Cognitively Demanding Tasks: Supporting Students and Teachers during Engagement and Implementation

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ARTICLE INFO	ABSTRACT
	A clear relationship exists between the consistent selection and implementation of cognitively demanding tasks and students' conceptual understanding of mathematics. However, mathematics teachers often struggle to maintain the cognitive demand of implemented tasks, with a number of factors identified as contributing to this decline. As many students have limited opportunity to engage with cognitively demanding tasks, the percentage of students proficient in mathematics remains low and unchanged. Given that the consistent opportunity to engage with cognitively demanding tasks is key to increasing the percentage of students proficient in mathematics, understanding the role of tasks in the mathematics classroom is of essence to both mathematics students and the field of mathematics education. As such, the purpose of this paper is to describe the role of cognitively demanding tasks in the mathematics classroom, the barriers to task implementation, and the supports for students and teachers related to either their engagement with or implementation of cognitively demanding tasks.

INTRODUCTION

The consistent implementation of cognitively demanding tasks has been shown to improve students' conceptual understanding of mathematics (Boaler & Staples, 2008; Stein & Lane, 1996; Tarr et al., 2008). However, teachers often struggle to implement these tasks in their classrooms (Boston & Smith, 2009; Jackson et al., 2013; Stein et al., 1996) and in contrast, their instruction includes few opportunities for students to think and reason about mathematics. From this, it is not surprising that only 33% of 8th graders in the United States are proficient in mathematics (National Center for Education Statistics [NCES], 2019). As the consistent opportunity to engage with cognitively demanding tasks is seemingly the key to increasing the percentage of students proficient in mathematics, understanding the role of tasks in the mathematics classroom is of essence to not only the field of mathematics education, but to each and every mathematics student. The purpose of this paper is to describe the role of cognitively demanding tasks in the mathematics classroom, the barriers to task implementation, and the supports for students and teachers related to either their engagement with or implementation of cognitively demanding tasks. We begin by discussing the role of cognitively demanding tasks in mathematics teaching and learning.

REVIEW OF LITERATURE

Cognitively Demanding Tasks in Teaching and Learning

We begin with an overview of how cognitively demanding tasks have been gradually centralized within national mathematics standards and reform documents. We then formally define cognitively demanding tasks and describe two frameworks, one used to identify the cognitive demand level of a task, and the other used to demonstrate how tasks progress from selection to implementation. We conclude with a discussion of how cognitively demanding tasks affect students' academic outcomes and the factors that affect the maintenance and decline of cognitively demanding tasks.

A call for cognitively demanding tasks in the United States

The role of problem solving has remained a consistent and important aspect of mathematics education for more than four decades. While guiding documents in the field of mathematics education have remained steady in their call to teach via problem solving, the use of cognitively demanding tasks as the means to teach via problem solving has been gradual. *An Agenda for Action*

(National Council of Teachers of Mathematics [NCTM], 1980) was the first nationally accepted reform publication in mathematics education and provided empirically supported teaching recommendations, the first of which focused on developing students' problem-solving abilities. Almost a decade later, NCTM (1989) published the *Curriculum and Evaluation Standards for School Mathematics*, which included both curriculum and content standards. Within these standards, the role of developing students as problem solvers remained, and recommendations for how to teach via problem solving were also included as the authors emphasized the use of problem situations to introduce mathematics topics (NCTM, 1989).

The first formal recommendation for teaching with tasks was included within the *Professional Standards for Teaching Mathematics* (NCTM, 1991), a publication intended to detail the teaching practices needed to support the demands set forth within *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). The first of six standards for teaching mathematics was "Worthwhile Mathematical Tasks" (NCTM, 1991, p. 25). The standard begins with the following: "The teacher of mathematics should pose tasks that are based on sound and significant mathematics; knowledge of students' understandings, interests, and experiences; knowledge of the range of ways that diverse students learn mathematics" (NCTM, 1991, p. 25). Additional recommendations included that worthwhile tasks be used to develop students' mathematical understanding and to promote problem solving. Three considerations for selecting tasks were included within the standard: (1) the mathematics, (2) the students, and (3) how students learn mathematics (NCTM, 1991). Although explicit discussion of cognitive demand was absent, NCTM (1991) recommended that when considering the mathematics of the task, the task should convey what it means *to do mathematics*, which they contrasted with tasks that only require students to produce correct answers.

Even after a call to teach with worthwhile tasks was formalized (NCTM, 1991), a disconnect between teaching via problem solving and the use of tasks remained. In NCTM's (2000) next set of standards, *Principles and Standards for School Mathematics*, while the use of tasks was discussed throughout the six principles for school mathematics, discussion of tasks within the *problem solving* process standard was not visible. Similarly, although it was recommended that worthwhile tasks be used to introduce and engage students with challenging mathematics, the discussions within the principles for school mathematics were not directly tied to ideas of problem solving.

It was not until the publication of *Principles to Actions* (NCTM, 2014), a document intended to help mathematics educators implement the *Common Core State Standards for Mathematics* (National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010), that tasks were clearly positioned as the means to promoting problem solving in mathematics instruction. Within this publication (NCTM, 2014), eight Mathematics Teaching Practices were identified, one of which was to *Implement Tasks That Promote Reasoning and Problem Solving*. The practice specified that "Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies" (NCTM, 2014, p. 17). More recently, in an effort to address the critical challenges facing mathematics education, these teaching practices were examined in light of ways in which the practices could be used to promote equitable mathematics instruction (NCTM, 2019). In the *Catalyzing Change* series (NCTM, 2019, 2020a, 2020b), recommendations included that tasks have multiple solution pathways, are culturally relevant, and allow learners to draw on their funds of knowledge (i.e., the experiences students bring to the mathematics classroom).

Defining cognitively demanding tasks

To more formally define cognitively demanding tasks, we start by examining how the term "task" has been conceptualized within some of the earliest literature. Doyle (1983) first formalized the idea of an academic task as follows:

> The term "task" focuses attention on three aspects of students' work: (a) the products students are to formulate, such as an original essay or answers to a set of test questions; (b) the operations that are to be used to generate the product, such as memorizing a list of words or classifying examples of a concept; and (c) the "givens" or resources available to students while they are generating a product, such as a model of a finished essay supplied by the teacher or a fellow student (p. 161).

In addition to focusing on the outcome of the task, Doyle (1983) included the "operations" or process(es) required for students to complete the task. Others have defined tasks to focus solely on the mathematics or content to be completed. For example, Stein et al. (2009) defined a task as a problem or set of problems that address a related mathematical concept or content. Most recently, NCTM (2014) defined a task to include both the mathematics and varying levels of cognitive effort—or processes—needed to solve the task; "Tasks can range from a set of routine exercises to a complex and challenging problem that focuses students' attention on a particular mathematical idea" (p. 17). For the purposes of this paper, the definition of task published by NCTM (2014) will be assumed throughout.

As visible in both Doyle (1983) and NCTM (2014), tasks vary by the cognitive demand, or the cognitive reasoning required to solve the task. While there are a number of frameworks that might be used to assign or assess the level of cognitive demand associated with a task (e.g., Bloom's taxonomy, depth of knowledge, etc.), the Task Analysis Guide (Stein et al., 2000) is used almost exclusively within the field of mathematics education (e.g., Boston & Smith, 2009; Charalambous, 2010; NCTM, 2014, etc.). The Task Analysis Guide includes four levels; the two lower levels are associated with tasks requiring low-levels of cognitive demand and the two upper levels are associated with tasks requiring high-levels of cognitive demand. The first level is *memorization* and only requires students to recall previously acquired mathematics knowledge (e.g., facts, definitions, etc.). The second level is *procedures without connections* and requires students to apply a specified procedure that lacks any connection to the concepts underlying the procedure. The third level is *procedures with connections* and includes the use of procedures in a way that focuses students' attention on developing a conceptual understanding of the related mathematics concepts. The fourth and highest level

of cognitive demand is *doing mathematics*, and these tasks lack a clear solution pathway, thus requiring students to explore and make sense of the mathematical processes within the task (Stein et al., 2000).

Mathematical task framework

For all tasks, multiple factors affect how a task progresses from "as represented in the curricular materials" to "student learning" (Stein et al., 1996). Unfortunately, although a task may begin as cognitively demanding, multiple studies have shown that tasks often decrease in cognitive demand once introduced and implemented in mathematics classrooms (Boston & Smith, 2009; Henningsen & Stein, 1997; Stein et al., 1996). To help make sense of how a task progresses through instruction, and where tasks may decrease in cognitive demand, Stein et al. (1996) developed the Mathematical Task Framework, which includes three phases.

The first phase is how the task is initially represented in curriculum or instructional resources, including teacher-created or adapted materials, published curriculum, supplemental materials, and resources shared online (Boston & Smith, 2009; Hsu & Silver, 2014; NCTM, 2014; Parrish & Martin, 2020). In examining tasks as represented in the curriculum, both the cognitive demand and features of the task should be considered (Stein et al., 1996). First, regardless of where the task originated, if a task does not begin as cognitively demanding, there is a low chance the task will be implemented as cognitively demanding (Jackson et al., 2013; Stein & Lane, 1996). Second, task features are those features that impact opportunities for student engagement and reasoning and include the presence of multiple solution strategies, the extent to which the task could be solved using multiple representations, and a press for explanation and justification (Stein et al., 1996).

The second phase is the task set up, which is how the task is introduced by the teacher. How the task is introduced—or launched—may vary from simply telling students they can get started, to more elaborate explanations of the task requirements, context, and available resources (Jackson et al., 2013; Stein et al., 2009). Although the degree to which teachers launch tasks may vary, Jackson et al. (2013) identified two key ways in which the launch impacts the implementation of the task. First, how the task is introduced will directly affect which students, and in what ways, they can engage with the task. Second, the task launch affects the type of work the teacher completes during the task implementation, from either reintroducing the problem to supporting students as they engage with the task. To guide mathematics educators in effectively launching a task, two sets of criteria should be considered. First, consider how the cognitive demand and features of the task will be announced or encouraged by the teacher (Stein et al., 1996). For example, although a task may require students to solve the problem using at least two different representations, the teacher might reduce this requirement during the launch and only require students to solve the task using one representation. In considering a second set of criteria, Jackson et al. (2013) identified four key aspects of a high-quality task launch:

1. Key contextual features of the task scenario are explicitly discussed.

2. Key mathematical ideas and relationships, as represented in the task statement, are explicitly discussed.

3. Common language is developed to describe contextual features, mathematical ideas and relationships.

4. The cognitive demand of the task is maintained over the course of the setup (p. 652).

Of these key aspects, discussion of key mathematical ideas and relationships and the maintenance of cognitive demand were identified by Jackson et al. (2013) as the most indicative of student learning. Similarly, explicitly discussing the mathematical ideas and relationships as represented in the task increased the number of students who could engage with the task during implementation (Jackson et al., 2013). Mathematics educators are likely in need of support in learning to conduct a high-quality task launch, as Jackson et al. (2013) found that only 6.7% of 460 observed lessons—from 165 teachers—included the key aspects of a task launch.

The third phase is task implementation, which is how students interact with the task. During task implementation, both the cognitive demand and task features reference how students are actually engaging with the content of the task. For example, as related to cognitive demand, are students actually thinking and reasoning about the mathematics content of the task as intended? For task features, are students actually using multiple solution strategies?

Student achievement and cognitively demanding tasks

Regardless of the cognitive demand, tasks inherently influence students in two ways; first, students will acquire the content or information associated with the task, and second, students will practice the processes required to complete the task (Doyle, 1983). Stein and Smith (1998) demonstrated this principle in the following:

> Tasks that ask students to perform a memorized procedure in a routine manner lead to one type of opportunity for student thinking; tasks that require students to think conceptually and that stimulate students to make connections lead to a different set of opportunities for student thinking (p. 269).

Given the role of tasks in students' understanding of mathematics, as well as how they perceive the discipline, we next report on research linking the type of tasks in which students engage with their learning outcomes.

In a first study, Ni et al. (2018) explored the relationship between features of cognitively demanding tasks and students' achievement gains across three outcomes: Computational fluency, conceptual understanding, and complex problem solving. Participants included 30 mathematics teachers and their 1,779 Grade 5 and Grade 6 students. Ni et al. (2018) found that 48% of

the tasks were cognitively demanding, 61% of the tasks required multiple representations, and 40% of the tasks encouraged the use of multiple solution pathways. When considering student achievement, results indicated a statistically significant improvement in student achievement for each of the three outcomes. However, connecting the learning gains to a particular aspect of the tasks was not always possible. For example, tasks that were of high cognitive demand and tasks that encouraged the use of multiple solution strategies were not independently found to positively impact any of the three achievement outcomes. In contrast, mathematical tasks that included multiple representations were found to be a statistically significant predictor of higher student achievement in solving complex problems (Ni et al., 2018).

Stein and Lane (1996) also examined the relationship between teachers' selection and implementation of cognitively demanding tasks and student learning. The mathematics teachers at four schools received professional development focused on instructional tasks and mathematics discourse. Each of these schools also served as project sites in which both classroom observation data and students' mathematics performance were collected over a three-year period. Stein and Lane (1996) reported that the greatest student learning gains were associated with classroom instruction that focused on tasks with high levels of cognitive demand. In contrast, there were minimal student learning gains in classrooms where tasks were procedurally based and could be solved with a single strategy. Lastly, moderate student learning gains were associated with classroom instruction in which tasks were introduced as cognitively demanding, but often declined during implementation (Stein & Lane, 1996).

In addition to the types of tasks implemented, the placement of cognitively demanding tasks within instruction—before instruction or after instruction—may affect students' learning outcomes (Russo & Hopkins, 2019). Three primary grade classes were randomly assigned to one of the three lesson structures: Task-First Approach, Discussion-First Approach, or Alternating Approach. Aspects of the task-first approach includes initial student work on a cognitively demanding task, teacher-facilitated discussion of the mathematical concepts, then student work on consolidating routine tasks. The discussion-first approach begins with a teacher-facilitated discussion of the mathematical concepts followed by student work on routine tasks, culminating in a cognitively demanding task toward the end of the lesson. For the Alternating Approach, one unit of instruction utilized the Task-First Approach, and the second unit of instruction utilized the Discussion-First Approach. Students were assessed for both fluency performance and problem-solving performance on two consecutive units of instruction—patterning and addition. Russo and Hopkins (2019) reported a statistically significant difference between the discussion-first scores and the task-first scores for students' fluency performance, with higher scores reported in the discussion-first lessons. In contrast, there was not a statistically significant difference for students' problem-solving performance. Lastly, the authors reported a statistically significant difference with respect to time (i.e., pre vs. post), which suggests that teaching with cognitively demanding tasks, regardless of their placement within instruction, largely influenced both students' fluency performance and problem-solving performance (Russo & Hopkins, 2019).

Research also suggests that features of mathematics instruction and curriculum impacts student achievement (Boaler & Staples, 2008; Tarr et al., 2008). Boaler and Staples (2008) examined the impact of two types of instruction, traditional and reform-oriented instruction, on students' mathematics achievement and their perceptions of the discipline. The study was conducted over a period of five years and included three schools. At two of the schools, Hilltop and Greendale, instruction was often provided through more traditional methods of direct instruction and with tasks that would be characterized as requiring low levels of cognitive demand. At the third school, Railside, instruction was characterized as reform-oriented and included more conceptual problems that would be characterized as requiring higher levels of cognitive demand. At the beginning of high school, students attending Hilltop and Greendale scored considerably higher than those students attending Railside; after two years, the scores of those students receiving reform-instruction at Railside exceeded the scores of those students receiving traditional instruction at either Hilltop or Greendale. Furthermore, the achievement gap between students of different ethnic and cultural groups was lower among Railside students than those students attending Hilltop or Greendale. Students attending Railside also viewed mathematics more positively and were more likely to plan a future that involved mathematics than those students attending Hilltop or Greendale (Boaler & Staples, 2008).

In a last study, Tarr et al. (2008) examined the impact of both the mathematics curriculum and learning environment on students' mathematics achievement. Mathematics curriculum within the study was identified as traditional or NSF funded. The NSF funded curriculum was designed to reflect the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and focused on student engagement and opportunities to learn through problem solving. Regardless of the implemented curriculum, the degree to which classrooms reflected a standards-based learning environment (SBLE) (e.g., focus on conceptual understanding, etc.) was also assessed. Two assessments were used to measure students' mathematics achievement: The TerraNova Survey, which focuses both on skills and concepts, and the Balanced Assessment in Mathematics, which assesses problem solving and reasoning skills. Tarr et al. (2008) did not report a statistically significant difference between the type of textbook used and students' mathematics achievement, as measured using the Terra Nova Survey. However, in classrooms where both an NSF-published textbook was used, and the instruction was characterized by Moderate or High SBLE scores, students' performance on the Balanced Assessment in Mathematics was higher. Although a NSF-curriculum and a Moderate or High SBLE score extends beyond the selection and implementation of cognitively demanding tasks, Tarr et al. (2008) provide evidence that the types of mathematics tasks in which students engage—even if embedded within the curriculum—and the degree to which the classroom environment reflects best teaching practices impact students' mathematics abilities and problem-solving skills.

Factors affecting the implementation of cognitively demanding tasks

Given that students' consistent engagement with cognitively demanding tasks is key to improving their mathematics achievement, we next consider factors affecting the implementation of these tasks. In general, teachers have demonstrated their ability to successfully maintain the features of a task throughout task implementation; Stein et al. (1996) found that "when preceded with the appropriate set up, students were found to actually use multiple-solution strategies and multiple

representations and to produce explanations and mathematical justifications in the majority of cases" (Stein et al., 1996, p. 483). In contrast, although the percentage of tasks introduced as cognitively demanding tasks varies widely between studies, from 36% (Jackson et al., 2013) to 74% (Stein et al., 1996), there is agreement that a much lower percentage of cognitively demanding tasks are maintained during implementation (Boston & Smith, 2009; Henningsen & Stein, 1997; Stein et al., 1996). Maintaining the cognitive demand of higher-level tasks is inherently challenging as these tasks lack a clear solution path, are more complex, and often take longer for students to solve than lower-level tasks.

To better understand the specific factors associated with either the maintenance or decline of cognitive demand during task implementation, 144 tasks—each associated with a classroom observation—were collected from four schools, across three years, and were examined in two studies (Henningsen & Stein, 1997; Stein et al., 1996). Of the 144 task implementations, 74% of the tasks were set-up such that students would engage in high levels of cognitive demand, 33% of these tasks were identified as *procedures with connections* and 40% of these tasks were identified as *doing mathematics* (Stein et al., 1996). When students actually engaged with these tasks, the cognitive demand decreased drastically. Of those tasks that were cognitively demanding during the launch, 57% of the *procedures with connections* tasks and 62% of the *doing mathematics* tasks, declined during implementation (Stein et al., 1996). When considering the factors that influenced the decline of the cognitive demand, most commonly, the challenging aspects of the task were simplified (64%), either by the students pressing the teacher to suggest a specific procedure or by the teacher taking over and completing the task for the students. Other factors included (1) the task was inappropriate with respect to student interest, prior knowledge, or motivation (61%), (2) the focus was shifted to the correctness of the solution (44%), or (3) the amount of allotted time was either too much or too little (38%) (Stein et al., 1996).

In a second study, Henningsen and Stein (1997) more closely examined the factors affecting the maintenance or decline of those tasks requiring the highest level of cognitive demand, *doing mathematics*. Of the 144 tasks within the study, 58 were identified as *doing mathematics*, with the cognitive demand declining for 36 of those tasks (62%) (Henningsen & Stein, 1997). Henningsen and Stein (1997) identified three categories for how implemented tasks declined and examined the factors within each category. The three categories included tasks declining into: *procedures without connections* (n = 8), unsystematic exploration (n = 11), and no mathematical activity (n = 10). Of those tasks that declined into *procedures without connections*, most commonly, challenges became non-problems as teachers either specified a solution pathway or took over the problem-solving process. Next, unsystematic exploration was assigned when students were not able to make systematic and sustained progress in developing meaning and understanding throughout the task exploration (Henningsen & Stein, 1997). Within this category, the most common factors for decline included inappropriateness of the task, an inappropriate amount of time, or challenges becoming non-problems. Inappropriateness of the task included lack of student motivation, lack of prior knowledge, or lack of clear task expectations. Lastly, of those tasks that declined into no mathematical activity, factors most commonly included inappropriateness of the task, classroom management problems, and an inappropriate amount of time. Henningsen and Stein (1997) reported that common across all three categories was an inappropriate amount of time, with inappropriateness of the task also common across two of the three categories.

Across both studies, the most common factors influencing the maintenance of cognitive demand included: the task built on students' prior knowledge, an appropriate amount of time was allotted, a high-level performance was modeled by the teacher or peers, the teacher maintained pressure for explanation and justification, and tasks were scaffolded in a manner that allowed students to make progress on the task without reducing the cognitive demand (Henningsen & Stein, 1997; Stein et al., 1996). Based on these findings, there exists a clear dichotomy between those factors that influence the decline of cognitive demand and those factors supporting the maintenance of cognitive demand. For example, with respect to the appropriateness of the task, if the task is inappropriate with respect to students, there is a chance the cognitive demand will decline during implementation. In contrast, if the task is appropriate with respect to students' prior knowledge, motivation, and interest, there is a higher likelihood of the cognitive demand being maintained during implementation (Henningsen & Stein, 1997; Stein et al., 1996).

In other cases, even when the factors identified above were satisfied, the cognitive demand of implemented tasks have been shown to decline for other reasons, such as classroom norms or the habits and dispositions of the teacher and students (Stein et al., 1996). Classroom norms reference how academic work typically gets done within the classroom, by whom, and to what standard. The habits and dispositions of the teacher and students references the teaching and learning behaviors that influence how the teacher and students engage in classroom events. For example, how long are students willing to engage with a challenging task, what types of support does a teacher typically provide a struggling student, and how long is a teacher comfortable letting a student struggle before stepping in to help?

Styliandies and Styliandies (2008) report on a case wherein the more typical factors affecting the decline of cognitive demand were satisfied, but where the classroom norms and habits and dispositions of the teacher and students caused the decline of cognitive demand. The authors observed a seventh-grade mathematics teacher weekly over a four-month period. Within this time period, a focal lesson was selected to serve as the basis of this study. In the focal lesson, students were completing a task that required them to use provided travel notes to create a table, graph, and to explain how each of the six travel notes contributed to the construction of their table and graph. The cognitive demand of the task was lowered by the teacher in two ways: (1) by providing students with a table to organize their data, including a scale to use for the independent variable, and (2) by requiring students to include drawings on their poster in place of explaining how they used the travel notes to create their table and graph. Of the six groups of students in this focal class, only two groups produced graphs representative of the expectations set forth at the start of the task. Even so, near the conclusion of the class, the teacher identified all posters as excellent, with no opportunity to compare and discuss the varying task products (Styliandies & Styliandies, 2008).

To further analyze the implementation of the task, Stylianides and Stylianides (2008) considered two components: (1) teacher awareness of potential changes throughout task implementation, and (2) classroom context and culture in which the task was

being implemented. With respect to providing students with a table to organize the data, as well as providing a scale, it did not appear the teacher realized how this adaptation lowered the cognitive demand of the task. In contrast, the authors believed that the teacher was aware that four of the six groups produced incorrect graphs. In considering the culture of the classroom, Styliandies and Styliandies (2008) note multiple norms over their four months of observations:

> (1) the teacher highly appreciated students' efforts and avoided critical comments on their work, trying to instill in them a feeling of accomplishment and to maintain a positive environment in her class;

> (2) student motivation was considered a vehicle to learning;

> (3) possible student failure in completing the assigned tasks would be attributed primarily to the teacher's poor articulation of her expectations; and

> (4) assigned tasks were most of the times completed in the small groups, without whole class discussion for sharing and analysis of the intellectual constructions of small groups (p. 872).

While the real-life context of the task aligned with the second aspect of the classroom culture as students were engaged and motivated to produce aesthetically pleasing posters, the first aspect of the classroom culture, avoids critical comments, explains why the teacher characterized all of the posters as excellent. Next, as the cognitive demand of the task contributed to variation in the mathematical correctness of some of the groups' posters, a whole-group discussion and critical analysis of how different groups approached the graph differently would possibly undermine the work of some groups. With respect to classroom culture, this could cause issues with both the first aspect, avoid critical comments, and the third aspect, teacher's poor articulation of her expectations (Stylianides & Stylianides, 2008). In this case, the fourth aspect of classroom culture "saved" the situation as it allowed the task to end without a whole-group discussion. Findings suggest that the classroom culture, specifically the social practices within the classroom, was a factor in the decline of the cognitive level of demand of the task (Stylianides & Stylianides, 2008).

While reform documents, standards, and research support the consistent selection and implementation of cognitively demanding tasks, teachers struggle to maintain the cognitive demand of these tasks for a number of reasons (Henningsen & Stein, 1997; Stein et al., 1996; Stylianides & Stylianides, 2008). As both the cognitive demand and features of a task have been shown to affect students' mathematics achievement (Boaler & Staples, 2008; Ni et al., 2018; Russo & Hopkins, 2019; Stein & Lane, 1996; Tarr et al., 2008), identifying ways in which to support students and teachers either in their engagement or implementation of cognitively demanding tasks is needed.

Supports for Selecting and Implementing Cognitively Demanding Tasks

To improve the frequency at which cognitively demanding tasks are selected and maintained through implementation, targeted support for both students and teachers should be considered. We start with describing strategies or interventions that have been shown to support students prior to or during engagement with cognitively demanding tasks. We then report on the various means by which teachers have been supported in improving their knowledge or skills specific to selecting and implementing cognitively demanding tasks. Equal emphasis will be placed on reporting the details of both the support and the impact of the support on either students or teachers with respect to cognitively demanding tasks. The **Appendix** provides a description and reference(s) for each included support.

Student supports for engaging with cognitively demanding tasks

In an effort to better support students as they engage with cognitively demanding tasks, possible student supports include building prior knowledge through preteaching, providing the support of a conceptualization scaffold just before task implementation, and providing enabling and extending prompts during task implementation. Each of these supports are discussed below.

Preteaching. Preteaching is defined as the advanced introduction of a concept or skill corresponding with upcoming instruction (Bouck et al., 2019; Watt & Therrien, 2016). Preteaching has been used in mathematics in two ways. First, preteaching has been used to introduce students to the concept prior to classroom instruction on that same concept; preteaching in this manner allows students an opportunity to familiarize themselves with the specific content prior to instruction. Second, preteaching may be used to provide advanced instruction specific to the prerequisite skills needed to engage with upcoming mathematics content (Watt & Therrien, 2016). It is this second use of preteaching that is of particular interest in supporting students during their engagement with cognitively demanding tasks. As described first, the use of preteaching to provide an advanced introduction of content prior to classroom instruction, especially through explicit or direct instruction, has been shown to cause interference with how students later engage with cognitively demanding tasks specific to that topic (Lalley & Miller, 2006; Pesek & Kirshner, 2000).

Two studies have examined the effect of preteaching the prerequisite skills needed to engage with the mathematics of the general education instruction (Watt et al., 2013; Watt & Therrien, 2016). Watt et al. (2013) first examined the implementation of preteaching with the same group of ten fifth-grade students, but across two different units of instruction, geometry and fractions. The participating students were those already receiving support in mathematics. Prior to each unit of instruction, the ten students received preteaching instruction through six, 30-minute sessions, spread across two weeks. Students in the treatment group scored significantly higher on the posttest administered at the conclusion of preteaching, but before the whole-class unit began. Following the implementation of the unit, although the scores of the treatment group were not significantly different from those

that did not receive preteaching, there was a small to moderate effect on the treatment students' ability to generalize prerequisite skills on the unit exam. Preteaching was also shown to have a moderate to large effect on treatment students' ability to maintain the learned prerequisite skills on the maintenance exam administered two weeks following the unit exam (Watt et al., 2013).

In a second study, Watt and Therrien (2016) implemented a similar preteaching model as described in Watt et al. (2013) but with 32 sixth-grade students who were also already receiving support in mathematics. Two weeks prior to the unit, the treatment group received daily preteaching sessions on the identified prerequisite skills. Following the two weeks of preteaching, but before the unit began, students in the treatment group scored significantly higher on a posttest with a large effect. Even so, after the whole-class unit on fractions, there were no significant differences between the treatment and control groups on either the unit exam or the maintenance exam administered a few weeks later. When examining the approaches of students in each group, students that received preteaching only made procedural errors on the posttest whereas students in the control group made both procedural and conceptual errors (Watt & Therrien, 2016).

Conceptualization scaffolds. DiNapoli (2018, 2019) examined how the use of a common conceptualization scaffold, given prior to beginning the task, supported students in improving their ability to persevere on cognitively demanding tasks. Ten ninth-grade students completed five cognitively demanding tasks, about one per week, with the conceptualization scaffold being randomly assigned to three of these five tasks. The conceptualization scaffold was the same for each of the assigned tasks and read as follows: "Before you start, what mathematical ideas or steps do you think might be important for solving this problem? Write down your ideas in detail" (DiNapoli, 2019, p. 1388). As the goal of the study was to capture how a conceptualization scaffold supported perseverance from a student perspective, think-aloud interviews were used to capture how students engaged with the task. Over time, participating students improved in their perseverance on scaffolded and non-scaffolded cognitively demanding tasks, but improved at a much higher rate across scaffolded tasks. On scaffolded tasks, students continued to persevere after reaching an impasse, whereas on the non-scaffolded tasks, students would stop working once they reached a similar impasse; "the conceptual thinking recorded after engaging with the scaffold prompt acted as an organization toolbox from which to draw a fresh mathematical idea, or a new connection between ideas" (DiNapoli, 2018, p. 896). From exit interviews, six of the ten students attributed the opportunity to conceptualize their ideas before beginning the task to their improved work. DiNapoli (2019) also concluded that it was not the opportunity to work with cognitively demanding tasks alone that led to improved perseverance on tasks but providing students an opportunity to conceptualize their ideas prior to beginning the task that influenced improvement in students' perseverance.

Providing enabling and extending prompts. In addition to supporting students prior to engagement with the task, enabling and extending prompts support students while they are engaging with the task (Sullivan et al., 2009). Enabling prompts allow students struggling with the primary task an alternative, but related task that provides opportunities for engagement with the mathematics topics and still allows them to engage in the concluding whole-class discussion around the original task. Sullivan et al. (2009) described enabling prompts in the following way:

> These prompts can involve slightly lowering an aspect of the task demand, such as the form of representation, the size of the number, or the number of steps, so that a student experiencing difficult[y] can proceed at that new level, and then if successful can proceed with the original task (p. 22).

In contrast, extending prompts are intended for those students that finish the original task early and provide opportunities for further engagement with the same mathematics topic in a manner that does not make the students feel like they are simply getting more problems (Sullivan et al., 2009).

Russo and Hopkins (2017) embedded these enabling and extension prompts within a larger framework intended to reduce students' cognitive load while engaging with cognitively demanding tasks. The Cognitive Load Approach to Shaping and Structuring Challenging Tasks (CLASS Challenging Tasks) framework includes multiple steps. To start, teachers should first select a learning objective, and a cognitively demanding task aligned with that learning objective. Teachers then identify secondary learning objectives that are inherently embedded within the task. To determine these objectives, the authors recommended asking yourself the following question: "What other skills and knowledge am I assuming that students either have, or will develop, through engaging in this task as it is currently presented?" (Russo & Hopkins, 2017, p. 23). Following, teachers sort the secondary learning objectives as either intrinsic cognitive load—those connecting directly to the content of the primary learning objective—or extrinsic cognitive load—those less relevant to the primary learning objective. Teachers then redesign the task to remove the demands of those secondary learning objectives identified as relating to the extrinsic cognitive load or deemed as not relevant to the overall learning goal. For example, if an original task focused on understanding that "doubling" is a rule that makes values grow large quickly and included the initial scenario as a written description, the revision might include representing this scenario in a table and including more than the initial value or stage. This revision removes the need for students to translate the written mathematics problem into tabular or pictorial representations, a clear secondary objective not directly related to the primary objective. Next, teachers create both enabling and extending prompts, as previously described by Sullivan et al. (2009). Lastly, teachers should conclude the lesson with a two-minute summary statement that includes a restatement of the primary learning objective, as well as student work that illustrates this primary learning objective. Russo and Hopkins (2017) noted that this framework occurs within the context of the classroom, which is often in contrast with other interventions that pull one or more students out of the classroom for remediation.

Teacher Supports for Selecting and Implementing Cognitively Demanding Tasks

Support for teachers—preservice and inservice—in their enactment of cognitively demanding tasks can be broadly categorized in five different ways: professional development, post-secondary methods of teaching modules, lesson observations, lesson

studies, and learning communities. Note that these categories are not mutually exclusive, and in some studies, there is an overlap of supports (i.e., lesson observations in the context of a larger professional development). Each category of support is discussed below, with equal attention to both the ways in which teachers were supported in their enactment of tasks and the impact or implications of this support on their knowledge or implementation of tasks.

Professional development. In examining the use of professional development to support teachers' enactment of cognitively demanding tasks, multiple frameworks or activities were common within many of these trainings, including the Task Analysis Guide (Stein et al., 2000), the Mathematical Task Framework (Stein et al., 1996), and task sorts (Stein et al., 2004). In other cases, teachers were supported as they engaged with elements of instruction, such as solving cognitively demanding tasks as mathematics learners, or planning for their own implementation of cognitively demanding tasks.

In a first study, Arbaugh and Brown (2005) facilitated a year-long professional development focused on the role of mathematics tasks. The professional development included readings and activities that supported teachers in assessing the cognitive demand of tasks using the Task Analysis Guide (Stein et al., 2000), as well as an introduction to the Mathematical Task Framework (Stein et al., 1996). Participating teachers were also supported in finding cognitively demanding tasks that would align with their curriculum and adapting lower-level tasks to require a higher level of cognitive demand. A central activity within the training included teachers participating in a task sort, which required teachers to sort 20 middle-school mathematics tasks into categories of their choosing—ways in which task sorts can be used as a source of teacher learning are described further in Stein et al. (2004). In comparing participants' initial and final task sorts, the teacher-created categories changed drastically. During the initial task sort, nearly 80% of the teacher-created categories were based on superficial characteristics of the tasks. In contrast, on the final task sort, approximately 56% of the teacher-created categories were based on the cognitive demand requirements of those tasks. In comparing the tasks used in teachers' classrooms from the first and last week of the training, there was no significant difference found in the number of low-level and high-level tasks used between these two weeks. However, when looking at how task implementation differed between individual teachers, two of the seven teachers showed significant improvements in the number of high-level tasks implemented. Although not all teachers demonstrated an increase in their selection of cognitively demanding tasks, Arbaugh and Brown (2005) concluded that the training supported teachers in "thinking more deeply about the relationship between mathematical tasks and the work of students in their classes" (p. 525).

Second, Boston and Smith (2009, 2011) and Boston (2013) reported on the effects of a two-year professional development targeting teachers' selection and implementation of cognitively demanding tasks. The Enhancing Secondary Mathematics Teacher Preparation (ESP) project included a number of supporting frameworks. In addition to using the Task Analysis Guide (Stein et al., 2000) and the Mathematical Task Framework (Stein et al., 1996), teachers were also introduced to the "Thinking through a Lesson Protocol," a lesson planning protocol focused on planning for the implementation of cognitively demanding tasks (Smith et al., 2008). The primary text of the training was *Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development* (Stein et al., 2000), which includes each of the above-mentioned frameworks and various case study examples around task enactment. Boston and Smith (2011) further described the ESP training as follows:

> At the heart of the ESP sessions, and the task-centric approach to professional development, were ongoing opportunities for teachers to solve and assess the cognitive demands of mathematical tasks (so teachers could recognize high-level tasks), to analyze the implementation of mathematical tasks during instructional episodes (so teachers could begin to identify classroom-based factors that support or inhibit high-level student engagement), and to enact and reflect on the use of cognitively challenging tasks in their own classrooms (so teachers could experience and come to terms with the challenges associated with this work) (p. 967).

Participating teachers submitted classroom artifacts—implemented tasks and students' work on these tasks—across three times points within the training, Fall 2004, Winter 2005, and Spring 2005 (Boston & Smith, 2009). Specific to the cognitive demand level of selected tasks, the mean increased from 2.54 in Fall 2004, to 3.01 in Spring 2005, which was statistically significant. When specifically examining the number of selected tasks identified as high-level, the number of high-level tasks increased from 44% in Fall 2004 to 73% in Spring 2005. As evidenced by student work, the mean cognitive demand of the implemented tasks increased from 2.27 in Fall 2004 to 2.86 in Spring 2005, which was also statistically significant. Likewise, the percentage of tasks in which the cognitive demand was maintained during implementation increased from 24% in Fall 2004 to 67% in Spring 2005.

Two years later, Boston and Smith (2011) examined the residue of this initial professional development on how teachers were able to maintain their selection and implementation of cognitively demanding tasks over time. Seven of the 18 original teachers submitted classroom artifacts during Spring 2007. From Fall 2004 to Spring 2007, the percentage of high-level tasks selected improved from 44% to 86%, and the percentage of high-level tasks maintained during implementation improved from 24% in Fall 2004 to 71% in Spring 2007; both of these results were statistically significant (Boston & Smith, 2011).

Boston (2013) also considered how the ESP project impacted teachers' knowledge of cognitively demanding tasks, as well as which aspects of the professional development led to changes in knowledge. Similar to Arbaugh and Brown (2005), participating teachers completed a task sort before and following the first year of training; teachers in a control group also completed the task sort. From the beginning to the end of the training, the increase in the teachers' knowledge of cognitively demanding tasks was statistically significant; likewise, the participating teachers had significantly higher knowledge when compared to the control group (Boston, 2013). The increased knowledge was attributed to the participating teachers being introduced to the levels of cognitive demand through the Task Analysis Guide (Stein et al., 2000), opportunities to compare and categorize mathematics tasks with respect to cognitive demand, and opportunities to discuss the cognitive demand of tasks they solved as mathematical learners; these discussions and opportunities were included in five of the six full-day workshops (Boston, 2013).

Third, Foley et al. (2012) examined the effects of a year-long professional development targeting teachers' use of cognitively demanding tasks to teach statistics. Similar to other task-based professional development, the training made use of both the Task Analysis Guide (Stein et al., 2000) and the Mathematical Task Framework (Stein et al., 1996). At the conclusion of the summer institute, participants were also supported in using cognitively demanding tasks to plan and implement a mock statistics lesson. Following the start of the next school year, participants reported increased comfort and confidence in selecting and implementing cognitively demanding tasks although their self-reported use of such tasks only ranged from 5% to 30%. Other changes attributed to the training included resequencing instruction to begin with more challenging tasks before moving to procedural computations, rewriting exam questions to increase the cognitive demand, and an increase in classroom discourse. When considering constraints to implementing tasks, four of the five participants identified time as the largest barrier to implementation (Foley et al., 2012).

Fourth, Sullivan et al. (2015) supported both primary teachers (students aged 5-12) and secondary teachers (students aged 12-14) in selecting and implementing cognitively demanding tasks using a two-day training. In the training, the teachers were provided ten age-appropriate lessons to work through, followed by a discussion of possible student solutions and effective pedagogical strategies. The provided lessons also included supporting materials, such as a lesson rationale, suggestions for introducing the task, intentions for student learning, and enabling and extending prompts (Sullivan et al., 2009) for students with varying abilities. Following the training, teachers implemented the lessons with their students in alignment with their typical pacing or curriculum guidelines. Overall, the teachers found the lesson structure and supports helpful, including the additional prompts to support students who were struggling, as well as those who were ready to move on to more challenging tasks. The authors also noted that the consistent manner in which tasks were introduced minimized negative responses from students related to engaging in challenging, open-ended tasks. The researchers suggest that providing professional learning opportunities that focus on working through challenging tasks, creating and implementing lessons based on cognitively demanding tasks, and supporting students in productive struggle is beneficial to classroom teachers (Sullivan et al., 2015).

Lastly, Ader (2020) investigated the impact of a year-long professional development on teachers' implementation of cognitively demanding tasks. The professional development program included four preliminary workshops, the use of video recordings of the participating teachers implementing the tasks, and reflections shared in the professional learning community established among the teachers and the program facilitators. The preliminary workshops were conducted a week prior to the beginning of the new school year. The focus of these workshops was on the main concepts of effective and high-quality implementation of cognitively demanding mathematics tasks. Throughout the school year, video-recorded lesson observations were completed biweekly with individual and group interviews conducted weekly. During the interviews, the teachers reflected on the previous lesson observations—task implementation quality—and planned the upcoming week's tasks. The quality of task implementation was determined by the cognitive demand level of the task, attention to student thinking during the task, and intellectual authority of the student. While there was no significant relationship between the time teachers spent in the professional development program and attention to student thinking or intellectual authority during observations, there was a significant relationship between the time in the professional development program and the cognitive demand of enacted tasks. As noted in the teacher interviews, the continued opportunities to implement tasks and reflect on the recorded lesson observations in the professional learning community, coupled with the focus on the quality of task implementation throughout the program, contributed to the improvement of their practice (Ader, 2020).

Post-secondary methods of teaching modules. The next means of support included learning activities designed for and implemented within methods of teaching mathematics courses at the post-secondary setting. Even so, the described activities and instruction could be adapted in ways to support teacher learning in other settings. The first two studies focus on launching cognitively demanding tasks while the last study focuses on assessing students' cognitive level during task engagement.

Wilhelm and Woods (2020) examined the ways in which the *explore phase* of the teacher learning cycle allowed inservice mathematics teachers to reflect on launching cognitively demanding tasks. In the *explore phase*, teachers read about the aspects of an effective task launch (Jackson et al., 2012), reflected on ways in which they might implement a task launch in their own instruction, and had opportunities to analyze a video of a task launch alongside their peers and course instructor. Wilhelm and Woods (2020) also included a lesson plan within the appendix of their manuscript detailing how they used the teacher learning cycle to engage teachers with the practice of launching a cognitively demanding task. How teachers reflected about launching cognitively demanding tasks varied with respect to their individual instructional vision, or how they viewed ideal classroom practice. Those teachers who viewed their role as "deliverer of knowledge" described ways in which they might incorporate specific aspects of an effective task launch within their own instruction (e.g., developing a common language among students), but not in alignment with how the practice is intended. Those teachers who viewed their role as "facilitator of knowledge" were able to identify ways in which they were already implementing some aspects of an effective launch within their practice, but that the activities within the *explore phase* would serve as motivation to improve their practice in ways that would fully align with their instructional vision (Wilhelm & Woods, 2020).

Next, Creager et al. (2021) designed and evaluated a unit specific to supporting preservice teachers in their understanding of how to launch a cognitively demanding task. The unit was organized using a pedagogies of practice framework as the unit included representations of practice through both written and video examples, opportunities to decompose the practice using the aspects of an effective task launch identified by Jackson et al. (2013), and lastly, opportunities to plan and rehearse a task launch among their peers. Creager et al. (2021) included an outline of their unit, alongside supporting links and resources, within the appendix of their manuscript. In assessing the effectiveness of the unit, from the start of the unit to the final exam, the preservice teachers demonstrated improvement in their ability to notice mathematical relationships and contextual features, but they still struggled with noticing the development of common language and the maintenance of cognitive demand. Most notably, the students often

struggled with how much to "tell" while developing common language during the launch of the task and still maintain the level of cognitive demand (Creager et al., 2021).

Research suggests that preservice teachers may also use knowledge gained from assessing the level of cognitive demand of mathematical tasks to inform their instructional use of cognitively demanding tasks and support students in persevering in completing those tasks (Norton & Kastberg, 2012). In this study, 17 preservice teachers were paired with high school algebra students for a twelve-week, letter-writing exchange. Each letter included a mathematical task shared with the student that was then completed and returned in the same letter. The classroom teacher allowed approximately 20 minutes per week for the students to individually complete the task. Each week, the preservice teachers would record the expected level of cognitive demand of the task as well as the assessed level of cognitive demand after the solution was returned on the assessment form. The preservice teachers would use students' written responses and the researcher's feedback about possible revisions to restructure the previous task, or design a new task, and then share with the student the following week. The purpose of this study was to determine how the preservice teachers accounted for discrepancies in the expected level of cognitive demand and the assessed level of cognitive demand and how they adapted tasks to elicit a higher level of cognitive demand. Two preservice teachers were selected for this case study as the cognitive demand of more than one of their tasks was identified at the highest level of *doing mathematics*. Data included the preservice teachers' written tasks, student written solutions, and the preservice teachers' assessment forms. Two themes emerged related to the discrepancies noted in the expected and assessed level of cognitive demand: student effort and engagement. When adjusting for the level of student effort in completing the task, supports and hints were provided for the students so that they felt more confident in persevering to complete the task. To adjust for disparities based on time on task and student engagement, tasks were restructured based on student responses that pressed for more reasoning and generalizations. The preservice teachers' evolving knowledge of student thinking was used to design or restructure tasks that were more appropriately challenging for the students with whom the letters were being exchanged. Overall, a focus on students' mathematical thinking was found to be an important factor in implementing cognitively demanding tasks and encouraging student persistence in completing the tasks (Norton & Kastberg, 2012).

Lesson observations. Lesson observations were also a common means of support as teachers were positioned to focus on the implementation of the tasks and not on the delivery of the lesson. To start, Russo and Hopkins (2019) supported three elementary teachers in a "lead teacher, co-teacher" model. In this model, the researcher served as lead teacher and was responsible for designing and delivering instruction, thus, allowing each elementary teacher the opportunity to focus on the delivery of the lesson and how their students were engaging with the task (Russo & Hopkins, 2019). Three themes emerged in how the participating teachers perceived students' responses to engaging with implemented cognitively demanding tasks: (1) students persisted with challenging problems and engaged in productive struggle, (2) the classroom culture of productive struggle was established through classroom norms and high student expectations, and (3) increased engagement led to more purposeful student learning (Russo & Hopkins, 2019). Although the three participating teachers perceived their students' response to cognitively demanding tasks positively, they varied in their willingness to continue instruction using these tasks. In contrast, the teachers were in agreement on the challenges of planning a cohesive unit of cognitively demanding tasks without help from more knowledgeable others, as well as the time-intensive nature of the implemented units. Russo and Hopkins (2019) concluded that the participating teachers felt an obligation to "cover" each topic in the standards, and that this obligation trumped any similar responsibility to teach for depth and understanding.

Next, Clarke et al. (2014) explored instructional strategies that could be used when teaching with and engaging students in cognitively demanding tasks. To do so, 12 teachers were provided an opportunity to participate in one or more lesson observations. For each of the lesson observations, the researcher began with an overview of the lesson and provided an opportunity for teachers to ask clarifying questions. During each lesson, the teachers recorded the following: teacher behaviors that encouraged student perseverance, additional behaviors the teacher could have done to encourage persistence, and cognitively demanding features of the task. Each lesson observation concluded with a focus group in which the teachers discussed the notes they made while observing the lesson. During the focus groups, participants identified enabling prompts that were offered to students that were unable to move forward and extension prompts that were offered to those who finished early. The participants also discussed the classroom culture of persistence that included the teacher facilitating and monitoring student progress while offering encouragement and praise to the students throughout the task; the teacher was also reported to continually discuss the importance of engaging in challenging tasks with the students. These teachers also emphasized the "time" students were provided to work on the task, think about the task, and to share their thinking with others. Clarke et al. (2014) compared this lesson observation approach to the professional development described above by Sullivan et al. (2015) wherein teachers had opportunities to work through lessons together in a community setting. The authors noted that the most obvious limitation to the lesson observation approach was the single opportunity to observe a demonstration lesson for some participants and the lack of on-going support to follow. This is in contrast to those teachers who attended the professional development (Sullivan et al., 2015) and were able to maintain the support of their professional learning community and the project team members during the training (Clarke et al., 2014).

Lastly, Tekkumru Kisa and Stein (2015) examined the influence of video-based professional development on shifting teachers' views of teaching and student thinking. Participating teachers engaged in seven professional development sessions of noticing. While viewing recorded instruction, participants were asked to first describe what they noticed in the video, identify the cognitive level of student thinking, and determine whether or not the level of cognitive demand was consistent throughout the lesson. Interviews with each participant were conducted both at the start and completion of the professional development. Although the participants' noticing of pedagogy and student thinking began at a high level, there was a significant increase in participants' comments about specific and more explicit teacher actions that were closely tied to student ideas and actions. From baseline to exit interviews, a major shift occurred in the way that the participants viewed teaching—more as an interaction between the

student, the teacher, and the cognitively demanding task. This suggests that teachers need to not only attend to student thinking but also monitor their own actions in efforts to maintain high levels of cognitive demand throughout the lesson (Tekkumru Kisa & Stein, 2015).

Lesson Study. One study examined the impact of Lesson Study on primary teachers' ability to maintain the cognitive demand of mathematics tasks (Estrella et al., 2020). The Lesson Study spanned eight weeks, and the participants—three researchers and four primary school teachers—met for two hours weekly. As part of the Lesson Study, the participants developed a lesson plan, reflected on the enactment of the lesson plan, and discussed how to improve the lesson plan accordingly. The lesson plan was implemented following the fourth and fifth Lesson Study sessions; the sessions following each implementation were used to reflect and analyze the implemented lesson. The first implementation of the plan included multiple instances where the cognitive demand of the task declined, namely as challenging aspects of the task became routinized by the teacher (Estrella et al., 2020). During reflection and analysis of this first lesson, changes were considered prior to the second implementation, such as creating and providing a scaffolding document to the students. In the second implementation, the cognitive demand was maintained and attributed to appropriate scaffolding, a consistent press for student justification, a means for students to monitor their own progress, and the modeling of high-level performance. Although the Lesson Study participants sought to plan a lesson that made use of cognitively demanding tasks, it was not until the participants had an opportunity to reflect on an initial implementation of their lesson that changes were made in a manner that allowed for the cognitive demand of the task to then be maintained (Estrella et al., 2020).

Learning community. Lastly, engagement with a learning community is a means of support that might provide opportunities for teachers to improve in their knowledge and dispositions related to selecting and implementing cognitively demanding tasks. Parrish and Martin (2020) examined an informal, online learning community to determine if and how the community provided key learning opportunities around cognitively demanding tasks. Wilhelm (2014) previously identified the knowledge and conceptions that impact teachers' ability to successfully select and implement cognitively demanding tasks which included: mathematics knowledge for teaching (MKT), visions of high-quality mathematics instruction (VHQMI), and views about how to support struggling students (VSSS). From this, Parrish and Martin (2020) specifically examined if and how learning opportunities related to MKT, VHQMI, and VSSS were included within the MathTwitterBlogosphere (MTBoS), an online community with a mission to help mathematics teachers improve their practice ('Profiles of Math Teachers Who Blog and Twitter', n.d.). The MTBoS primarily engages within the social media platform Twitter and the connected blogs of those who participate with the community; "searching '#MTBoS' is one way to find tweets from the MTBoS community related to teaching and learning mathematics; many of these tweets include a link to a related blog post, while others do not" (Parrish & Martin, 2020, p. 3). Parrish and Martin (2020) found that content within the MTBoS community primarily addressed MKT and VHQMI (82% of the content), while VSSS was addressed sparingly (3% of the content). Of particular interest were the amount of cognitively demanding tasks located within the content of the community; of the 186 included tasks, 85% were identified as cognitively demanding when analyzed using the Task Analysis Guide (*procedures with connections* or *doing mathematics*) (Stein et al., 2000). With respect to interviews with members of the MTBoS, they too felt the community supported their development of MKT and VHQMI but provided much less support for developing their VSSS (Parrish & Martin, 2020).

DISCUSSION AND CONCLUSIONS

Cognitively demanding tasks have been well positioned as both the means to teach mathematics via problem solving (NCTM, 1991; 2014) and to improve students' mathematics achievement (Boaler & Staples, 2008; Russo & Hopkins, 2019; Stein & Lane, 1996). A shift towards teaching with cognitively demanding tasks is of imminent need as only 33% of eighth graders in the United States are proficient in mathematics (NCES, 2019). When considering the ways in which tasks progress from "as represented in the curriculum materials" to "student learning," it is clear that teachers struggle to both launch and implement cognitively demanding tasks. Although the percentage of teachers able to maintain the cognitive demand of a task during the launch varies between studies (Jackson et al., 2013; Stein et al., 1996), multiple studies have shown that an even lower percentage of teachers are able to maintain the cognitive demand of a task during implementation (Boston & Smith, 2009; Henningsen & Stein, 1997; Stein et al., 1996).

When selecting a cognitively demanding task, although doing so might be perceived as the least challenging phase of the Mathematical Task Framework, there are key implications that should be considered during task selection. As task appropriateness with respect to students' prior knowledge and motivation was a prominent factor in why tasks declined during implementation (Henningsen & Stein, 1997; Stein et al., 1996), teachers must not only consider the cognitive demand and features of the task but must also consider the characteristics and knowledge of the students that will engage with the implemented task. Furthermore, although selecting a cognitively demanding task does not guarantee that students will engage with the task as intended, tasks that begin as cognitively demanding, but then decline during implementation, are of greater benefit to students than those tasks that lack cognitive demand from the start (Stein & Lane, 1996).

As tasks are then introduced and implemented in the mathematics classroom, support for both students and teachers are needed if students are to engage with the tasks at the intended level of cognitive demand. Multiple factors indicate that students need support prior to and during task implementation. For example, as task appropriateness was a common reason for task decline, supports intended to improve students' prerequisite knowledge—better alignment with grade-appropriate cognitively demanding tasks and students' mathematics knowledge—would likely minimize the prevalence of this factor. Similarly, if students continue to struggle while engaging with a cognitively demanding task, the use of a conceptualization scaffold would provide access to new ideas (DiNapoli, 2018; 2019), while the availability of an enabling prompt would allow students to engage with a

more accessible task, but still allow them to participate in the concluding, whole-class discussion (Russo & Hopkins, 2017; Sullivan et al., 2009). Per NAEP reports (NCES, 2019), if such a small percentage of students are proficient in mathematics, it can be expected that attention to prerequisite knowledge will be needed if tasks are going to be appropriate with respect to students' prior knowledge.

When examining ways in which teachers have been supported in their selection and implementation of cognitively demanding tasks, although multiple studies have made use of common frameworks, the discussed studies would indicate the teachers also need opportunities to think about and reflect on the knowledge accessible within these frameworks. The degree to which teachers are able to directly apply the knowledge within these frameworks to their own classroom instruction relates to the improvements that teachers are able to make in their own selection and implementation of cognitively demanding tasks. Within the five categories of teacher support, the supports ranged in how connected they were to teachers' classroom practice. In some studies, teachers were provided opportunities to engage with cognitively demanding tasks absent the context of classroom instruction, either through formal task sorts, adapting low-level tasks to high-level tasks, planning and teaching mock lessons, engaging with an online community, or solving tasks as a mathematics learner. When the primary support only included the opportunity to engage with cognitively demanding tasks absent the context of classroom instruction, while teachers might have improved in their knowledge or dispositions related to the tasks, there were generally limited or no effects on their subsequent selection and implementation of such tasks (Arbaugh & Brown, 2005; Foley et al., 2012).

A next series of supports provided an increased connection to classroom instruction through observations, either recorded or live. Observations provide teachers opportunities to focus on how a task is being implemented, as well as how students are responding and thinking about these tasks, which is not often possible when the teacher is solely responsible for delivering instruction (Russo & Hopkins, 2019). Additionally, observations allow teachers opportunities to begin recognizing those classroom and teacher actions that either support or cause the cognitive demand of the task to decline (Clarke et al., 2014; Tekkumru Kisa & Stein, 2015). Teachers participating in observations viewed student engagement with tasks positively, were able to identify teacher actions that supported the implementation of tasks, improved in their ability to identify the inclusion and exclusion of key aspects of a task launch, and identified ways in which they intend to shift their own instruction. Even so, there was limited evidence that observations alone supported teachers in their actual selection and implementation of cognitively demanding tasks (Clarke et al., 2014; Russo & Hopkins, 2019; Tekkumru Kisa & Stein, 2015). For example, although the teachers in the Russo and Hopkins (2019) study were able to observe both how their very own students persisted with challenging tasks and how this engagement with tasks led to improved learning, the degree to which teachers planned to use cognitively demanding tasks in their future instruction varied.

Lastly, some supports provided teachers with opportunities to implement and reflect on their use of cognitively demanding tasks. Boston and Smith (2011) indicated that providing teachers with opportunities to implement and then reflect on their use of cognitively demanding tasks allowed them to fully understand the challenges associated with enacting cognitively demanding tasks. Although we have indicated that engaging with and observing the implementation of cognitively demanding tasks have resulted in limited effect on teachers' selection and implementation of cognitively demanding tasks, both types of support are important in improving teachers' knowledge and dispositions prior to task implementation. In one study (Ader, 2020), teachers attributed their improved task implementations to both the consistent focus on the quality of task implementation within the training and the continued opportunities to implement and reflect on this instruction. Evidence also suggests that the opportunities to reflect on the initial implementation of a cognitively demanding task—especially when the cognitive demand declines during implementation—are key in identifying modifications to the task or task implementation; in Estrella et al. (2020), it was not until modifications were made and the task was implemented a second time that the cognitive demand of the task was maintained.

To some degree, each type of described support is important in teachers increasing their ability to select and implement cognitively demanding tasks. Even when teachers were provided opportunities to implement cognitively demanding tasks as part of a professional development, they were also provided opportunities to engage with tasks absent the context of instruction, and in some cases, to observe the implementation of tasks (Ader, 2020; Boston & Smith, 2009; 2011; Sullivan et al., 2015). Even though teachers participating in the ESP project were provided opportunities to implement and reflect on their enactment of cognitively demanding tasks, they attributed their increased knowledge of tasks to engaging with the various frameworks, completing task sorts, and solving tasks as learners (Boston, 2013). From this, it is clear that engagement with the various types of described support is needed for teachers to improve in their selection and implementation of cognitively demanding tasks. While teachers may be able to engage with task-specific frameworks or observe instruction independently or informally with their peers, formal opportunities to implement and reflect on tasks may not always be available. In these cases, teachers may still benefit from being intentional in their implementation and reflections. This could include viewing their recorded instruction through the lens of an observation protocol, such as the Instructional Quality Assessment (IQA) Academic Rigor instrument, that focuses on aspects of cognitive demand (Boston & Smith, 2009; 2011).

It is well documented that maintaining the cognitive demand of mathematics tasks is challenging but essential to supporting student learning of mathematics with understanding. In an effort to increase the number of cognitively demanding tasks both selected and maintained throughout implementation, we have identified a number of supports for both students and teachers. Even so, student supports, including preteaching and conceptualization scaffolds, were only provided to a select group of students and beyond the normal instruction of a general education classroom. Future research should examine ways in which both of these supports might be implemented with all learners and as part of normal instruction. Likewise, if the percentage of students proficient in mathematics is going to increase as a result of their consistent engagement with cognitively demanding tasks, teachers are going to need regular opportunities to reflect on their enactment of cognitively demanding tasks in both preservice and inservice contexts (Ader, 2020; Boston & Smith, 2009; 2011; Estrella et al., 2020; Sullivan et al., 2015). For preservice teachers,

opportunities within teacher preparation programs to enact cognitively demanding tasks should be provided to determine if these experiences equip preservice teachers with the knowledge and skills needed to transfer this practice to their future classrooms. For inservice teachers, opportunities for task implementation and reflection are traditionally embedded within formalized and funded professional development, which is not often available to all teachers. As these limited opportunities exist, future research should consider ways teachers might implement and reflect on their task selection and implementation outside of formalized professional development. For example, research may examine if engagement in school-based or online learning communities provides the support and structure needed to improve this aspect of their practice.

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APPENDIX

Student and Teacher Supports related to Cognitively Demanding Tasks

Support	Description	Reference
Student supports		
Conceptualization scaffolds	A prompt students respond to prior to engaging with a cognitively demanding task; the scaffold prompts students to consider what mathematical concepts or processes might be used prior to beginning the task.	(DiNapoli, 2018; 2019)
Enabling and extending prompts	Enabling prompts allow students struggling with the primary task an alternative, but a related task that still allows students to engage in the concluding whole-class discussion. Extending prompts are intended for those students that finish the original task early and provide further engagement with the same mathematics topics.	(Russo & Hopkins, 2017; Sullivan et al., 2009, 2015)
Preteaching	An advanced introduction of the prerequisite skills needed to engage with upcoming mathematics content.	(Watt et al., 2013; Watt & Therrien, 2016)
Teacher supports		
Compare and sategorize mathematics tasks	Opportunities for teachers to compare and categorize tasks with respect to cognitive demand.	(Boston, 2013)
Explore phase of the teacher learning cycle	Opportunities for teachers to read, reflect on how they might implement, and analyze the practice.	(Wilhelm & Woods, 2020)
Implement and reflect on task enactment	An opportunity to implement a cognitively demanding task within their own classroom, followed by an opportunity to reflect on the implementation. Implementation and reflection allow teachers to fully understand the challenges associated with enacting cognitively demanding tasks.	(Ader, 2020; Boston & Smith, 2009, 2011)
Implementing standards-based mathematics instruction: A casebook for professional development	A text that includes multiple frameworks related to cognitively demanding tasks, as well as a series of case study examples of task enactments.	(Stein et al., 2009)
Instructional Quality Assessment (IQA) academic rigor protocol	Rubric used to assess both the potential of the task, as well as the implementation of the task, both with respect to cognitive demand.	(Boston & Smith, 2009, 2011)
Lead-teacher, co-teacher lesson observation	Another instructor or researcher serves as the lead teacher and is responsible for designing and delivering instruction, while the co-teacher has the opportunity to focus on the delivery of the lesson and how the students were engaging with the task.	(Russo & Hopkins, 2019)
Lesson Study	An opportunity to plan, implement, and reflect on the enactment of a lesson plan; in this lesson study, the lesson was enacted twice.	(Estrella et al., 2020)
Lesson (task) observations	Opportunities to observe the implementation of cognitively demanding tasks, either live or through video, such that teachers have an opportunity to identify teacher actions or classroom factors that support or inhibit the implementation of cognitively demanding tasks.	(Boston & Smith, 2009, 2011; Clarke et al., 2014; Russo & Hopkins, 2019; Tekkumru Kisa & Stein, 2015)
Letter writing exchange	A weekly letter exchange in which preservice teachers present students with cognitively demanding tasks; student responses are analyzed and used to design or restructure tasks to elicit a higher level of cognitive demand.	(Norton & Kastberg, 2012)
Mathematical Task Framework (MTF)	A three-phase framework that details how a task progresses through instruction, from how tasks are initially represented in the curriculum to student learning.	(Stein et al., 1996)
MathTwitterBlogosphere	An online community—that engages on Twitter and the connected blogs of those participating teachers—with the mission to help mathematics teachers improve their practice.	(Parrish & Martin, 2020)
Pedagogies of practice	A module of instruction for teacher learning that includes representations of the teaching practice, opportunities to decompose the practice using research-supported criteria, and an opportunity to rehearse the practice in the context of peers.	(Creager et al., 2021)
Planning and implementing mock instruction	An opportunity to plan and then implement a lesson that includes cognitively demanding tasks, but in the context of other teachers.	(Foley et al., 2012)
Solve mathematics tasks	An opportunity for teachers to determine task features associated with tasks of varying levels of cognitive demand.	(Boston & Smith, 2009, 2011)
Task analysis guide	A four-level framework used to assign the level of cognitive demand associated with a task.	(Stein et al., 2000, 2009)
Task sorts	An activity that requires teachers to sort a selection of tasks into categories of their choosing.	(Stein et al., 2004)
Thinking Through a Lesson Protocol (TTLP)	A lesson planning protocol focused on supporting teachers in planning for the implementation of a cognitively demanding task.	(Smith et al., 2008)