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# COMPUTER GRAPHICS AS AN INSTRUCTIONAL AID IN AN INTRODUCTORY DIFFERENTIAL CALCULUS COURSE

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**ABSTRACT.** Mathematicians in general claim that the Computer Algebra Systems (CAS) provide an excellent tool for illustrating calculus concepts. They caution, however, against heavy dependency on the CAS for all computational purposes without the mastery of the procedures involved. This study examined the effect of using the graphical and numerical capabilities of Mathematica as a supplemental instructional tool in enhancing the conceptual knowledge and problem solving abilities of students in a differential calculus course. Topics of differential calculus were introduced by the traditional lecture method to both the control and experimental groups comprised of students enrolled in two sections of the Business and Life Sciences I course. Mathematica was used only by the students of the experimental group to reinforce and illustrate the concepts developed by the traditional method. A content analysis was conducted using the qualitative data obtained from students' explanations of the derivative of a function. The quantitative data, the students' test scores, were analyzed using ANCOVA. The results showed that students in the experimental group scored higher than students in the control group on both the conceptual and the computational parts of the examination. The qualitative analysis results revealed that, compared to the control group, a higher percentage of students in the experimental group had a better understanding of the derivative.

**KEYWORDS.** Computer Algebra Systems, Computer Graphics, Numerical Computation, Supplemental Instructional Tool, Black-box Syndrome.

## INTRODUCTION

### Background

The need for a reform in content and objectives of the introductory calculus course in view of the available technology, such as the Computer Algebra Systems (CAS), has been advocated by researchers. Issues relating to the integration of technology and calculus instruction have drawn enormous attention from mathematics educators for the last decade (Armstrong, Garner, & Wynn, 1994; Boyce & Ecker, 1995; Cipra, 1988b; Dubinsky, 1992; Gordon, 1993; Heid, 1988; Hundhausen, 1992; Judson, 1992; Kolata, 1988; Kowalczyk & Hausknecht, 1994; Lefton & Steinbart, 1995; Leinbach, 1992; Nowakowski, 1992; Porzio, 1995;


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Ralston, 1991; Schoenfeld, 1992; Smith, 1992; Solow, 1991; Steen, 1988; White, 1990; Zorn, 1992). According to Kolata (1988) and Hundhausen (1992), calculus courses must renew emphasis on conceptual understanding rather than rote learning and manipulation of formulas. The value of skill-based calculus courses has been questioned since computers and calculators can perform most (if not all) of the manipulative procedures taught in such courses (Gordon, 1993; Steen, 1988; Tall, 1987). In view of the large number of students taking calculus, much concern has been expressed over the rote, manipulative learning that takes place in the course (Cipra, 1988a; Douglas, 1986; Steen, 1988; White, 1990). In order to make the content more applicable and concept-oriented, calculus courses are now being designed to include a technological component, such as the Computer Algebra Systems (CAS) or programming, with the traditional lecture approach of teaching.

In its 1989 report, the National Research Council advocates that technology should be used not because it is seductive, but because it can enhance mathematical learning by extending each student's mathematical power. Calculators and computers are challenging tools that should not be used just as a substitute for hard work. Schoenfeld (1992) and Bennet and Whittington (1986) advocated in favor of using technology to prompt students for detailed investigations of mathematical concepts and to teach through experimentation. The symbolic, computational, and graphical capabilities of the CAS could provide opportunities for exploration of mathematical concepts (Hundhausen, 1992), discovery learning (Hundhausen, 1992; Solow, 1991), and treatment of real world problems in ways that had not been feasible without computers (Boyce & Ecker, 1995; Hundhausen, 1992). Kowalczyk and Hausknecht (1994) described two effective applications of the CAS in calculus instruction: CAS as a demonstration tool in classroom and CAS as a laboratory tool for students to experiment on their own. With symbolic, graphic, and numeric computing, Zorn (1992) claimed that mathematical ideas can be presented from different perspectives and more directly than when taught the traditional way. Calculus educators have found the computer and calculator-enhanced learning environments to be relevant, meaningful, and intuitive for each individual student for specific topics of calculus (Rochowicz, 1996).

A wide range of calculus concepts that cause problems for students have been identified by researchers. In particular, students' difficulties with the abstract concepts of rate of change (Orton, 1984), limit (Francis, 1993; Tall & Vinner, 1981), and function (Dreyfus & Eisenberg, 1982; Even, 1993) have been mentioned. Researchers have recognized the great potential of the CAS in enriching, enlightening, and expanding students' learning. The possibility of adding graphical and numerical viewpoints to the traditional symbolic ones has been seen as the most important advantage of using technologies. A little mechanical help, used judiciously, could reveal clearly the insights behind the mathematical ideas being addressed (Gordon, 1993; Zorn, 1992). Computer graphics have made possible the visual presentation of many of the dynamic phenomena studied in calculus. This capability has suggested many pedagogical possibilities (Kowalczyk & Hausknecht, 1994; Steen, 1988). Hundhausen (1992) showed how the CAS could be effectively used to illustrate Euler's method to calculate antiderivatives and to compute the Riemann sum. According to Zorn (1992), symbolic computing could provide a concrete approach to infinite series. As compared to traditional version of courses, students' disposition towards mathematics and the use of the computer has been found more positive in a computer laboratory calculus course (Fredenberg, 1993; Lefton & Steinbart, 1995; Park, 1993; Park & Travers, 1998).

## Black-box Syndrome

Mathematics educators, in general, have recognized that the numerical and graphical capabilities of the CAS provide an excellent tool for illustrating calculus concepts. However, heavy dependency on the CAS without the mastery of the procedures of calculus could promote its use as a black- box and generate another type of rote learning (Hundhausen, 1992). Dependence on the CAS, according to Hundhausen (1992), might hinder development of abilities to do theoretical work and analyze problem results.

This study, adopting the principles of Hundhausen (1992) and Zorn (1992), examined the effect of computer graphics and numerical computation used as a supplemental instructional tool in enhancing the conceptual knowledge of the limit, derivative, and its applications in certain types of optimization problems. It also focused on whether computer graphics would enhance students' problem solving abilities, particularly in finding critical numbers, inflection points, and relative extrema. Both Hundhausen (1992) and Zorn (1992) discourage students' heavy dependency on the CAS and recommend its use in a judicious and balanced form.

The research questions of interest were:

1. Do the computer graphics and numerical computation used as a supplemental instructional tool enhance students' conceptual understanding of differential calculus?

2. Is there a difference in students' abilities to find critical numbers, inflection points, and relative extrema if the computer graphics is used as a supplemental instructional tool?

## Limitations of the Study

The subjects were not randomly selected because two intact classes were used in the study. By using intact classes as sample sets, disruption of the subjects' schedule was avoided. This also facilitated in obtaining the departmental permission to conduct the study. A second limitation was the small sample size which was comprised of only two groups of students enrolled in an introductory calculus course designed for Business and Life Science majors. Therefore, ability to generalize the results of the study, as well as the internal validity of the experiment, was limited.

# METHOD

## Subjects

This study was quasi-experimental with a nonequivalent control group design. Eighty-eight students, enrolled in two sections of an introductory calculus course MA 1613 "Calculus for Business and Life Sciences I" at Mississippi State University were the subjects of this study. The control group and the experimental group consisted of 42 and 46 students respectively. None of the students were mathematics majors. Initially, there were 17 males and 25 females in the control group, and 32 males and 14 females in the experimental group. By the end of the semesters, students withdrew from both the control and experimental groups. Of the 42 initially enrolled in the control group, 9 (21%) males and 18 (43%) females completed the study. Of the 46 initially enrolled in the experimental group, 22 (48%) males and 9 (19%) females completed the study. To determine the group equivalency prior to this study, the control and experimental group students' ACT (American College Test) math subscores were compared. Since the ACT subscores for two experimental group students (one male and one female) were not available, these two students were also excluded from this study. Therefore, the sample sizes for the control and the experimental groups reduced to 27 and 29, respectively. Although gender was not a factor investigated in this study, the effect of gender on students' performance on a calculus achievement test had been found to be statistically non-significant in a study conducted by Fredenberg (1993).

The subjects had no prior knowledge about this study. Both the control and experimental group classes met for three lecture hours per week and were taught by this author. To determine the consistency in how the course contents were being taught, two professors of the Departments of Mathematics and Statistics visited the control and experimental group classes twice during the semester. These visits were unannounced and were made once around the mid-term examination and then at the end of the semester before the final examination. Both professors expressed satisfaction over the instructional approach and the students' active participation in the process. The contents of the course included limits, continuity, derivatives, and their applications. The only prerequisite for the course was (MA 1313) College Algebra or (MA 1303) Quantitative Reasoning.

## Course Structure

As stated by Zorn (1992), calculus is algebra-intensive. Most of the main objects of calculus (such as derivatives, series, and integral) are defined via algebraic combinations of functional expressions. Therefore, the first two to three class periods were used to review the fundamentals of algebra in both the control and experimental sections. Emphasis was placed particularly on defining the domain and range of some frequently used functions in the text book, Brief Calculus with Applications (Larson,Hosteler, & Edwards), such as linear, quadratic, cubic, rational, absolute value, and square root functions and their graphs. In addition, a small amount of time was spent in the experimental group to introduce the syntax and commands of the CAS Mathematica for defining functions, plotting the graph of one or more functions on the same set of axes. The students were introduced to some Mathematica functions, including the table function that generates a table of functional values. Students were also provided with necessary tutorial worksheets. With the notebook feature of the software, it was easy to execute a Mathematica statement, edit the statement, and then re-execute it. Although no computer terminal was available in the lecture hall, students had access to the 2.2.3 version of Mathematica in a well-equipped computer lab. Every section of the calculus text book included some technology-dedicated problems marked as *technology* suggesting that a computer algebra systems or a graphic calculator might be helpful in solving the problems.

Development of calculus concepts and applications were the main objectives of the teaching method. Mathematica was used only by the experimental group students to enhance problem solving abilities and to reinforce and deepen the concepts developed by the traditional method of teaching. The graphical and numeric capabilities of Mathematica allowed the students to explore calculus concepts that were not easily accomplished without the computer. However, students did not use the computer to evaluate limits and find derivatives symbolically. Their use of Mathematica was limited to generating graphs and tables of function-values. The topics of differential calculus, including limit, derivative, monotonic functions, concavity and inflection points, were introduced in a traditional lecture method in both the control and experimental groups. With simple examples involving functions that were easy to graph by hand, a graphical interpretation of the mathematical phenomena was given whenever possible. The same problems from the textbook were used to introduce a topic and the same problems were assigned for homework in both the control and experimental groups. Because of their access to Mathematica, students in the experimental group were required to do most of the homework problems analytically, graphically, and also numerically if it helped them understand the concepts. The students in the control group, on the other hand, had to rely mostly on the analytical method to solve those problems. Their graphing skills, until the major part of the course sequences were completed, were very limited. For example, the notion of limit was introduced analytically, graphically, and numerically in both the sections. Students in the experimental group were assigned homework problems requiring them to approximate the limit by all the three methods; analytically, numerically, and graphically. However, most of the limit problems were done by the control group students using only analytical methods for the reasons discussed earlier.

A major difference between the control version and the experimental version of the calculus course centered on visualization of concepts through graphs. Graphs played a vital role in reinforcing concepts in the experimental group. Although students in the experimental and control groups did not construct graphs by hand until limit, derivative, and their applications in curve sketching were introduced, the experimental group students analyzed a large variety of

Mathematica generated graphs. The experimental group students were able to analyze how some important properties of a function's graph were reflected in the graphs of its derivative and the second derivative functions by graphing the three functions on the same set of axes. In optimization problems, particularly those related to Business and Economics, graphs played a key role in development of concept in the experimental group. The experimental group students interpreted the computer generated graphs of the revenue and cost functions on the same axes in an applied sense. They also used the graphs of the revenue and cost functions to locate sales levels for a given profit value. They used computer generated graphs to compare optimal sales levels for corresponding revenue and profit functions. They graphed the marginal revenue and marginal cost functions on the same set of axes to note that the maximum profit occurs at the intersection of the two curves. By graphing the revenue function $R(x)$ and its second derivative function $R''(x)$, the experimental group students located the point of diminishing returns (the point at which the concavity of the graph of $R(x)$ changed from upward to downward). On the other hand, since most of the application problems in Business and Economics were modeled through symbol manipulations using polynomial functions with fractional coefficients, these functions were difficult to graph by hand. Hence, students in the control group had to depend on the analytical method of solving those problems. One of the fundamental topics covered in calculus was to find the slope of the tangent line. Students used the derivative $f'(x)$ to find the slope of the line tangent to the graph of *f* at the point $(x, f(x))$. The experimental group students found the slopes of the tangent lines of a variety of functions at the indicated points analytically, wrote the equations of the tangent lines, and then graphed the lines and the corresponding functions on the same axes to convince themselves that the lines were actually tangent to the graphs of the corresponding functions at the required points. For example, they graphed the function $f(x) = \frac{x^4}{x^3 + 1}$ and the line on the same axes to note that the line actually appeared to be tangent at the graph of *f* at the point $\left(1, \frac{1}{2}\right)$. Also, by drawing several secant lines through a given point on the graph of *f*, they approximated the slope of the tangent line. However, the visual representation of these mathematical phenomena was limited for the control group students who found the slope of functions analytically. The experimental group students, because of their access to Mathematica, did most of the homework problems analytically and graphically and some of the limit problems numerically also, whereas the control group students relied mainly on the analytical approach to solve those problems. Table 1.1 briefly lists the major differences in approaches to solving homework problems assigned from various topics between the experimental and control group students.

**Table 1.** Major difference in approaches between groups to solving homework problems

| Topics | Experimental group | Control group |
|---|---|---|
| Limit | Most of the limit problems were done analytically and graphically. A few of them were also done numerically. | Most of the limit problems were solved analytically. |
| Tangent line | An equation of the tangent line to the graph of $f$ at the indicated point was found analytically. The result was then verified by sketching the graph of $f$ and the tangent line. Also, by drawing a sequence of secant lines through a given point on the graph of $f$, the slope of the tangent line was estimated. | An equation of the tangent line at a given point was found analytically. |
| Application problems | With computer generated graphs of $f, f', f''$ on the same set of axes, the critical numbers of $f$, and the inflection points, if any, were found. The behavior of $f$ relative to the signs of $f'$ and $f''$ was observed. Then the relative extrema of $f'$ if any, were found. The results were also verified by the analytical method.<br>Analytical solutions to optimization problems, particularly relating to Business and Economics, were also graphically explained and verified. | Critical numbers, inflection points, and relative extrema were found analytically.<br>Optimization problems relating to Business and Economics were done analytically. |

## Instrumentation

The instrument used to collect the data contained six concept-oriented questions (see Appendix A), designed to test the conceptual understanding of limit, derivative and their applications, and three computational questions (see Appendix B), designed to test the computational skills of students. The concept-oriented questions were either taken from the textbook or developed by the author and required little or no computation to answer them. For example, answers to question numbers 1, 4, and 6 followed directly from definitions and theorems. These concept-oriented questions were included in the four class tests, including the final examination, conducted during the semester in both the groups. The computational questions were taken from the text book and were given as a take-home assignment. The experimental group students were allowed to use computers to do the take-home assignment. For the qualitative analysis purpose, a question asking students to explain what they understood about the derivative of a function was included. All of the class tests and the take-home assignments that were to be graded for the control and experimental groups were approved by the same professors of the Mathematics Department who had visited the experimental and control group classes twice during the semester. The test and assignment questions for both the experimental and control groups were either the same or similar in nature.

**Data Analysis**

Statistical analysis of the data was conducted using SPSS. An independent sample t-test analysis was used to determine if there was statistically significant difference between the students of the experimental and control groups in their prior mathematical knowledge as measured by the mathematics ACT subscores . Analysis of covariance (ANCOVA) was conducted on the mean scores for the conceptual as well as the computational parts of the examinations respectively to analyze the difference in scores between the students of the two groups. The ACT math subscores were used as the covariate. A content analysis was conducted using the qualitative data obtained from the various answers given by the students of the two groups explaining the derivative of a function.

## RESULTS

With an  of .05, the results of an independent sample t-test conducted on the ACT math sub-scores indicated no significant difference in mathematical knowledge prior to this study between the students of the experimental group (M =21.7586, SD = 3.988) and the control group (*M* = 21.9630, *SD* = 5.741), t (54) = .16, p = .877.

The test questions (see Appendix A) designed to test the conceptual understanding of limit, derivative, and their applications required little or no computation to answer them. The experimental group students (M = 7.65, SD =1.47) scored significantly higher than their control group counterparts (*M* = 4.88, SD = 1.70), F(1,53) = 48.40, p < .01.

On the computational part of examination (see Appendix B), the students of the experimental group (M = 17.06, SD = 3.15) also scored significantly higher than the control group students (*M* =10.60, SD = 6.70), F(1, 53) = 21.31, *p* < .01.

The passing rates, based on the course final grades which were computed on a 4-point scale (Table 1.2), were 90% and 85%, respectively for the experimental and the control groups. However, for the computation of the final grades, students' scores on the evaluation of objectives such as exponential and logarithmic functions, velocity and acceleration, had also been taken into account. Questions on these objectives were not included on both the conceptual and the computational parts of the examination since the experimental group did not use computer to do homework problems relating to these objectives.

**Table 2.** Final grades of students of both the groups

| Groups | N | A | B | C | D | F |
|---|---|---|---|---|---|---|
| Experimental | 29 | 5 | 11 | 5 | 5 | 3 |
| Control | 27 | 1 | 7 | 6 | 9 | 4 |

**Content Analysis**

The qualitative data, obtained from the explanations of the derivative of a function given by the students of the two groups, was analyzed using a content analysis. The derivative in this course was seen as the slope of a tangent, as a rate of change, and as a function. Student responses were classified in one of five categories:

1. as the slope of a tangent and as a rate of change,
2. as the slope of a tangent,
3. as a rate of change,
4. required to find certain useful information (i.e. critical numbers, inflection points, extrema), and
5. no answer or an unsatisfactory answer.

A summary of the results are given in the Table 3.

**Table 3.** Number and percentage of students' answers explaining derivative

| Responses | Experimental | Control |
|---|---|---|
| 1. As the slope of a tangent and as a rate of change | 14 (48 %) | 3 (11 %) |
| 2. As the slope of a tangent | 6 (21 %) | 4 (15 %) |
| 3. As a rate of change | 4 (14 %) | 8 (30 %) |
| 4. Required to find certain useful information such as critical numbers, inflection points, and extrema. | 4 (14 %) | 3 (11 %) |
| 5. No answer or unsatisfactory answer | 1 (3 %) | 9 (33 %) |
| Total Group Responses | 29 (100 %) | 27 (100 %) |

Given below are some typical quotes from the explanations of the derivative of a function given by the students of two groups:

A student in the experimental group described derivative both as the slope of a tangent line and a rate of change.

*The derivative of a function is the rate of change of the function. It can be used to find velocities, critical points, marginal profit and a list of other real world problems. Also derivative can be used to find the slope of a tangent line which is the best approximation of the slope of a graph at a given point.*

For a control group student, derivative was just the slope of a tangent line.

*The derivative is a function that gives you a formula for finding the slope of the tangent line at any point on the functions graph.*

Another student in the control group found derivative useful for finding critical numbers, inflection points, and the relative extrema.

*The derivative of a function is used for many different mathematical operations. The first derivative is used to show relative extrema, critical points and open intervals for which f(x) is increasing or decreasing is also found here. The second derivative is used to show concavity and inflection points.*

Another student in the experimental group described derivative as a rate of change.

*Derivative seem to be the basis of almost all calculus. Through the use of orders of derivatives, one can tell anything about the graph of a function, making it extremely easy to find one graph of values from another. For instance, the derivative of a position function is velocity, and the derivative of a velocity is acceleration. Derivatives can also be applied to the business world. you can find max. profit from the first derivative and point of diminishing return from the second derivative. Derivative is a rate of change.*

A student in the control group gave this unsatisfactory explanation of derivative:

*Derivative is an easier way to analyze a function.* $f(x) = x^2 + 4$ *then* $f'(x) = 2x$

A higher percentage of students in the experimental group showed more evidence of understanding the notion of derivative than the students in the control group (Table 3). The answers given by most of the students in the experimental group were more explanatory than those in the control group. For example, one of the six students in the experimental group who described derivative as the slope of a tangent line also added that the slope of the line tangent to the graph of a function at a given point is the slope of the graph of the function at that point. Two of the four students in the experimental group who saw the first and the second derivatives as functions useful to find the critical numbers and inflection points that are useful for determining the concavity of the graph and relative extrema, also explained how the notion of concavity was related to the concept of diminishing returns in economic problems. They indicated that the point of diminishing returns, where the concavity of the graph of the (advertising) cost function changes from upwards to downwards, could be found by using the inflection points. Nine (33 %) of the control group as compared to one (3 %) of the experimental group either did not answer the question or gave an unsatisfactory answer. Among those nine, three were unable to answer the question. The one in the experimental group gave an unsatisfactory answer. As far as writing abilities were concerned, students in both the groups had problems expressing themselves in appropriate mathematical terms.

## DISCUSSION AND CONCLUSION

The statistically non-significant difference between the experimental and the control groups as measured by the ACT math subscores revealed that both the groups appeared to be equivalent in their mathematical knowledge prior to this study. However, in a quasi experimental design, the possibilities of pre-existing differences in mathematical abilities between the groups can not be completely ruled out. The statistical procedure of the analysis of covariance takes into account any such pre-existing differences between the groups while comparing their means. So for further analysis of the data, the analysis of covariance was used with the ACT math subscores used as the covariate.

The experimental group scored higher than the control group on both the conceptual and the computational parts of the examination. The differences in the mean scores of the two groups on both parts of the examination were statistically significant.

Questions 1, 3, and 6 of the conceptual part of the examination were derivative related questions posed through graphs. The students' answers required no computation. The application problem, Question 5 asked for the point of diminishing returns. The repeated visualization of these mathematical phenomena through graphs might be a major contributing factor to a clear understanding of the mathematical process involved in the solution of these problems, thus resulting in a better performance by the experimental group on the conceptual part of the examination.

On the computational part of the examination which was assigned to both the groups as a take - home exam, students were asked to determine the critical numbers, the open intervals where the graph of a given function *f*, is increasing or decreasing, is concave upward or concave downward, and the relative extrema. The students of the experimental group were allowed to use computers to solve these problems. Once the derivative and the second derivative functions of a given function are graphed, solutions to these questions almost become obvious. Hence, answering these questions with the help of computers turned out to be an easy routine for most of the students in the experimental group. But for many students in the control group, finding the roots of some of the equations arising from these problems (in the process of finding the critical numbers or the inflection points) appeared to be the greatest hurdle standing in the way of solving these questions. The higher course passing rate for the students in the experimental group (90%) as compared to those in the control group (85%) can not be solely attributed to the use of Mathematica as an instructional tool. As stated earlier, students in the experimental group did not use computers to solve the homework problems relating to exponential and logarithmic functions, velocity, and acceleration. For the computation of the final grades, students' scores on the evaluation of these objectives were also taken into account.

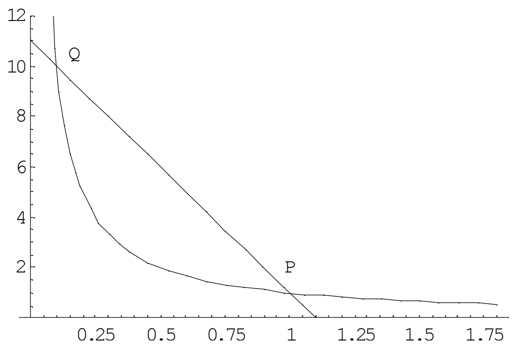
The findings of this study support the previous research that adding graphical and numerical viewpoints to the traditional symbolic ones gives a clear insight behind the mathematical ideas being addressed. Computer graphics appeared to be very effective in visualizing functions or relations between functions and their derivative functions, and variables. As a result, the abstract analytical solutions of many of the application problems become meaningful to students.

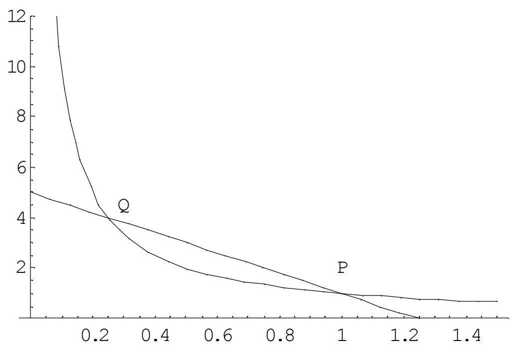
# APPENDIX A

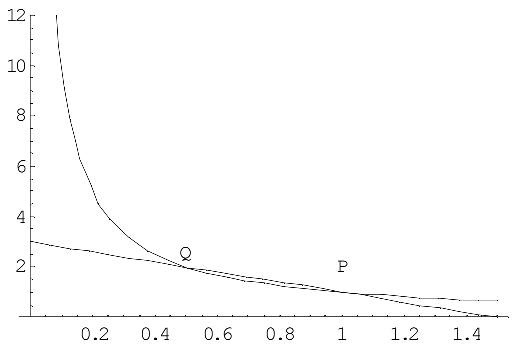
CONCEPTUAL PART OF THE EXAMINATION FOR THE

EXPERIMENTAL AND THE CONTROL GROUPS

1. Consider the slope of a line through Q and P. What is the limit of the slopes of the secant lines as Q approaches P along the curve? (Answer in words)

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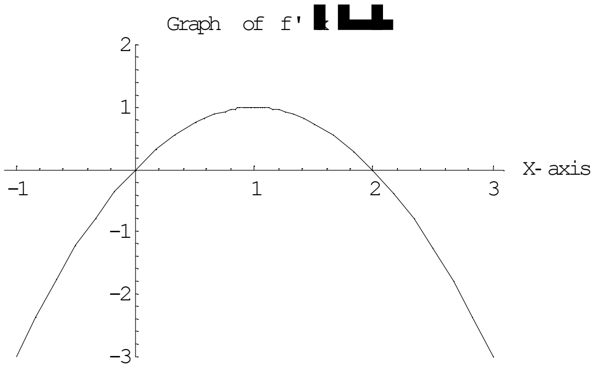
2. Let $f'(x)=3x^3+3x^2-6x$ . Note that $f'(-2)=f'(0)=f'(1)=0$. Identify the open intervals on which $f$ is increasing or decreasing.

3. You are given the graph of *f'(x)* .

i) Find the open intervals on which the graph of *f(x)* is increasing or decreasing.

ii) If *f(x)* represents the revenue function then find the number of units *x* that will maximize the revenue.

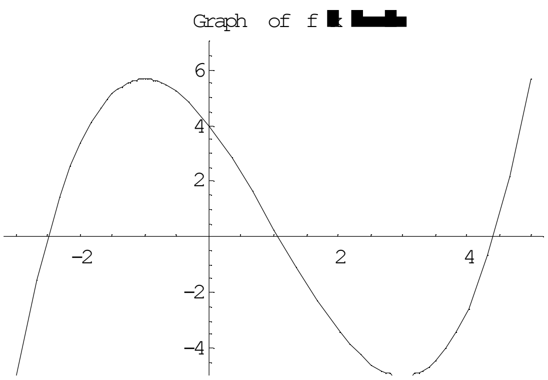



4. What are the asymptotes (if any) of the graph of *f* if

$$\lim_{x\to 1^-} f(x) = -\infty, \lim_{x\to 1^+} f(x) = +\infty, \text{ and } \lim_{x\to \pm\infty} f(x) = 1?$$

5. Given $R(x) = -\frac{4}{9}(x^3 - 9x^2 - 27), 0 \le x \le 5,$ where R is the revenue and x is the amount spent on advertising. Identify the point of diminishing returns for the input-output function.

6. Given below is the graph of f. The sign of *f"* at the point ( 3, -5 ) is:
(a) Negative (b) Positive (c) Zero (d) The sign can not be determined.



## APPENDIX B

COMPUTATIONAL PART OF THE EXAMINATION FOR THE

EXPERIMENTAL AND CONTROL GROUPS

In problems 1 - 3, find all critical numbers and inflection points (if any) of the graphs of the given functions. Then find the intervals on which the functions are increasing or decreasing. Also identify the intervals on which the graphs are concave upward or concave downward.

1) $f(x) = x^4 - 8x^3 + 18x^2 - 16x + 5$

2) $f(x) = \dfrac{x}{x^2 + 1}$

3) $f(x) = x\sqrt{4 - x^2}$

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