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Investigating Fifth-Grade Students' Construction of Mathematical Knowledge through Classroom Discussion

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ABSTRACT

This study is descriptive-qualitative in nature, aims to examine, (1) How do the students construct the mathematical knowledge (i.e. statistics for elementary school) through classroom discussion?, (2) What role are the students and the teacher playing in the discussion?, (3) What impact do their contributions have in the construction of new mathematical knowledge? As subjects of this study are 21 fifth-grade students of SD Labschool Unesa. Data were collected by using observation sheets, and by videotaping the class lessons with three cameras. The results suggest that both the teacher and the students participated actively in small group discussion, and played the majority of roles offered in an effective discussion. At the end of the lesson, there was a whole class discussion that functioned as meaning negotiation – to facilitate the students to share solutions and strategies with the whole class, and promote student reflection on the different strategies. This suggests that even 'difficult materials' can be successfully constructed by fifth-grade students using classroom discussion. Therefore, it is suggested that classroom discussion can be chosen as one of learning methods in a mathematics classroom in which the teacher provides appropriate mathematics contents and uses productive strategies to facilitate the learning processes.

KEYWORDS
Classroom discussion, Mathematical knowledge, Statistics, Elementary school.

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Introduction

Nowadays, some mathematics teachers are still worried about content coverage, which can be distinguished by their efforts to tell the students everything they need to know. However, as a consequence of the implementation of curriculum 2013, which is better known as 'K-13' in Indonesia, there are teachers who perhaps are already convinced that students should possess

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creativity, critical thinking skills, communication and collaboration, which are called as the 'Four Cs' (NEA, 2016; As'ari, 2016). In an effort to focus on providing 4C skills, mathematics learning should elaborate aspects of critical thinking and problem solving, communication and collaboration, and creativity and innovation (Partnership for 21st Century Skills, 2011). The aspect of communication and collaboration is meant that students know how to articulate thoughts and ideas effectively, through oral, written, or nonverbal communication. Students should also be able to listen effectively to translate or decipher the meaning of knowledge, values, attitudes, and goals. They should also be able to communicate on a wide range in different groups and environments.

Parallel to this point of view, NCTM (2000) and Nathan and Knuth (2003) stated that learning mathematics is viewed as a social endeavor in which mathematics classroom functions as a community where thinking, talking, agreeing, and disagreeing are encouraged. The teacher provides students with powerful mathematics problems to solve together and students are expected to justify and explain their solutions. The primary goal is to extend one's own thinking as well as that of others (Hufferd-Ackles, et.al., 2004).

One of the learning methods which can promote those valuable skills is classroom discussion. However, classroom discussion is often implemented in a wrong way. It means that the teacher does not prepare the material to be discussed properly, or the teacher forms the small group randomly. It often happens that, if group members are not heterogeneous or students are not in their zone of proximal development, the class discussions will not work optimally.

The term of *Zone of Proximal Development* (henceforth, ZPD) refers to the most well-known and largely adopted theory of Vygotsky, who places more emphasis on the social context of learning (Mercer, 1995). Reviewing Vygotsky's (1978) theory, Williams & Burden (1997) refers to the two levels of development in ZPD: the *actual* development level and the *potential* development level. The *actual* development level refers to the individual's ability to perform certain activities independently of another individual. In the *potential* level the individual can perform the activities when help or support is given. The difference between the two levels is the ZPD which Vygotsky defines as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance of and in collaboration with more capable peers." Lev Vygotsky (1896–1934) stated that a child follows an adult's example and gradually develops the ability to do certain tasks without help. Vygotsky and some other educators believe that the role of education is to give children experiences that are within their zones of proximal development, thereby encouraging and advancing their individual learning.

Similarly, Lantolf (2000) synthesizing Vygotsky's ZPD theory puts forward that ZPD is "the difference between what a person can achieve when acting alone and what the same person can accomplish when acting with support from some artifacts." In fact, Vygotsky's work put emphasis on the importance of having students work with one another in addition to receiving instruction from adults.

The ZPD theory results in a shift in the mathematics teaching and learning. Teachers should implement a suitable method or strategy to shift the students' potential development level to their actual development level. In other words, how to help students move into and through their *zone of proximal development* is of particular importance. For this purpose, one of the learning methods which can be implemented in the teaching and learning process is Classroom Discussion.

This study is fundamentally rooted in the perspective that knowledge is essentially constructed by individuals rather than transmitted from one person to another. Constructivism is one of the 'approaches' or 'movements' in cognitive psychology which is concerned with the way in which humans think and learn. Constructivism emphasizes the ways in which individuals try to bring a sense of personal meaning to their worlds.

There are two types of constructivism, Individual and Social Constructivism. The former is based on the work of developmental psychologist Jean Piaget. Piaget's theory of cognitive development suggests that humans cannot be given information which they automatically understand and use. They must construct their own knowledge. They have to build their knowledge through experience (Kaplan, 2002). The latter, Social Constructivism is the theory developed by Lev Vygotsky (Kaplan, 2002). For social constructivists, education is associated with social transformation and value is placed on the sociocultural context indicating that constructivism reflects a theory of human development that puts the individual within a sociocultural context. Individuals develop through social interactions within which cultural meanings are shared by the group and eventually internalized by the individuals. Social constructivists claim that with the assistance from teachers, adults or more advanced peers, students can grasp concepts and ideas that they cannot understand on their own.

Constructivists view learning as the transformation of knowledge which requires students' active participation, largely in collaborative small groups (Nunan, 1999). Moreover, constructivists believe that the learning process is initiated by the learners themselves. Learners construct knowledge for themselves. Learners individually construct meaning as they learn. Nobody else can plant this knowledge into the learners; they are to do it themselves. Learners construct new knowledge based on the existing knowledge already possessed by them (Kaplan, 2002). Constructivists also claim that it is not possible to absorb new knowledge without having some previous knowledge to build on. It recognizes the construction of new understanding as a combination of prior learning and new information.

Another principle of constructivism concerns the role of interaction in the construction of knowledge. Students learn by interaction with others. Learning is claimed to be intimately associated with connections with other human beings – teachers, peers, family, as well as casual acquaintances. Related to this particular notion, Mercer (1995) suggests strongly that knowledge exists as a social entity and not just an individual possession.

A challenge faced by mathematics teachers is how to engage students in their mathematical content through rich discussion or discourse. In classrooms where there is high-quality mathematical discourse, teachers and students ask challenging and thought-provoking questions, and there is skillful facilitation of

meaningful discussions focused on the mathematics. The discussions emphasize reasoning, proof, evaluation, and justification. Students learn from one another and value the thinking of other students. The focus of the conversation is not simply the answer to the problem, but also the students' strategies, discoveries, conjectures, and reasoning.

The Curriculum 2013, which has been implemented in Indonesia since 2013, places a strong emphasis on meaningful process of learning. As a consequence, students are expected to construct viable arguments and express their opinion as well as critique the reasoning of others. In the mathematics classroom, meaningful discussions rely on purposeful instructional moves from the teacher, as well as a clear understanding of the lesson materials that are placed on students.

The use of small group work in mathematics classes as can be seen in classroom discussion has both academic and social benefits to students (Boaler, 2008; Cohen, 1994; Slavin, 1980; Yackel, Cobb, & Wood, 1991). Although there are many ways that group work can and has been used in mathematics classrooms, one very typical way that some teachers utilize this instructional practice is for students to work on an often rich and open-ended task in small groups, often for a substantial period of time within a single lesson.

Despite its promise, the use of group work poses particular instructional challenges. For example, (a) Students may not work cooperatively; (b) Academically heterogeneous groups may be difficult to manage; and (c) The teacher's role in monitoring and supervising group work may be confusing and uncomfortable. Furthermore, (d) The teacher must be mindful of the 'groupworthiness' of the task assigned (Cohen, 1994), (e) Create classroom norms to facilitate positive group functioning, and ensure that classroom management is kept under control while monitoring accountability and participation of all members of each group.

To manage the positive and negative features of small group work, in this study the researchers prepared a lesson plan and students' worksheet in which the teacher's activities as well as those of the students are clearly managed. It should be noted that the definition of group work in this study aligns with the term cooperative group work, in which individual students accountable for learning collectively without competition; the teacher creates roles for students, facilitates small group work, intervenes the group work if only needed, and leads the whole class discussion in the stage of meaning negotiation, as well as explicitly teaches students the social skills to work together productively (Cohen, 1994; Sharan & Sharan, 1992).

Ding, Li, Piccolo, and Kulm (2007) noted that interventions during group work can be at the individual level (e.g., teacher has a one-on-one conversation with one member of a group), the small group level (e.g., teacher has a conversation with one of the student groups in the class), and whole class level (e.g., halting group work to bring the whole class together); each of these types of interventions can help answer questions that many groups have, clarify the tasks, and reduce confusion to promote students' thinking.

Similarly, Dekker and Elshout-Mohr (2004) observed that students benefit when teachers' intervention during group work can focus on both the dynamics of group functioning (*process-help interventions*) and the mathematical content (*product-help interventions*). In accordance with these research results, we

created a lesson plan that was expected to help the teacher in helping students either as process-help interventions or product-help interventions.

In this study, we observed a mathematics lesson implementing the method of classroom discussion on the material of statistics for elementary school. The learning contents comprised of reading the data presented in a table or diagram form, defining median data with the students' own words; and determining the median of a group of data.

The aim of this study is to answer the following questions, (1) How do the students construct the mathematical knowledge (i.e. statistics for elementary school) through classroom discussion?, (2) What role are the students and the teacher playing in the discussion?, (3) What impact do their contributions have in the construction of new mathematical knowledge?

Method

This study is descriptive-qualitative in nature. As the participants of this study were 21 students of class 5B of SD Labschool Unesa, a private elementary school owned by "Dharma Wanita Persatuan Unesa." This school is located in the campus of Unesa, Ketintang, Surabaya. As the teacher of this study was Mrs. Mardiyati, one of senior teachers at Labschool Unesa.

The researchers prepared a lesson plan on the materials of statistics which consisted of reading the data presented in tabular form, writing the meaning of median data, and determining the median of a group of data. Besides, the researchers also provided student worksheets, and administered assessments of student learning.

The data collection method in this study involved video taping lessons with three cameras. One camera focused on the teacher, one camera focused on a group of students, while another camera focused on the whole classroom. This three-camera technique rendered this classroom data set an excellent resource for studying how was the teaching and learning process going on, and what were the teacher activities and the students activities in classroom discussion in order for the students to construct the learning materials. Besides, by affording visual and auditory access to both teacher and students concurrently, it became possible to conduct a detailed examination of how mathematical knowledge (i.e. statistical contents) was constructed by the students, as the product of the their cognitive processes.

To enhance the analysis and fully explore the progress of classroom discussion, the language used, as well as the mathematical contents, a complementary methods of video analysis was employed to provide a means of closely examining the mathematics lesson for both the non-verbal content (e.g. gestures, body positioning) and their mathematical artifacts.

In the process of classroom discussion, the observed classroom conversation was divided into teacher's talk and student's talk. Teacher's talk refers to all speech acts by the teacher that were addressed to individual student, to small group of students, or to the whole class. While students' talk refers to all student utterances directed to the teacher or to other students in a group, or to other students in the whole classroom.

Results and Discussion

In this study the researchers observed the teacher activities in implementing the prepared lesson plan to conduct mathematics instruction

using classroom discussion. She conducted learning processes that make classroom discussion more manageable by applying the teacher's role as suggested by Bruce (2007) and Chapin et.al, (2003). İt is based on reported research results that if students are left to their own devices, they will not necessarily engage in high-quality math-talk. Consequently, the classroom discussion should be well-managed by the teacher.

We have special reasons why in this study we use statistical materials for our research. First, the material of statistics is one of the mathematics contents in Curriculum 2013 to be taught in the even semester. Then, statistical knowledge is indispensable not only for individuals to learn the content, but it also has social impact. Therefore, the statistical content functions as either a scientific knowledge or as a practice of scientific production.

It can be seen in the curriculum 2013 that the contents of statistical materials are primarily collect, organize, read and interpret data or information. Thus, the purpose of learning statistics is giving opportunities to the students in constructing procedures to collect, organize, communicate data using tables, graphs and representations that are often seen in their daily lives, as well as to interpret statistical data using some statistical measures, such as mean, median and mode.

The following is an overview of the teaching and learning process at Class 5B, SD Labschool Unesa. The teacher began the lesson by conducting an introduction, i.e. motivating the students, stating the purpose of learning the materials, and relating the materials with the ones they have learned before (i.e. apperception). The teacher motivated the students by conveying the usefulness of statistics in daily life, i.e. by providing contextual problems associated with the presentation of data in the form of tables and diagrams. In addition to drawing on their knowledge of mathematical content, the teacher must also bring to classroom discussions an understanding of their students' prior knowledge and experiences. This is what we usually called as 'apperception.' In the second step of instruction, the teacher assigned tasks in the form of worksheet that required students to work together to do mathematical activities, and to develop joint solutions and problem-solving strategies.

As can be seen in Figure 1, the worksheet contains mathematical tasks that are appropriately challenging and enhance students' learning. The mathematical tasks investigate important mathematical ideas and relevance for students, as can be seen in the various informal definitions of median data which were written by the students during the learning process. In addition, the mathematical activities and the problems posed encourage investigation, promote reasoning, and require students to provide justifications for their thinking. Ultimately, the mathematical tasks in the worksheet are worthy of student discussion and emphasize important mathematical concepts.

The student worksheet also consists a multilevel task, and was created by considering that scales of group work learning tasks must be set at an appropriate level to encourage group working. The tasks were not set at too low or too high of students' cognitive level, because it would discourage student participation.

In the third step, the teacher divided the students into five small groups whose members were heterogeneous. This is parallel to the findings of studies by

Galton and his colleagues which suggest that students are more likely to succeed in undertaking cognitive tasks when they work in pairs or small groups.

Table 1. Area of classroom and the Number of fifth-grade students

Name of classroom	Area (m^2)	Number of students
Class 5 A	43.3	10
Class 5 B	70.0	25
Class 5 C	80.1	25
Class 5 D	64.2	23
Class 5 E	61.1	24
Class 5 F	61.8	24
Class 5 G	62.4	22

Data in Table 1 can be split into two kinds, i.e., area of classroom, and the number of student. Sort the data from the smallest to the largest.

Table 1.1 Area of Grade 5 Classroom

Number	Name of classroom	Area (m^2)
1		
2		
3		
4		
5		
6		
7		

Note the sequence of the data in Table 1.1. In this table there are 7 data, and the 4th order data is in the middle position.

4th Data = ...

In Table 1.1, the number of data is 7 (odd), so it can be concluded that the middle data lies in the 4th data. What if the number of data is even?

Continue your activities by looking at Table 1.2.

Table 1.2 Area of Grade 6 Classroom

Number	Name of classroom	Area (m^2)
1		
2		
3		
4		
5		
6		
7		

Which data is the middle data? Give your reason._____

$$\frac{\_\_\_\_th\_Data + \_\_\_\_th\ Data\_\_}{2} = \frac{+}{2} = \frac{\_\_\_\_}{2} = \ldots\ldots..$$

The value obtained from the above calculation is **Median** data.
Express in your own words what is called **Median** data.
If the number of data is odd, median data can be defined as

_____

Conversely, if the number of data is even, median data can be defined as

_____

Figure 1. An example of mathematical activities on the material of describing the meaning of median data.

The small groups consisted of 4 (four) or 5 (five) students which was in line with the recommended size for the pursuit of cooperative and collaborative tasks, with the tasks involving enrichment and incremental learning. Then, after small-group work, the teacher also implemented whole class discussion that was expected to provide a context for the wider transmission of knowledge.

However, to gain effectiveness of group working, the students were encouraged to establish positive relationships between group members that allow for sensitivity to others, trust of others and effective communication. It is because learning processes related to cognitive development (either new knowledge or application of knowledge) will be enhanced by effective social communication and support. The figures below illustrate the situation of mathematics learning at Mrs. Mar's classroom.

Figure 2. As a motivation for the students, Mrs. Mar chose two students to solve an introductory problem related to data presentation in the form of tables and diagrams.

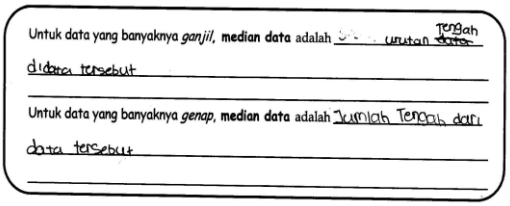
Figure 3. The body position of these students implied that they actively involved in the discussion.

The most important thing a teacher should do to ensure the success of discussions is to ask meaningful questions and facilitate the dialogue among students. Mrs. Mar practiced this recommendation since the beginning of the instruction. She asked meaningful questions not only to students who seemed actively involved in the discussion, but also to student who was not active as can be seen in Fig.2.

Figure 4. The teacher should build in opportunities for independent work and partner or small group work.

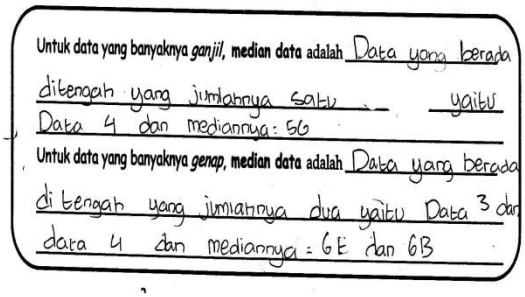
Figure 5. In classroom discussion, a student still has to construct the new knowledge individually as the product of his/her cognitive processes.

In Fig.2, we can see that the body position of the students implied that they involved in a fruitful discussion. However, the results of small group work for this question are not always satisfying. This is because the mathematics content to be learned is considered as 'difficult material', according to Mrs. Mar. The following are two examples of students' works to prove Mrs. Mar's opinion.



In English:
(If the number of data is odd, median data is *'middle position of the data'*).
Conversely, if the number of data is even, median data is *'the sum of the middle data'*)

Figure 6. Example of student' work (a)



In English:
If the number of data is odd, median data is datum in the middle which is only one, that is the 4th data, so the median is 56.
Conversely, if the number of data is even, median data is 'the two data in the middle, i.e. 3rd and 4th data, so the median is 6E and 6B.

Figure 7. Example of student' work (b)

In Fig. 6 and Fig. 7, we see that the students do not fully grasp the notion of median data, if the number of data is even. The only notion they remember is 'the sum of two data in the middle.' They do not realize that the sum of two data in the middle must be divided by two. Even though the students have mentioned 'the sum of two data in the middle,' they are still confused in differentiating the order of data' (i.e. as ordinal numbers), and the value of data (i.e. as cardinal numbers). In addition, one important thing that the students forgot is that before determining the median, the data must be sorted from the smallest to the largest or reversely.

Figure 8. After finishing the small-group discussion, the students were asked to publicly present their solutions in front of the class.

In this step, the teacher facilitated the sharing of solutions and strategies with the whole class, and promoted student reflection on the different strategies. The teacher urged students to explain and compare their solutions and solution strategies with peers. In addition, the students were encouraged to be both supportive and challenging with peers.

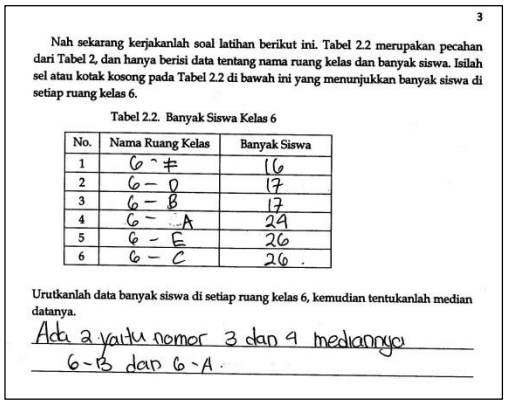
Once the task had been conducted, Mrs. Mar was ready to handle the different strategies that the students propose. As Anderson (2003) stated that talking and thinking together can help all students understand mathematics better. While waiting for the representative of each group wrote their solution and strategies on the white board, Mrs. Mar asked the remaining students to compare their solutions with those of other groups written on the whiteboard. She asked the students several times by using the following expressions: "_____*'s strategy was similar to mine because…",* or *"*_____*'s strategy was different than mine because…"*

In this phase of negotiation of meaning, Mrs. Mar facilitate the students to re-think about their solutions by comparing with the different solutions of other students on the whiteboard. We observed a fruitful discussion in the whole class setting on this phase.

In order for students to openly share their thinking and risk-making mistakes in front of the other students, it is imperative that there is a supportive classroom environment. Everyone should understand their role in the classroom through the development of classroom norms. It should be noted that Mrs. Mar asked their students to raise their hands up before asking question or expressing opinion, or speaking up in the classroom. These classroom norms were proven to be very useful in keeping the classroom condition.

We also observed that Mrs. Mar posed thought-provoking questions, support students' conversations, listen carefully to monitor students' understanding and misconceptions, encourage student participation in discussions, and promote student reflection about the learning experience.

The teacher also asked the students who were listening to the explanation of their peer in front of the class to be attentive to the thinking of others, and reflect on the ideas they have heard to evaluate their efficiency. Besides, the students need to determine whether they agree or disagree, whether they understand the thinking of their peers, and what similarities and differences they see between their own thinking and the thinking of others.



3

Nah sekarang kerjakanlah soal latihan berikut ini. Tabel 2.2 merupakan pecahan dari Tabel 2, dan hanya berisi data tentang nama ruang kelas dan banyak siswa. Isilah sel atau kotak kosong pada Tabel 2.2 di bawah ini yang menunjukkan banyak siswa di setiap ruang kelas 6.

Tabel 2.2. Banyak Siswa Kelas 6

No.	Nama Ruang Kelas	Banyak Siswa
1	6 ~ #	16
2	6 – D	17
3	6 – B	17
4	6 – A	29
5	6 – E	26
6	6 – C	26

Urutkanlah data banyak siswa di setiap ruang kelas 6, kemudian tentukanlah median datanya.

Ada 2 yaitu nomor 3 dan 4 mediannya 6-B dan 6-A.

Figure 9. This student's work tells us that he/she cannot differentiate between the order of data, and the value of data.

As chosen students defended their solutions and shared arguments for their strategies, the teacher ensured active listening and reflection through the use of guiding questions. For instance, Mrs. Mar asked the student: Why did you answer 3 and 4 (see Fig. 9)? Where did 6-B and 6-A come from? How should we determine the median if the number of data is even? Why must the sum of the two data in the middle be divided by two?

At the end of this phase of negotiation of meaning, it seemed to us that the students understood the meaning of median whether the number of data is odd or even. We believed it fully after we got the result of quiz that was very good, the arithmetic mean of students' scores was 86. As the teacher of Class 5-B, Mrs. Mar was very glad because of it.

Conclusions

In the new curriculum (K-13), learning mathematics is viewed as a social endeavor in which mathematics classroom functions as a community where thinking, talking, agreeing, and disagreeing are encouraged. It is parallel to the recommendation that encourages educators to prepared the students with the skills needed to live in the 21st century which are called as 'Four Cs' (creativity, critical thinking skills, communication and collaboration).

In mathematics classroom, teacher provides students with powerful mathematics problems to solve together and students are expected to justify and explain their solutions. Classroom discussion is a learning method which can promote those valuable skills. In this study, statistical contents were proven to be appropriate to learn using classroom discussion. However, before the discussion, the teacher should prepare a lesson plan and worksheet in such a way that they will be suitable to promote students' active participation in the discussion as well as to construct new mathematical knowledge.

In implementing discussion on a mathematics classroom, a teacher should choose the appropriate materials, because there is no learning method that is suitable for all materials of mathematics. İn doing so, the teacher should understand the specific characteristics of mathematics contents as well as the characteristics of classroom discussion method. One important thing that should be considered is that the mathematical contents must be within the students' *Zone of Proximal Development,* that is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance of and in collaboration with more capable peers. It often

happened that, if group members are not heterogeneous or students are not in their zone of proximal development, the class discussions will not work optimally.

In terms of the construction of statistical materials in this study, we noted that until the middle of the discussion, the students encountered difficulties in describing the notion of median, or to determine the median of data if the number of data is even. They did not realize that the sum of the two data in the middle must be divided by two. It is might be caused by the usual approach used in learning mathematics, i.e. 'mechanistic' approach. The students were able to do the calculations without knowing the reason or the justification behind their conduct. The followings are some findings in terms of mathematics content in the observed lesson: (1) There is no students who mentioned the need for data to be sorted first before determining the median of the data; (2) Students do not distinguish the order of data and the value of data; (3) Students know that the median data is the middle data, but less aware of one datum in the middle or two data in the middle (whether the number of data is odd or even); (4) There is no students who mentions that if the number of data is even, so the sum of the two data in the middle must be divided by two. It seems to us that students do it mechanically, not with a deep understanding.

In order to help students summarize and understand their thinking as well as the thinking of others, the teacher provide opportunities for students to "turn and talk" about their solutions. Teacher should also facilitate the sharing of strategies with the whole class, promote student reflection on the different strategies. This gives students practice constructing arguments, providing justifications, and critiquing the thinking of others. By implementing these useful strategies in leading the whole class discussion at the end of the lesson, the teacher was successfully help students in meaning negotiation. The students revised their work that can be seen by the much use of 'stypo' in their worksheet.

In summary, how successfully a teacher facilitates a discussion drives how mathematically rigorous the work is for students. Equally important is that students know how to listen to the thinking of others, and pose questions, as a way of deepening their mathematical understanding. The success of these small group and whole class discussions rests on the ability of the teacher to facilitate purposefully.

Disclosure statement

No potential conflict of interest was reported by the authors.

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