International Electronic Journal of
Mathematics Education
Volume 2, Number 2, July 2007 www.iejme.com

LINKING SCHOOL MATHEMATICS TO OUT-OF-SCHOOL MATHEMATICAL ACTIVITIES: STUDENT INTERPRETATION OF TASK, UNDERSTANDINGS AND GOALS

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ABSTRACT. This article considers one class of high school students as they worked on a task given to them by a company director of a haulage firm. The article provides details of students' transformation of the given task into a subtly different task. It is argued that this transformation is interrelated with students' understandings of mathematics, of technology and of the real world and students' emerging goals. It is argued that the students did not address the company director's task. Educational implications with regard to student engagement with realistic tasks are considered..

KEYWORDS. Goals, Mathematics, Out-Of-School, School, Tasks, Understandings.

INTRODUCTION

This article reports on the work of one class of high school students in a project1 which investigated ways of linking school mathematics to out-of-school mathematical activities. The class worked on one task, identifying when a vehicle is in a specific area, given to them by a director of a transport/haulage company. This article presents detail of what the students did and argues that they did not address the company director's task because they transformed the task and that this transformation is interrelated with their understandings and their goals.

The next section briefly considers the literature on the use of mathematics in and out of school and the words 'understanding', 'goals' and 'activity'. This is followed by a description of the wider project and this classroom project. Results for students working in groups are then detailed. The Discussion section attends to student transformation of the task and the interrelations of this with their understandings and goals. The final section considers educational implications.

ISSN: 1306-3030

[1] See http://www.education.leeds.ac.uk/research/cssme/outofschool.php for the project description.

THE USE OF MATHEMATICS IN AND OUT OF SCHOOL

This section serves to position the classwork considered in this article with regard to related research and to introduce constructs employed in the Discussion section. It does not aim to present a comprehensive review of literature in the field.

30 years ago there appeared to be a general acceptance, in mathematics education literature, that the relation between mathematics, school mathematics and out-of-school mathematics was relatively unproblematic. Since then 'school mathematics' has been a focus of inquiry. For example, with regard to ‘mathematics’ and school mathematics, Chevallard’s notion of ‘didactical transposition’ where “mathematics in research and in school can be seen as a set of knowledge and practices in transposition between two institutions, the first one aiming at the production of knowledge and the other at its study.” (Lagrange, 2005, 69). The school mathematics considered in this article, however, is somewhat different from the school mathematics Chevallard considers as the students in the series of lessons were not formally studying mathematics; it is school mathematics by virtue of its situation, a mathematics classroom, but this, I argue later, is important.

With regard to the relation between school and out-of-school mathematics, most research shows a strong discontinuity between school and out-of-school mathematical practice. According to early work on situated cognition, e.g. Lave (1988), this discontinuity is a consequence of learning in and out of school being two distinct social practices. School mathematics, moreover, is often not suited to out-of-school practices: in some cases out-of-school problems are only apparently similar to school mathematics problems, but in reality there is a range of explicit and implicit restrictions which makes school methods unsuitable, and thus other methods are used (Masingila et al., 1996); in other cases (Scribner, 1984) work mathematics may appear to be simple, but there are no simple algorithms or methods to solve the problem and school-learnt procedures are of no use.

Despite this evident discontinuity some authors have observed an interplay between school and out-of-school mathematics. Pozzi et al. (1998) found cases of nurses looking for a mathematical explanation for ‘simple’ mathematical procedures used in their daily practice. Magajna and Monaghan (2003), in a study of technicians designing moulds for bottles, found evidence that, in making sense of their practice, the technicians resorted to a form of school mathematics. The focus of this article is a sort of converse to the focus of these studies: what sense do school students make of an out-of-school problem within their school practice?

Understanding, goals and activity

In this sub-section I briefly attend to how I use the words ‘understanding’, ‘goals’ and ‘activity’ in this paper. This is important for clarity and because these words are used in different ways in everyday and in academic discourse.

I use the word 'understanding' in its everyday sense. Davydov (1990) would refer to this as 'empirical thought', as opposed to 'theoretical thought', connecting features of reality rather than reproducing reality. When I refer to students' understandings, I refer to how they connect features of their perceptions of reality rather than their theoretic reproduction of reality.

'Goals' is another everyday word which is used with specific but variant meanings in academic discourse. To Leont'ev (1978) goals are the raison d'être of an activity but Saxe's (1991) emergent goals are little 'must do' things that come into being during activity and often, but not always, pass away. Emergent goals that do not pass away, however, can and do develop into conscious (and bigger) goals. This development of goals is an important aspect of the task-goal transformation considered in this article.

'Activity' is, of course, central to activity theory and, in education-speak, to student activity in classrooms. In this article I use it in the education-speak sense. There is, however, a tentative connection between these two senses of 'activity' (and, correspondingly, between Leont'ev's and Saxe's goals). Activity theory at present is often viewed with regard to activity systems (see Daniels, 2001, pp.83-94). Activity systems are large scale systems. A school qualifies as an activity system, a sequence of lessons does not. This remark applies to Leont'ev's version of activity theory too, though Leont'ev acknowledges that activities are nested. There is a sense, however, that continued student work over several weeks, such as the student work considered in this article, represents an activity and that students' goals, when they shape students' subsequent work over a period of time, can be considered as the goals of this activity.

THE 'LINKING SCHOOL MATHEMATICS' PROJECT

The wider research, of which the classroom work reported on here was one of 20 classroom projects, worked with high school mathematics teachers to explore ways in which school mathematics might be 'made real' – be linked to out-of-school mathematical activities. Research questions were formulated around four interrelated research themes: the design of tasks; the use of resources; teachers' perceptions; and student learning. Participating teachers were selected from a wider group of applicants on the basis that they perceived links between school and out-of-school mathematics to be a problematic issue and on their avowed intentions to undertake substantial classroom activities. Teachers were to work with the author and the project research assistant but were free to pursue activities of their own choice. The project ran for two school years.

This classroom project

The class teacher was involved in the wider project because the project reflected what he wanted to move on to in some of his lessons. He has special status as an 'advanced skills' teacher and his school is a high attaining state school which accepts students of all attainments.

The class, 29 students, is a Year 10 set 2, i.e. students are 14-15 years of age and are high, but not the highest, attaining students in mathematics in that year group.

The teacher had carried out two previous classroom projects within the wider project. For both he consulted with people in industry to find real life tasks. For this classroom project he consulted with a company director of a large transport company with a fleet of over 250 large refrigerated trucks which distribute frozen food from wholesalers to retailers. The problem, one the company director was actually considering at the time that he was approached, was how to obtain an automatic computer record of the time when each vehicle reaches and leaves a destination, e.g. a specific supermarket depot. The company had a map of each delivery site, so the problem reduced to knowing when a truck is within a specific geographical location. Each truck is fitted with a global positioning system (GPS) which reported the truck's location at a regular time interval to the company's central computer.

The teacher and I visited the company director at his work where he explained about the company and his problem to us. Following the meeting the teacher and I exchanged (and rejected) various ideas, many quite technical such as transforming latitude and longitude readings into map references, before the teacher suggested "Let's just let him present the problem and let the kids find their own solution." This is what happened.

The teacher allocated seven 60 minute consecutive lessons to this work. The company director would present the problem in the first lesson and the students would present their solutions in the last lesson. The students would work in self-selecting groups. The company director was present for the first, fifth and last lesson. The project research assistant joined the team for data collection. The teacher and the two university researchers were present in all lessons.

The company director made his presentation (an abridged transcript of the presentation is provided in Appendix 1). After the presentation the students formed into seven groups of three-five students. The remainder of the classwork is presented in Results section.

Methodology for this classroom project

The approach to researching this series of lessons was to record participants actions, motives and interactions. Meetings of the teacher and the company director were audio recorded. Meetings between the teacher and the author were written up. The teacher and the company director were interviewed prior to the lessons. Two digital video cameras positioned in different parts of the classroom recorded all the lessons. Six audio recorders were used: five on student group's desks and one with the author. During the lessons teacher, company director and student group interviews were conducted (group interviews in an adjacent room, others mainly in-class). The two researchers kept fieldnotes which were later written up. Students' hand written work was photocopied and copies of their computer files were made. Still photographs of students' work were taken. The only known gap in data collected was that not every group was audio recorded in every lesson – there were five audio recorders for this and seven groups; missing data was partially reconstructed from fieldnotes. All audio recordings were transcribed. Physical and electronic files were compiled for each group and for whole class activity.

Data analysis was a lengthy and uneven process over many months. It used elements of a grounded theory approach (Strauss and Corbin, 1998) but this research did not attempt to follow all the rubrics of grounded theory. The data collected on these students' work could be analysed to support a number of related foci, e.g. collaborative group work or argumentation or student modelling. The focus on students' goals, understandings and their transformation of the task in this article was chosen because it is closely linked with the overarching aim of the project – to investigate how school mathematics may (and may not) be linked to out-of-school activities.

The main dataset for which detailed analysis was conducted were transcripts of students talking (student work in groups and interviews with students). Other data was re-examined but was not subjected to formal analysis. Selected transcripts were analysed by the researchers, first independently and then results were compared, codings refined and transcripts re-examined using: open coding á là Strauss and Corbin (1998), but stopping short of selective coding; Artzt and Armour-Thomas' (1992) cognitive – metacognitive categories; an interim category system. These transcript codings should be viewed as exercises which helped the two researchers make sense of the data. The interpretation made of this data was discussed with the class teacher but, in the end, is just one interpretation.

RESULTS

This section outlines what the seven groups of students did in these lessons and reports on their work as groups. This is done partly for reasons of economy of reporting and partly because the work of students in groups may be regarded as "taken as shared rather than shared" (Cobb et al., 2001, 119). I outline what each group did, how this approach arose and developed and the accuracy of their work[2]. This section ends with an overview of what the students felt they learnt in these lessons. First, however, I comment on an aspect of 'time' and the role of adults in students' work.

Student activity (actions and discourse) can be considered over different time intervals: micro (seconds); meso (minutes); and macro (hours). This article considers all three levels but there is an analytic danger in doing this because micro, meso and macro analyses of student activity are likely to produce different results due to differing levels of adult interference (often not present at the micro level; always present at the macro level). Adult intervention was considerably less in this work, compared to much 'traditional' classwork, but I nevertheless guard against this danger by recording adult intervention in accounts of student work. A

[2] The accuracy of each solution was tested by writing a spreadsheet programme for each group's solution and testing it with four points which were, with regard to the depot's perimeter: 'well' outside, 'well' inside, 'just' outside and 'just' inside. The students were aware of this programme but did not know where the test points were.

simultaneous consideration of all levels is regarded as important for the focus of this article because, as shall be argued in section 5.3, of the influence of emergent goals (at the micro level) on goals (at the macro level).

Each group interacted with adults (teacher, company director and researchers). The adults claimed that they did not direct students to adopt specific approaches and attended only to practical problems with student approaches and help with specific mathematical techniques. As shall be discussed, however, adult intervention did appear to influence student work.

Student group work

The first half of lesson 1 was devoted to the company director's presentation. Thereafter the students split into groups. In the remainder of lesson 1 each group 'brainstormed' ideas.

Group A consisted of five girls who covered the map of the depot with five rectangles (see Figure 1). In lesson 1 they asked a number of questions:

> Are we allowed to change it if we want?
>
> ….
>
> Surely you can't actually do that without having like programmed stuff into the GPS.

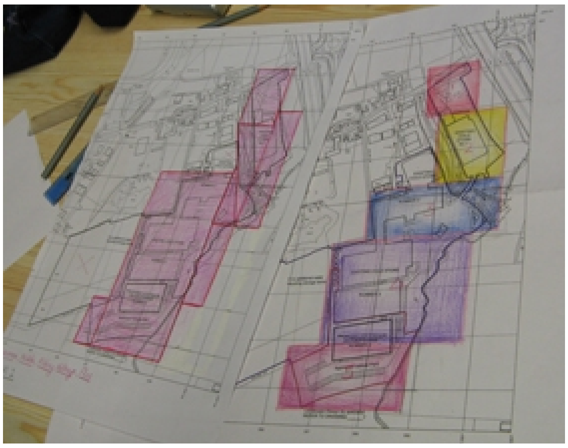

Figure 1. Group A, five rectangles

A researcher said that they could make changes and emphasised that anything they did must be able to be programmed on to a computer. He also said that they could "find a simpler problem which is in the same ball park". In lesson 2 they started by covering the map with geometrical shapes that made up the polygon but quickly moved on to covering the map with rectangles and triangles which contained the area of the depot because "we wanna make it simple". Triangles were eventually rejected because they were complicated. Rectangles were orientated so that they were parallel to lines of latitude and longitude "to make calculations similar to each other". This solution strategy formed the basis for the remainder of their work and subsequent lessons developed computational methods based on using rectangles. Lessons 5 and 6 were devoted to preparing a Powerpoint presentation and "just doing some examples and some test points to prove that our theory works". Technical discussions included how to calculate using Northings and Eastings and the accuracy required (they eventually decided to use "simple numbers just to show the basic idea"). Their approach was straightforward to program although they did confuse Northings and Eastings. Once the coordinates were corrected the solution was successful.

Group B consisted of four boys who covered the map of the depot with four rectangles (see Figure 2). During lesson 1 and the first 20 minutes of lesson 2 their main focus was on polygons and whether polygons had to be used. They started by considering whether they had to use the given polygon. One mentioned finding the "equation for a polygon" but another commented that that would be "horrendously complicated". One mentioned "separate polygons or rectangles" but another commented "I don't like polygons, can we use triangles?" and in the ensuing discussion several commented that triangles, rectangles and pentagons were all polygons but that polygons themselves were very complicated. The last reference to a "complicated polygon" was followed by "Can we just have a square that fits here?" After this comment they started working looking at squares, then bringing coordinates into squares and later (but still in lesson 2) widening their consideration to rectangles, settling on four rectangles. This solution strategy formed the basis for the remainder of their work. Subsequent lessons attended to the problem of not including the motorway within a rectangle "that's such a problem" (lesson 3), developing computational methods and preparing a Powerpoint presentation (the group split into two pairs from lesson 3 onwards, one pair attending to technical issues and the other preparing the presentation). Their approach was straightforward to program although they did calculate some coordinates incorrectly. Once the coordinates were corrected the solution was successful.

Figure 2. Group B, four rectangles

Group C consisted of four girls who constructed a six step algorithm which determined whether a specific square metre was within the boundary of a shape:

1) Take the largest of x and y regardless of the sign.

2) See if there is a number that is greater than or equal to that number, with the same sign, on the same axis.

3) If not, then the co-ordinates are outside the area.

4) If yes, then see if there is a number that is less than or equal to the chosen number on the same axis.

5) If no, the co-ordinate is outside the square.

6) Take the second number in the co-ordinate and repeat steps 2 to 5.

Work on this algorithm started in lesson 2. In lesson 1 they considered "technical tags" at the entrance and exit of every depot to record the arrival and departure of lorries and said "It would be easier not to do it with the GPS". In lesson 2 they discarded this and related approaches, e.g. barcodes on the sides of the vehicle, because "none of them really used much maths". There approach was focused on squares (subdivided into 10000 squares) because the shape can be subdivided into squares and "if you had 10,000 squares I don't think it would take a computer long to check it, matter of seconds". A lot of classroom time in lessons 3, 4, 5 and 6 was spent attending to the logic of the algorithm: the word 'work', as in "does it work" appears on every page of the transcripts of these lesson; when asked during their presentation "what was the main thing that directed you towards that solution?" the reply was "Just logic". This theoretical solution could not be programmed as no coordinates were calculated. The students realised this but were satisfied with their method.

Group D consisted of three girls who enclosed the map of the depot with a quadrilateral (see Figure 3). In lesson 1 they had two ideas: having a box by the entrance to a depot where the lorries waited until the company computer registered their arrival; and covering the map of the depot with enclosing rectangles. In lesson 2 the rectangles idea was pursued and gradients were mentioned (earlier mathematics lessons had covered the equation of a straight line). In lesson 3 they considered the diagonals of their rectangles and focused on the equations of these diagonal lines. This led them to look at how to tell which side of a straight line (given by an equation) a given point was on. This determined the focus for subsequent work. Their reasoning was:

> We have four equations of the lines and using an equality to it we can find out if it's, which side of each line it's on and so you work out an equation and you fit the corners to the four equations and if it works for every single one of them then it's inside the shape.
>
> Interview, lesson 5

Adults provided mathematical assistance but the method of determining which side of the line a point was on came from the students. They did not prepare for the presentation until lesson 6. Their solution contained an error with the equation of one of the lines but once this was corrected it was straightforward to program and was successful.

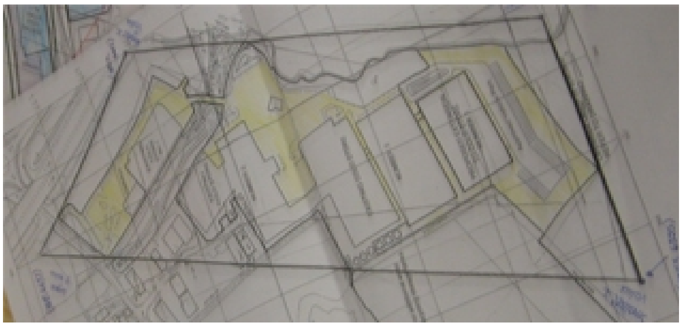

Figure 3. Group D, quadrilateral

Group E consisted of four boys who considered an octagon (see Figure 4). In lesson 1 they considered “something near the entrance of one so that when it drove past it registered there” but rejected it because “maybe the lorries wouldn’t be in the entrance long enough for it to register”. In lesson 2 they consider a square, “from (1, 1) to (2, 2) and if both points are between 1 and 2 then we know it’s inside the square”. The teacher asked them how they would know if the square was within the depot. They considered ways that the shape of the depot could be split up. The group focused on regular octagons, which they referred to these as “right angled triangles and squares”, because “any polygon can be made up of right angled triangles and squares”. Subsequent lessons focused on a specific octagon with simple coordinates which were not related to map references of the depot. In lesson 4 the teacher encouraged the group to start considering the presentation and two students took this as their main focus of activity. The solution used equations of lines to represent the sides of the octagon. It was possible to program their solution but: (i) simple coordinates rather than map references had to be used; (ii) all points within the square enclosing the octagon (produced from extending the hypothenuses of the triangles) registered as being within the shape.

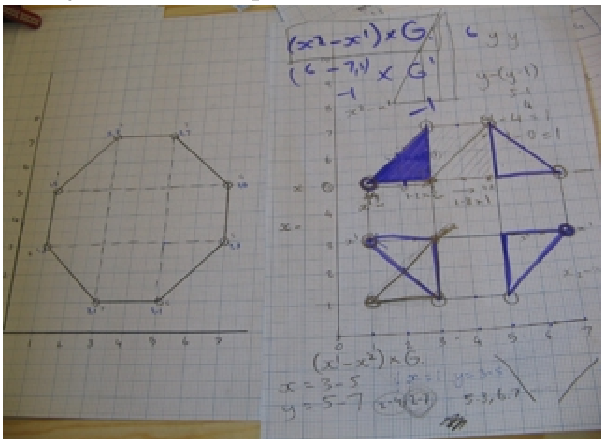

Figure 4. Group E, octagon

Group F consisted of five boys who covered the map of the depot with seven circles (see Figure 5). The group did not initially see why the company director had a problem “Can the lorry drivers just like phone say when they’re in the depot?” and “When the GPS reports back … it kinda lights up the area and makes a big beeping noise so we know it's in there". A researcher eventually persuaded the students that neither solution was desirable because 24 hour automation was required. The students then wondered if there was "formula" for the shape but did not know how to proceed. In lesson 2 they started thinking about "four different shapes … it would be easier to work out if a point was in each shape … it needs more development thought 'cos it's like complicated because latitude and longitude lines are not like on the rectangle". This led to their focus on circles (which is, mathematically, quite sophisticated[3]) because "you don't have the problem of it being over, like crossed over, where I mean however you turn a circle it's always gonna be the same shape on the grid", i.e. a circle does not need to be orientated to be parallel to latitude/longitude. Thereafter circles were the focus of their work. The group struggled with relevant school circle maths (pi, arc, area, 360 were mentioned) and the teacher provided mathematical assistance, $(x-a)^2+(y-b)^2=r^2$,which he felt took its cue from student-generated ideas. This was new mathematics to the students but they were quickly able to apply it to map coordinates. The teacher encouraged the group to prepare for the presentation at the start of lesson 4 but it was not until midway through lesson 5 that they began this as they were engrossed in working out the details, to 6 significant figures, of what they proudly called their "360 theory". Their solution was tailored to the company depot but they made recommendations for further use, "We suggest that if this theory is used, one or two sizes of circle should be used instead of several to save time and complications." The solution provided by this group was mathematically correct and easy to program. It required no alterations to ensure it worked correctly.

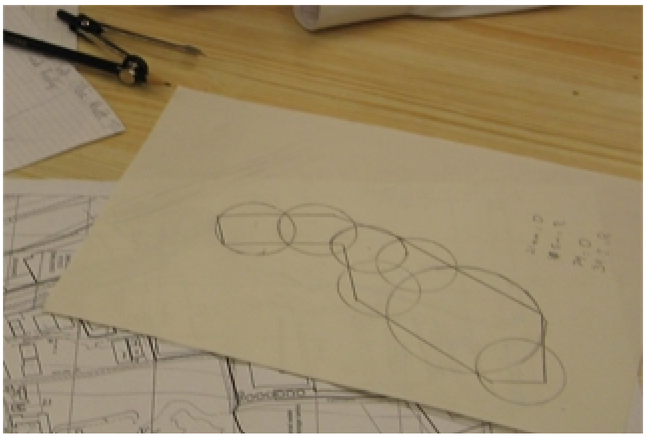

Figure 5. Group F, seven circle

Group G consisted of four girls who covered the map with lots of thin rectangles (see Figure 6). They had two ideas in lesson 1, split the polygon into simpler shapes and to have an oval shape with a central point which was somehow symmetrical. But in lesson 2 they stated that they did not really see what the problem was "you've already got the polygon … points so can't you just use the polygon … the points that you've got?" An extended dialogue with a researcher led the students to see the problem. Nevertheless the group remained uncertain as to how to

[3] I think several of the students realised that I was impressed when they explained this to me. I mention this because this could have influenced their goals/motives. This is a very difficult matter to state with any certainty but "this guy thinks we're clever" could have encouraged them to continue along these lines.

proceed until lesson 3, where 'straight lines' were considered and developed as an approach:

> Like just have a simple straight line around the edge … Or you could like have lines across the actual thing …
>
> …..
>
> … lots of lines all over it just so that you could like see how [tape unclear]
>
> …..
>
> So you put the co-ordinates of the line in

This is taken further but is then dropped as an idea. They reconsider their oval shape but know another group is using circles and reject their oval approach because it is not original. In lesson 4 they still do not have a plan. They discuss using rectangles and this connects with their lesson 3 'straight lines' idea – instead of using straight lines they could use "loads of thin rectangles" and this idea is pursued for the remaining lessons. The group works on the details but time is short and although it should be programmable (and overall correct) they are far from completing calculating the coordinates for all the rectangles by the end of lesson 6 and do not make a presentation.

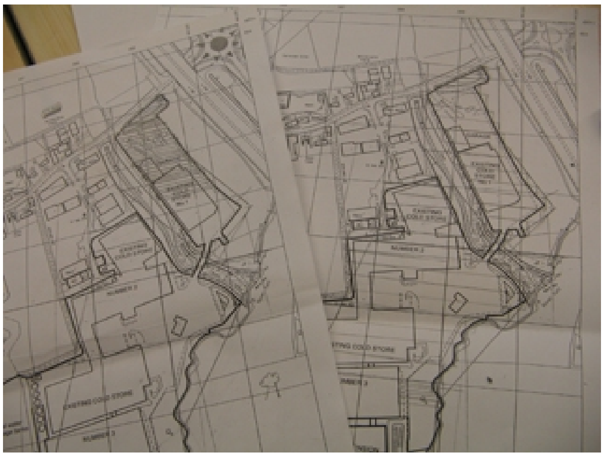

Figure 6. Group G, thin rectangles

What the students felt they learnt in these lessons

I briefly comment on what the students felt they learnt in these lessons as I comment on this later. In the lesson following the presentation 25 students (all who were in that lesson) were asked to write what they felt they learnt in these lessons. The mean word length of their responses was 40 words. Their responses were coded using open coding. All students listed more than one thing they learnt. Their responses are summarised below.

The largest category (19) was 'use of mathematics'. Comments to the effect that they learnt that mathematics can be used, or arises, in real life. In two cases students seemed a little surprised at this:

> I have learnt that mathematics can be used in everyday situations and still be helpful.
>
> I never really thought that coordinates could be used in a real life situation.

Then came learning about business (9):

> given me an insight into the way that businesses work

including mathematics can be used in business

> how closely connected it can be in the efficiency of running a business.

Other categories with frequencies greater than 1 were: GPS (5); team work (5); different ways to attack a problem (4); specific mathematics learning (4), e.g. “equation for a circle”; about mathematics (3), e.g. “mathematics can be computerized”; problem solving (2); and understanding over time (2).

DISCUSSION

In this section I consider the following three-part proposition.

1) These students transformed ‘the task’ into ‘their task’.

2) What students wanted to do (their goal) was interrelated with where they were (a mathematics classroom), their interpretation of the task and their understandings (of mathematics, of technology, of everyday life).

3) Students’ goals and their interpretation of the task interacted and developed over time; there was a task-goal trajectory.

Students’ interpretations of the task set

I restate the task the company director gave the class. He concluded his presentation with,

> What we need to be able to do is to be able to identify for a particular latitude and longitude whether it is within the boundary of that shape … … The best solution ... so far is to use a polygon … it very difficult to identify when a point is within a polygon.

He identifies a point being within the boundary of a shape with a point being within a polygon and the other adults present interpreted his task as determining when a point is inside a polygon. None of the final student solutions, however, used polygons. With the exception of the group that focused on octagons, all the groups, in one way or another, enclosed a polygon in a set of simpler shapes. In an in-class interview during lesson 5 the company director commented that it “will be easier if we just, if we can approach it on the basis of a polygon as opposed to breaking it down into shapes”.

Why did the student solutions not address the polygon problem? Well, polygons are somewhat complex shapes and students from all groups and several adults mentioned, at one time or another, the need to simplify the problem. Only Group B pursued polygons for some time: during lesson 1 and the first 20 minutes of lesson 2. They did not mention polygons again except during the lesson 2 interview (after the focus on squares had emerged) where they rationalised that the company director "didn't want complex shapes so if you break it down into simple ones instead of having a big polygon which is complex". Such development in interpretation is an example of what I mean by *they transformed 'the task' into 'their task'*.

The complexity of polygons also featured in the considerations of Group A but this was not the only reason for task transformation. Appendix 2 summarises the development of the groups highlighting their ideas, foci, goals and interpretations of the task. To further understand these transformations of the task I turn to students' goals and understandings.

Goals and understandings

Motives and goals are inextricably linked. Goals are what one wants to do and motives are why one wants to do it. Goals were easier to locate in students' discourse than motives. For this reason I focus on goals. In this sub-section I consider students' goals in relation to the situation of the activity (a mathematics classroom) and their understanding of mathematics, of technology and of everyday life.

In a mathematics class

This out-of-school activity was conducted in-school and this matters. These students were in a mathematics class and they were, obviously, aware of this. Students expect to do 'school mathematics' in a mathematics class. This was implicit in some cases, e.g. Groups D and E used equations of straight line graph, a topic studied in mathematics classes just prior to this work. In the discourse of Groups A and C the expectation to use school mathematics was explicit, e.g.:

> Shall we draw this in a graph?
>
> Why?
>
> Cos that's normally what you do with co-ordinates
>
> Group A, in-class discourse, lesson 2

> We had lots of ideas but then we realised none of them really used much maths
>
> Group C, interview during lesson 2

To spell out what may be obvious I draw attention to "Shall we draw this in a graph" suggests a goal and "none of them really used much maths" suggests that some goals were not pursued and that suggesting or avoiding these goals was related to students' understandings of mathematics and of being in a mathematics class. There was, however, no clear indication that what Groups B, F and G did was related to their work being done in a mathematics class.

In a workplace, however, when mathematics is used, it is not used for the sake of using mathematics, it is used to get something done (a goal). There is also the issue of 'the mathematics' used: in the workplace 'invisible' mathematics may be more relevant in an activity than 'visible' mathematics (Noss et al., 1996) but it is likely that the Group C student (see quote immediately above, "none of them really used much maths") thought of 'mathematics' as 'visible mathematics'.

Understandings of technology

Technological considerations were central to the task set as a central computer would receive a GPS reading and register whether a vehicle was in a depot and students' understandings of technology interrelated with their developing goals. I focus on two student understandings of technology which impinged on their goals/solution strategies, perceptions that computers 'know' and perceptions of their calculating prowess.

Many students in England are familiar with technology which will tell them, unprompted, say, if there is a wireless network in the area, and doing things for them, e.g. automatic correction of spelling. Without knowledge that there are programs behind such features it is not unreasonable for students to assign intelligence to technology to do things beyond than that which it has been programmed to do. Two groups, C and G, did not initially see what the problem was because they thought a computer would 'know' when a vehicle was in a depot:

> We didn't see why they had a problem because it shows on the board ... there's the little dot[4]. Can't you just have like a shape of the depot marked out and then you know when it's gone in.
>
> Group C, interview during lesson 2

> We were wondering what the actual problem is because you've already got the polygon ... surely they just need to update it so that they can just type in the co-ordinates and it can know.
>
> Group G, interview during lesson 2

One possible goal in such cases is to find out what the problem is. Another possible goal is to appear to be doing something (because you are supposed to be doing something). It is difficult to tell from the transcripts quite what the goals of Groups C and G were in this period.

None of the 29 students in the 30+ hours of audio recordings for these lessons appeared to consider data processing limitations of the company's central computer. This is not unreasonable for the company director did not mention this in his presentation and it is popular

[4] The "little dot" is the dot representing a vehicle in the company director's real-time computer display (see Appendix 1).

mythology that 'computers can do it in a nanosecond'. This is a case of technological considerations not influencing students' goals when, to the company director, such considerations would have been apposite. Indeed, during lesson 5 he commented, regarding all student solutions, that "it takes a lot of computing power to get through that"[5].

Everyday understandings

There is a case for students employing their knowledge of the world in real life problems but this assumes that this knowledge is pertinent to the problem at hand. This is not always the case as the following exchange between students shows:

> They could have heat sensors or maybe pressure actually then [sentence unfinished]
>
> Like on my dad's car where they could tell whether someone's sat on the seat or not
>
> Group C, in-class discussion, lesson 1

Or the following student who was responding to a researcher asking why other ideas of the group had been rejected:

> We were thinking about it but they weren't really, I dunno they weren't original and they didn't kind of appeal to us Group G, interview, lesson 5

Originality may be useful in, say, art and design but it was not important to the solution of this problem. This desire for originality appears to be linked to the situation, a school, where competition between the groups may be expected.

Understandings of technology were often intertwined with understandings from everyday life. For example, an early idea of Groups B, C and E was to have an electronic scanner at a depot's entrance (electronic scanners are common in clothes shops in England):

> We had an initial idea of putting a single line with two co-ordinates over the entrance and then when it passed it would register Group B, interview during lesson 2
>
> like having an infra red scanner like you have those tabs on clothes in shops and if you walk through the barriers it beeps at you Group C, interview during lesson 2

This is a sensible idea but, as the company director pointed out, he cannot insist that his clients have a scanner at their depots' entrances and there may be more than one entrance.

Of course a person's prior understanding will always influence their problem solving (in any domain) and these students engaged on this task were not exceptional in this. The point of this sub-section, however, is that the understandings of these young people (with no in-depth knowledge of the company, GPS or the working of a central computers) in a mathematics classroom interacted with their goals in ways that did not appear appropriate to a solution to the task given by the company director.

[5] The extreme case, with regard to the number of calculations, was Group C whose solution strategy considered each square metre of a depot. Imagine this information being processed by a computer every three minutes, for every depot for over 250 lorries and you may begin to see that it is not feasible.

Task-goal trajectory

In this sub-section I suggest that a relation sometimes holds between emergent goals and the goal(s) of a task and that the former can develop into the latter. I first attempt to clarify the relationship between task and goal.

With regard to school mathematics a task is something the students do, such as doing the questions in a section of a textbook. Chevallard (1999) views tasks as related to techniques[6] (used to solve tasks), technology (the discourse used to explain and justify techniques) and theory. Mathematics educators may judge a given task as 'good' or 'bad' but to the student it is something they do (because they are in a mathematics class). Why, their motive, and how students engage in a task may vary considerably, from wanting to complete the exercise before the end of the lesson to avoid homework, to having genuine interest in the mathematics in the task. Students with such extreme motives will have different goals and these will determine different actions. Out-of-school tasks are similarly things to do but in workplace settings (such as the director's company) it is expected that one does it quickly and correctly and this is an important and sometimes the only goal, though related goals such as 'finishing by 5.00' may co-exist. So tasks and goals may be extensionally identical or distinct.

The summary of the development of the groups' ideas, foci, goals and interpretations of the task in Appendix 2 suggests that, with each group, goals emerged which became the focus of subsequent work, i.e. emergent goals contributed to the transformation of the task (and when this happened the emergent goal was no longer an emergent goal but became the goal of the transformed task). I relate, below, the development of Group D with regard to these ideas but a similar narration is possible for the other groups.

In lesson 2 gradients were mentioned and the students returned to gradients in lesson 3 where they considered the equation of the diagonal line of a rectangle. This led them to consider how one can determine which side of a straight line (given by an equation) a specific point is on. This was an emergent goal but it became the goal of a new task which in their words was:

> we can find out if it's, which side of each line it's on and so you work out an equation and you fit the corners to the four equations and if it works for every single one of them then it's inside the shape.

There is a sense is which redefining tasks and setting new goals is unexceptional in student work such as this: problem solving, as meta-cognitive activity, can be characterized by an implementing-evaluating-refining cycle (Artzt and Armour-Thomas, 1992) and investigative school mathematics in England has been influenced by texts such as Mason et al. (1982) which encourage 'open' approaches. But work-based problems expect solutions that solve the given problem and, in my opinion and in the opinion of the company director, none of the student solutions solved the given problem; they transformed the problem/task into related but different tasks.

[6] The Group A student quote on graphs in section 5.2 "Cos that's normally what you do with co-ordinates" suggests that these students see school mathematics in a similar way to Chevallard.

EDUCATIONAL IMPLICATIONS

The upshot of the discussion above is that these students did not address the company director's task because their task-goal trajectories resulted in related but transformed tasks. This was only one school-based out-of-school project and care must be taken to not generalise from one case but student work in other activities in the wider project resulted, in my opinion, in similar task-goal trajectories. This raises the question as to whether out-of-school activities can be conducted in school as they would be in the work place. If the answer to this question is 'no', then does this mean there is no worth in doing such tasks in the classroom? I think not. The students clearly enjoyed the work[7] and most came up with reasonable solutions, in several cases exceptionally good school-based solutions. Further to this they felt they learnt many different things and most of them (19 out of 25) felt they learnt that mathematics can be used in out-of-school. Real tasks like this can also encourage 'rich equal exchange' (Sahlberg and Berry, 2003) in that all can contribute and the team outcome is likely to be much better than any one student could have done.

My final thoughts concern the role of adults/teachers in school work of this nature. Adults in these lessons, though they sometimes pointed out problems with students' ideas, were quite tolerant with regard to students' ideas. The adults let the students develop their task transformations. Perhaps adults/teachers in lessons of this kind should be less tolerant of students' ideas which are not seen as leading to practical solutions.

[7] 12 of the 25 students stated this in their comments on 'what have you learnt' even though they were not asked to comment on whether they enjoyed it.

ACKNOWLEDGEMENTS

This work was supported by the Economic and Social Research Council, Award Ref. No. RES-000-22-0739. I would like to thank Louise Sheryn, the students and the (sadly anonymous) teacher and company director for giving me the chance to examine a fascinating series of lessons.

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APPENDIX 1

The problem given by the company director. Extracts from a 22 minute presentation. '…' indicates missing text.

We have a business problem which is mathematically related and which we hope you will be able to come up with some good ideas about. ... We distribute frozen food on behalf of other people ... to supermarket depots all over the country ... we run 24 hours a day ...

A trailer tracking project ... to identify the positions of every one of our vehicles on the road ... The location of vehicles helps with our planning ... it reads positions constantly using GPS ... needs to record information about the time you arrived and departed from depots, how far we have travelled, the temperatures in the vehicle and other events as well ...

At this point the company director presents a real-time computer display on a large screen which shows a road map with a dot. The dot moves, indicating a new GPS reading.

So it is moving down the A1 [a major road], let's see where it goes now, ... right, it is here. One of the key things that this system has to be able to do is to identify when a vehicle is at a particular location, so the problem is to develop a method for the computer system to calculate from a GPS position when the vehicle is at a particular depot, and the example that I have picked out is our depot at ...

The company director brings the map below onto the large computer screen.

This is a map of the site and our land is highlighted. You see how irregular a shape it is. What we need to be able to do is to be able to identify for a particular latitude and longitude whether it is within that shape within the boundary of that shape ... we have tried various things ...we put a rectangular boundary around it and if you make it a rectangle ... but the problem is it is a very limited method of working it out, so we have a particular problem because we have the highway running very close by and if a vehicle gives us a point as it is passing by we don’t want a false reading, we don’t want it to say it has arrived in the depot when it hasn’t. ... Another possibility is a circle which is a slightly better solution, it is more complicated to work out when it is in a circle but it is not impossible but again you are limited, we might take in the highway with a circle. The best solution ... so far is to use a polygon which is perhaps the most sophisticated of the solutions, very flexible and will give us a reliable reading I hope, but the problem is that we are finding it very difficult to identify when a point is within a polygon.

APPENDIX 2

Group A initially attended to the polygon, then split the polygon into triangles and rectangles. Triangles were rejected because they were complicated. Rectangles were examined in lesson 2 and formed the basis for the remainder of their solution orientated work. Lessons 5 and 6 were largely devoted to preparing their presentation.

Group B's development was very similar to that of Group A. Considerable time was spent on "complicated" polygon. The focus then shifted to squares, which are easier, and then rectangles (lesson 2). From lesson 3 onwards they divided 'technical' and presentation preparation work between themselves.

Group C started with technical tags in lesson 1, rejected this and moved on to their algorithm, which formed the basis for all their subsequent work.

Group D had two ideas, a box by the entrance and enclosing rectangles. In lesson 2 the rectangles idea was pursued; gradients were mentioned. In lesson 3 consideration of the equation of the diagonals of rectangles led them to focus on which side of a line a given point was on, which determined the focus for subsequent work. Preparation for the presentation was in lesson 6.

Group E initially considered "something near the entrance" and moved on to consider squares and splitting the shape of the depot up in lesson 2.These two ideas led them to consider octagons which, together with preparing for the presentation from lesson 4, formed the basis for subsequent work.

Group F initially considered an electronic register. A consideration of 'four shapes' in lesson 2 led them to focus on circles which formed the basis for subsequent work. They resisted teacher incouragement to prepare for the presentation until lesson 5.

Group G had two initial geometric ideas but rejected these. They spent lessons 3 and 4 trying and rejecting ideas. The 'thin rectangles' idea forms as a fusion of two earlier ideas in lesson 4. Presentation considerations were minimal. Originality was cited as a factor in their foci.

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