for dealing with math problems that students can be trained in, and a specific strategy to illustrate the key to solving the mathematics problem. A specific strategy which is a key to solving a mathematics problem must not be confined to a specific pattern. Therefore, the presence of a general strategy such as a mind map for dealing with mathematics problems helps in creating multiple methods and strategies for solving.

Provision of feedback-corrective-enrichment that focuses on lower and higher cognitive processes

The findings demonstrated that evaluating students' thinking in dealing with mathematics problems can be considered a fundamental pillar in learning through metacognition. Findings pointed to a need for greater efforts in the approach to evaluation. One specific example of this was the provision of metacognitive activities, on which students could then be evaluated by the teacher in discovering their thought methods. It was also clear from the findings that the teacher's role should focus on evaluation and supervising the lesson, rather than reverting to traditional methods of rote teaching. This is consistent with Hogan et al. (2015) who outlined that the type of feedback, while just giving the student the right answer, fails to prompt them or suggest appropriate strategies for future problems. Hence, this traditional feedback style may be insufficient in assisting the learner to monitor, adjust or even to be aware of learning strategies and their effectiveness. In contrast, prompting is targeted at directing the learner as to when and why to apply a given strategy.

This view concurs with that of Mutekwe (2014) who discussed the need for cognition among students undertaking tasks, but added to this by pointing out that metacognitive skills often help students to understand how tasks are performed. Therefore providing a quality feedback structure is essential, as it helps to regulate, monitor and direct students. In this regard, the findings underlined the importance of evaluating students' thinking in dealing with mathematics problems from a peer. This is consistent with Mutekwe (2014) who mentioned rapid yet flawed feedback could be more effective than better thought-out feedback provided by an instructor at a much later stage. This evaluation for students in their dealing with mathematics problems cannot be undertaken successfully unless there is a prominent role for the student in the process of learning through metacognition. Thus, it is difficult to create learning based on metacognition when the student's role is limited to receiving information without participating in the search for it. A confirmation of the importance of the student's role in learning through metacognition was underlined by the findings of this study in both case studies. They revealed that the onus was upon students as a significant aspect in implementation. After all, metacognition itself is targeted at bringing students to the centre of the learning process, and giving them the responsibility to search for information so as to create a constructive learning atmosphere. This decentralization of teaching responsibilities encapsulates the difference between the traditional and metacognitive methods, and this was highlighted by interviewees as the new approach made students think, in contrast to memorizing and solving a problem by direct application.

Two other important requirements emerged from the findings regarding learning through metacognition. These fall into the context of evaluating the thought of students when dealing with mathematics problems. The requirements include the provision of sufficient time to practise and provision thorough preparation for mathematics activities – both regarding learning through metacognition. This is consistent with Sahin et al.'s (2013) research, which highlighted the importance of providing sufficient time for problem solving. They explained that students should be urged to take their time and be cautious in problem solving. Regarding long-term time allotment, Schraw and Gutierrez (2015) highlighted that programmes ranging from six weeks to several months tended to be more effective. This is because long-term programmes enable students to model, practise and automate strategies, while also enhancing conditional knowledge. Furthermore, another benefit is that instructors themselves improve their teaching and modelling of strategies over a lengthier period of time.

On the subject of practice, the findings showed that responsiveness to metacognitive techniques improved over time, which was attributed to providing a sufficient period for their implementation and practice. Students required much time to practise the four metacognitive skills, as well as creating solution strategies. The findings of this research are consistent with those of Grant (2014), as after intervention it was found that many students needed more time to absorb and enhance schemata after a new mathematics concept is presented to them. Some students were initially reluctant to use the method, but after witnessing benefits such as better understanding of mathematical concepts they realized its utility. A lack of familiarity was highlighted as an obstacle, yet participants explained that this faded with greater practice, as it was absorbed into their mathematics learning 'culture'. It was also underlined that metacognition should be maintained in practice for it to become a permanent feature in mathematics learning, rather than one applied in controlled circumstances, with one suggestion being its introduction into other subjects.

Furthermore, the findings demonstrated the importance of metacognitive instruction becoming habitual for the teacher. It was suggested that the technique should be used in more lessons so students could reap the maximum benefit. The results demonstrated that Mr. Fallatah tried to implement the IMPROVE programme to a greater extent, and this was not limited to the occasions in which I was present. Thus, the importance of metacognitive instruction becoming habitual for the teacher was demonstrated.

Teachers' thorough preparation for mathematics activities is also required with metacognitive teaching. This assists in the process of evaluation for learning mathematics metacognitively – which the findings demonstrated. Preparing suitable activities for metacognitive teaching is essential in leaving a lasting impact on students regarding the method. Syllabus content should be consistent with metacognitive teaching, which would be method rather than solution-oriented. These findings are supported by a study carried out by Simons (1996), which explained that certain features that improved intervention became clear through analysing the beneficial impact of metacognitive training. First was the formulation of tasks relevant to students' experiences both within and outside the school environment. Activities which suggest that achieving mastery in school is possible tend to encourage students to perform better, particularly in tasks overseen and evaluated by teachers or parents. This involves a suitable difficulty level, as a task too easy may negate the purpose of monitoring and regulation, as students will simply invoke routine processes. In contrast, a task too difficult may discourage low-achieving students while high performers may persevere despite failures.

Conclusion

Several findings were drawn from the data by this study, the first of these being that the traditional method can hinder mathematics teaching and learning through metacognition. Secondly, although metacognitive mathematics instruction should be planned, the strategy that is introduced should be directly targeted at improving the monitoring and regulation of students' thought when dealing with mathematics problems. Thirdly, metacognition should be given priority to improve students' consciousness of the learning processes. This is because conscious reflection enables students to develop an ability to choose the most appropriate strategies for learning concepts and solving mathematics problems. The findings underlined the importance of the student's role in learning through metacognition.

Disclosure statement

No potential conflict of interest was reported by the author/s.

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