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An Analysis for the Qualitative Improvement of Education and Learning based on the Way of Learner Errors in Descriptive Questions

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ABSTRACT

This study proposes and examines an analytical method with the aim of improving the quality of education and learning by situating the answers to full descriptive questions in probability and statistics to make variables of learners' comprehension of learned content as answer characteristics, based on actual student mistakes. First, we proposed and examined a method for extracting answer characteristics from the answers to the questions in probability and statistics as variables. Second, we proposed a method for obtaining answer characteristics to accurately describe learners' comprehension of each problem and indicate learning and educational policies for learners to improve learning by using regression trees. In addition, the relationship between learners' general ability and answer characteristics was visualized in an item characteristic chart to indicate the general comprehension of the learners. Further, the relationship between learners' learning strategy and answer characteristics was structuralized using Bayesian network models, and effective learning strategies for both learners as a whole and individual learners were extracted and evaluated towards the qualitative improvement of their comprehension using probabilistic reasoning. Our findings showed that the effectiveness of a learning strategy varies with each concept treated in a given problem; with the degree of basical or applied answer characteristics, indicating that the required learning strategy varies according to a given learner's stage of learning. Moreover, the improvement of hours studying dispersion for both mid-term and final examinations was revealed as effective for a wide range of subjects.

Keywords: learning and education in probability and statistics, descriptive questions, extraction of answer characteristics, error analysis, Bayesian network analysis, probabilistic reasoning

INTRODUCTION

"Error analysis" is an analysis of how and why learners make an error, and has been applied in a wide range of fields, including simple questions involving four arithmetic operations. Rushton (2018) studied the effect of the teaching and learning mathematics through error analysis. Mathematics education pedagogy has relied on teachers, demonstrating correctly worked example exercises. However, in recent years, combining the use of correctly worked exercises with error analysis has led researchers to posit increased mathematical understanding. Durkin and Rittle-Johnson (2012) studied the effectiveness of using incorrect examples to support learning about decimal magnitude.

In addition, in the discipline of probability and statistics, various misconceptions exist, as concluded by Kahneman and Tversky (1982), Sulistyani (2019) showed that there were 4 stages of student errors in

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inferential statistics. 1) Errors in comprehension occurred because students could not read statistics tables or read outputs in the questions. 2) Transformation errors occurred because students were not appropriate in applying/selecting the type of test statistics used or writing hypotheses. 3) Process skill errors occurred because students were less careful in calculating and inability to interpret the results of calculations. 4) Errors in the encoding stage occurred because students did not answer correctly or inappropriately in drawing conclusions in hypothesis testing. These results point to properties peculiar to the content of learning in the fields of probability and statistics, suggesting a possible trend in the ways learners of probability and statistics make errors. It would therefore be useful to analyze the information related to the learning experiences of other students who made errors in a similar fashion.

Accordingly, this study proposes the extraction of answer characteristics as variables, based on the forms of error that characterize differences in the degree of learners' comprehension using full descriptive questions related to probability and statistics. Then, using these variables, we perform a regression tree analysis on learners' comprehension of the study content for each problem, as well as an analysis of an item characteristic chart to estimate general comprehension, and a relational analysis between the comprehension and study strategy using Bayesian networks. In so doing, furthermore we propose a method for extracting effective learning policies or learning strategies for learners as a whole, as well as individual learners, through a probabilistic reasoning based on Bayesian networks.

OVERVIEW OF DATA FOR ANALYSIS

This study used students' mid-term and final examination results in a "Probability and Statistics" course offered to sophomore students enrolled in M Engineering Division in J Department at D University, as well as the results of a questionnaire survey on learning situation that was administered during the course to collect data for analysis upon students' consent. One hundred-four students took the mid-term examination, and 99 took the final examination. Of all students who took both examinations and answered the questionnaire survey, consent for use of the data was obtained from 89; these data were then analyzed for this study. In a Probability and Statistics course, the key concepts and items based on the textbook were explained and exercises were implemented during the lesson. Participants received feedback during the exercises through which they were learning probability theory and conducting statistical estimation and testing.

Questionnaire Survey Design

The questionnaire survey was designed to meet the goal of this study based on items concerning self-efficacy, intrinsic value, cognitive strategy and self-regulation, in accordance with the approach adopted by Pintrich and De Groot (1990), to examine the level of learning situation, by such that the questionnaire items conformed to purpose of the study. Further, questionnaire items concerning study habit, study time, and the type of materials used for study were also created as important measures in the study of probability and statistics (**Table 1**).

Table 1. Items in the Questionnaire Survey

Name	Description	Answer form
Self-efficacy: Useful	I think I can appropriately use what I learn in the probability and statistics class	5-point scale assessment (1: Strongly disagree, 2: Disagree, 3: Undecided, 4: Agree 5: Strongly agree)
Self-efficacy: Within the scope of the course	I think I can solve the scope of the problems worked in the class.	
Self-efficacy: Outside the scope of the course	I think I can understand content outside of the scope of this class if I study it.	
Intrinsic value: Important	I think probability and statistics is an important subject for me.	
Intrinsic value: Interesting	I find probability and statistics interesting.	
Cognitive strategy: Understand in one's own words	I try to understand matters that strike me as important by putting them into my own words.	
Cognitive strategy: Identification of content	I think about how to locate the study content within the field of probability and statistics.	
Cognitive strategy: Association with one's knowledge	I try to make an association with my previous knowledge when I study something new.	
Cognitive strategy: Repetitive learning of key concepts	I repeatedly study concepts that seems important to me.	
Self-regulation: Comprehension check	I confirm my comprehension of the study content.	
Self-regulation: Self-directed learning	I work exercises by my self without being required to do so in class.	
Self-regulation: Rereading	When I read the textbook or a reference, I review what I have already read.	
Self-regulation: Uninterested topics	I study hard even for a topic in which I am not interested.	
Study habit: Daily preparation and review	I make it a daily rule to prepare for and review the classwork in the probability and statistics course.	
Study habit: Review before examination	I attentively review what I have learned in the probability and statistics course before an exam.	
Study habit: Study with similar problems	I work on problems that are similar to problems I had difficulty solving in the probability and statistics course.	
Study time: Central position	Cumulative study time on the center of distribution during the course	Fill out numbers
Study time: Dispersion	Cumulative study time on the dispersion of distribution during the course.	
Study time: Probability distribution	Cumulative study time on probability distribution during the course	
Study time: Relation between variables	Cumulative study time on the relation between variables during the course	
Study time: Statistical estimation	Cumulative study time on statistical estimation during the course	
Study time: Statistical testing	Cumulative study time on statistical testing during the course	
Prescribed textbook	Use of prescribed textbooks during study	Yes-no question (0: No , 1: Yes)
References: Probability and statistics	Use of references on probability and statistics during study	
References: Outside probability and statistics	Use of references on topics other than probability and statistics during study	
Web-based material	Use of web-based material during study	
Other course materials or materials from other universities	Use of other course materials or materials from other universities during study	
Video material	Use of e-Learning and video material during study	
Others	Use of other materials during study	

Problems in the Mid-term Examination

The mid-term examination consisted of four full descriptive problems on the probability theories that students had learned in the first half of the course. Students could use a calculator during the examination, but were prohibited from bringing in textbooks or associated materials.

Figure 1 shows Problem 1 as an example of the problems provided as test items in the examination. Problem 1 is composed of questions associated with the mean, variance, and conditional probability based on the simultaneous probability distribution of 2 variables, and was worth 30 total points. As shown in **Figure**

Problem 1: Suppose that the simultaneous probabilistic distribution of random variables X, Y follows the table below.

X \ Y	1	2	3	marginal distribution
1	0.050	0.100	0.050	0.200
2	0.075	0.150	0.075	0.300
3	0.125	0.250	0.125	0.500
marginal distribution	0.250	0.500	0.250	1.000

(1) Are random variables X and Y independent or dependent? State the answer while explaining the reason for it.
(2) Calculate the conditional distribution and conditional mean of Y given X=2 (Round up to 3 decimal places).
(3) Calculate E [XY] (Round up to 3 decimal places).
(4) Calculate V [X+Y] (Round up to 3 decimal places).

Figure 1. Problem 1 In Mid-term Examination

Problem1
(1) Lesd the maximum likelihood estimator of the parameter λ based on sample variables (X1, X2,…, Xn) from the population that follow the Poisson distribution.
Note that the probability distribution of the Poisson distribution is expressed by the formula.

$$f(x_i;\lambda)=\frac{\lambda^{x_i}e^{-\lambda}}{x_i!}\ (x_i=0,1,2,\dots)$$

(2) Show that the maximum likelihood estimator led in (1) has the properties of (2-1) unbiasedness and (2-2) minimum variance and (2-3) consistency.

Figure 2. Problem 1 in the Final Examination

1, a simultaneous probability distribution table for two random variables is given, and the students must answer four questions.

Question (1) requires an assessment of whether two random variables are independent or dependent and a description of the reason for the judgment; a correct answer receives 5 points. Question (2) requires calculating the conditional distribution and the mean; a correct answer receives 10 points. Questions (3) and (4) require calculations of the expected values and for the product and the sum of the two random variables; correct answers receive 5 and 10 points each, respectively.

Problems in the Final Examination

The final examination consisted of four full descriptive problems on statistical estimation and testing, which students learned in the second half of the course. **Figure 2** shows Problem 1 as an example. Problem 1 is associated with maximum likelihood estimation, a key concept in statistical estimation, and (1) requires drawing the maximum likelihood estimator of the parameter λ in a Poisson distribution. A correct answer receives 10 points. Question (2-1) requires the student to prove the unbiasness of the estimator led in (1), question (2-2) requires proving its minimum variance; and question (2-3) requires the student to prove its consistency; 5, 10, and 5 points are allocated to each question, respectively.

SCORING MODEL IN THE EXAMINATION AND EXTRACTION OF ANSWER CHARACTERISTICS AS VARIABLES

Here, the scoring system of the examination is examined and a method for making variables, which characterize the answers based on the mistakes students made when answering the full descriptive questions, is proposed.

Scoring Model for Full Descriptive Questions

First, regarding the multiple-choice questions, suppose p question items exist. Let Y_j express the correctness or incorrectness of the question j. Further, suppose that $\boldsymbol{Y}$ is a vector which express the correctness or incorrectness of the entire problem in the examination. Then,

$$Y = [Y_1, \ Y_2, \ \cdot\cdot\cdot, \ Y_p] \tag{1}$$

$$Y_j = 0 \text{ (Incorrect in Question j), 1 (Correct in Question j)} \tag{2}$$

and $\boldsymbol{Y} = \boldsymbol{y}$, which is obtained in the real examination, is named the item response data (Toyota, 2012). The score S in the examination is expressed by the formula (3), where each Y_j is multiplied by weight W_j and is then totaled.

$$S = \sum_{j=1}^{p} W_j Y_j \tag{3}$$

W_j was set by the creator of the test based on the importance of the content that is assessed in question j. In this scoring model, a learner's response is expressed by two values, which are correct or incorrect, for the test questions. For this reason, the difference in the comprehension of learners as they provide wrong answers (for example, the difference between a learner who arrives at an incorrect answer after working on the question and a learner who fails to arrive at an answer after working on the question), cannot be expressed in formulas (1) to (3).

Unlike this, the full descriptive problem includes questions that require indicating not only the result but also the calculation process or the writing of a proof, and is often evaluated in detail such that even if the answer is close to achieving full credit (or close to 0), some points may be deducted (or added). Thus, we consider that the scoring model for the full descriptive examination in which detailed scoring, such as point-deduction or addition, can be successfully carried out.

The score for the question i (i=1 to p), where learning content items A1, A2, …, or Aq are tested is expressed as $m_1, m_2, \dots, m_q$, respectively, and in circumstances in which is a partial point deduction is made for content items Ak, which is answered correctly, d_k points are deducted, and where there is a partial point addition for content items Ak, which is answered incorrectly, c_k points are added. Thus, Y_j can be expressed as follows.

$$Y_j = [Y_{j11}, Y_{j12}, Y_{j13}, \ \cdot\cdot\cdot, \ Y_{jq1}, Y_{jq2}, Y_{jq3}] \tag{4}$$

Y_{jk1}: 1 if the answer for Ak is correct, 0 if the answer is incorrect

Y_{jk2}: 1 if there is a point deduction in the answer for Ak, 0 if there is no point deduction

Y_{jk3}: 1 if there is a point addition in the answer for Ak, 0 if there is no point addition

and score S_j for question j can be expressed as follows.

$$S_j = \sum_{k=1}^{q} (m_k Y_{jk1} - d_k Y_{jk2} + c_k Y_{jk3}) \tag{5}$$

The multiple-choice test scoring model can be applied to a full descriptive test by expanding it into the full descriptive test scoring model, as shown in formula (5).

Extraction of Answer Characteristics based on Mistakes in the Full Descriptive Test as Variables

In this paper, each content item characterizing differences in the comprehension of learners is regarded as an answer characteristic for each question, and a method for expressing learner's answer characteristics is proposed and examined.

Figure 3 illustrates the concept of extracting answer characteristics. Suppose that a problem assesses the comprehension of study content A, which comprises 5 items—A1, A2, A3, A4, and A5, and answers from learners 1, 2, 3, and 4 are obtained. Learner 1 correctly answers A1 and A2, while learner 3 correctly answers all except A5. Further, suppose that learner 2 correctly answers all, while learner 4 incorrectly answers all. When these answers are obtained, the comprehension of learners 2 and 4 can be expressed by the correct or incorrect answer to the questions, whereas the comprehension of learners 1 and 3 reflects differences in their comprehension of A3 and A4. Further, the comprehension of learners 2 and 3 can be characterized by the

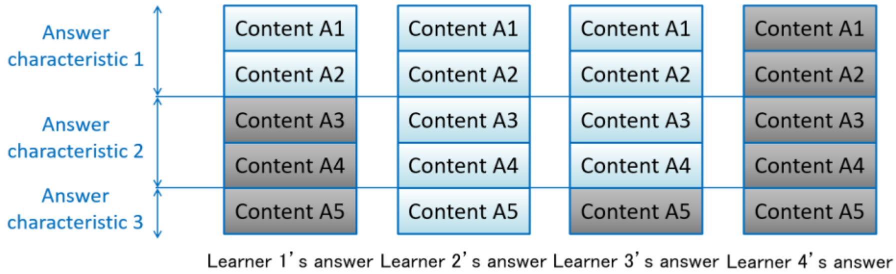


Figure 3. Conceptual Scheme for Extraction Of Answer Characteristics

Table 2. Expression Of Comprehension Via Answer Characteristic Variables

	Answer characteristic B1	Answer characteristic B2	Answer characteristic B3
Learner 1	1	0	0
Learner 2	1	1	1
Learner 3	1	1	0
Learner 4	0	0	0

When the random variables X and Y are independent, for all (X, Y), $\underbrace{\text{P(X, Y)=P(X)P(Y)}}_{\{1\}}$
From the table, when X=1 and Y=1,

$$\underbrace{P(X=1, Y=1)}_{\{2\}} = \underbrace{0.050}_{\{3\}},$$

$$\underbrace{P(X=1)P(Y=1)}_{\{4\}} = \underbrace{0.200 \times 0.250 = 0.050}_{\{5\}}$$

Thus, the formula is true.
$\underbrace{\text{Similar calculations reveals that the formula obtains for all (X, Y).}}_{\{6\}}$
Therefore, X and Y are independent.

Figure 4. Model Answer to Problem 1-(1) in the Mid-term Examination

difference in their comprehension of A5. In other words, the learners' comprehension can be characterized by "the comprehension of A1 and A2," "comprehension of A3 and A4," and "comprehension of A5." Thus, making variables of these as the answer characteristics B1, B2, and B3, respectively, allows us to characterize learners' answers in turn, as shown in **Table 2**.

The procedure for extracting the answer characteristics is explained below. First, of all n learners, suppose there are s learners who provided a partially false answer to question j without leaving it blank, and are expressed as learner i $(= 1, \dots, s)$.

(i) Closely examine the answers of learner $i = 1$, and let the characteristic Bk, which corresponds to each fallacy, be an answer characteristic extracted by $i = 1$.

(ii) Closely examine the answers of learners i $(= 2, \dots, s)$. (a) When the answers of learner i can be expressed by the answer characteristic variables extracted until learner $i-1$, express the answers of the learner i using the answer characteristic variables obtained by learner $i-1$.

(b) When the answers of learner i cannot be expressed by the answer characteristic variables extracted until learner $i-1$, add a new answer characteristic variable to express the answer of learner i.

(iii) Repeat step (ii) until i = s.

(iv) Express the answers from $i = 1$ to $i = s$ in terms of answer characteristic variables B1 to Br.

(v) For the remaining $n - s$ learners, set a value of 0 for all answer characteristic variables for the learners who did not answer the question, and set 1 for all answer characteristic 12 variables for the learners who provided a perfect answer.

Answer Characteristics for Each Question in the Mid-term Examination

Here, the variables that represent the answer characteristics extracted from Problem 1-(1) (independence and dependence) in the mid-term exam are shown. The model answer for the question and answer characteristics are shown in **Figure 4** and **Table 3**, respectively.

Table 3. Answer Characteristics In Problem 1-(1) In The Mid-term Examination

Variable name	Corresponding answer part	The number of incorrect answer	Example of incorrect answer
Understanding of formula for definition of independence	{1}	12	P(X,Y)=P(X)+P(Y)
Understanding of definition of probability	{3}{5}	14	$P(X=1,Y=1)=0.050^2$
Accurate reading distribution table	{3}{5}	10	P(X=1)=0.050
Correct numerical calculation	{3}{5}	10	P(X=1)P(Y=1)=0.33
Accurate notation	{1}{2}{4}	20	P(X×Y)=P(X)P(Y) (Corresponds to {1})
Clearly stated reason of judgment	{6}	53	(No statement for reason)

Since the population distribution can be expressed as $P(X=x_i)=f(x_i;\lambda)=\frac{\lambda^{x_i}e^{-\lambda}}{x_i!}\ (x_i=0,1,2,\dots)$

The likelihood function is expressed as $\underbrace{L(\lambda)=\prod_{i=1}^{n} f(x_i;\lambda)=\frac{\lambda^{\sum_{i=1}^{n}x_i}e^{-n\lambda}}{\prod_{i=1}^{n}x_i!}}_{\{1\}}$

Therefore,

$$\underbrace{\frac{\delta}{\delta\lambda}\log L(\lambda)=0}_{\{2\}}$$

$$\underbrace{\Leftrightarrow \frac{\delta}{\delta\lambda}\left\{\left(\sum_{i=1}^{n}x_i\right)\log\lambda-n\lambda-\log\left(\prod_{i=1}^{n}x_i!\right)\right\}=\frac{1}{\lambda}\sum_{i=1}^{n}x_i-n=0}_{\{3\}}$$

$$\Leftrightarrow \lambda=\frac{1}{n}\sum_{i=1}^{n}x_i$$

Thus, $\underbrace{\text{the maximum likelihood estimator of } \lambda \text{ is } \hat{\lambda}=\frac{1}{n}\sum_{i=1}^{n}X_i=\bar{X}}_{\{4\}}$

Figure 5. Model Answer to Problem 1-(1) in the Final Examination

Table 4. Answer Characteristics in Problem 1-(1) in the Final Examination

Variable name	Corresponding answer part	The number of incorrect answer	Example of incorrect answer
Understanding of the likelihood definition	{1}	7	$L(\lambda)=\sum_{i=1}^{n} f(x_i;\lambda)$
Correct formula manipulation	{3}	13	$1/\lambda$ -n=0
Accurate notation	{1}{2} {3}{4}	55	$\log(\sum_{i=1}^{n}x_i!)$ (Corresponds to {3})
Complete description	{1}{2}{3}{4}	11	No description on {3}
Aware of object of derivation	{4}	8	$\hat{\lambda}=\frac{\lambda}{n}$
Understanding derivation rules	{2}	11	No description on {2}

Answer Characteristics for Each Problem in the Final Examination

Here, too, the variables representing the answer characteristics extracted from Problem 1-(1) (maximum likelihood estimator) in the final examination are shown. The model answers for the question and answer characteristics are shown in **Figure 5** and **Table 4**, respectively.

ANALYSIS OF LEARNERS' COMPREHENSION AND EXAMINATION OF LEARNING AND EDUCATION POLICY BASED ON THEIR COMPREHENSION

This chapter analyzes learners' comprehension for each problem based on the answer characteristics variables extracted in Chapter 3, and proposes and examines a method for promoting qualitative improvement in the learning method and subject comprehension. First, a framework is given for each problem to analyze the relationship between the level of learners' comprehension (scores within the problem) and the qualitative difference (answer characteristics) using regression tree analysis. Then, the relationship between comprehension level, as characterized by the answer characteristic variables for each question and learners' total score, is analyzed using item characteristic chart.

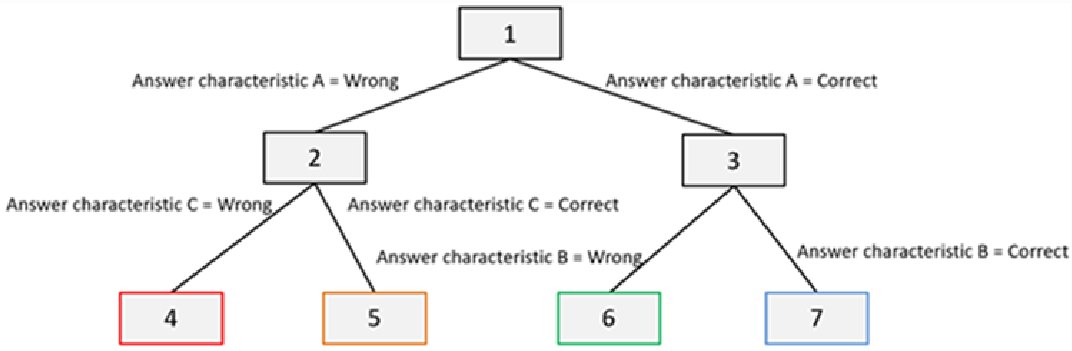


Figure 6. Answer Segmentation Via Regression Tree Analysis

Table 5. Relationship Between Answer Characteristic and Terminal Node

Terminal node	Answer characteristic			
	B1	B2	B3	B4
4	Wrong	-	Wrong	-
5	Wrong	-	Correct	-
6	Correct	Wrong	-	-
7	Correct	Correct	-	-

Segmentation of Learners by Question based on the Answer Characteristics Variables Using Regression Tree and Comprehension Analysis Using Item Characteristic Chart

In this section, learners will be segmented into groups by question using the answer characteristics variables.

Learner segmentation via regression tree analysis

Decision tree analysis is a procedure comprising the recursive segmentation and structuring of objects on a tree by conditionalizing the explanatory variables that most eminently characterize the difference in the objective variable. In our study, since the objective variable is an ordinal scale, a regression tree analysis will be performed (Targo, 2017).

$$S_T - (S_L + S_R) \quad \rightarrow \quad max \tag{6}$$

Note that S_T represents the sum of the squared deviation of the objective variable, S_L is the sum of the squared deviation of the objective variable in a left node after segmentation, and S_R is the sum of the squared deviation of the objective variable in the right node.

The segmentation of learners in the regression tree is explained below. In a tested problem from which four answer characteristics—B1, B2, B3, and B4—were obtained, learners are classified into four terminal nodes (nodes 4, 5, 6, and 7), as shown in **Figure 6**.

The score of the problem is provided as the objective variable, and the answer characteristics are provided as the explanatory variable. Answers with different characteristics are allocated to four terminal nodes according to the branching rule of the regression tree. At this point, note that the correctness cannot be uniformly determined for answer characteristics B2 and B4 in terminal nodes 4 and 5, or for answer characteristics B3 and B4 in terminal nodes 6 and 7. These answer characteristics are not represented on the tree structure; they may either be the parts of the answer that constitute smaller proportions of the total score or possess extreme difference between the number of correct answers and the number of incorrect answers of learners in the answer characteristics. Therefore, the distribution of explanatory variables must be understood when examining the results of the regression trees.

One goal of the classification of answers using a regression tree is to categorize the answers themselves based on quantitative (points) and qualitative information (answer characteristics) and to extract problems that arise in the learning or educational methods. Educators are able to gain qualitative information that reveals "what kind of mistake is found in common" via the explanatory variables reflected in the regression tree as routes from the root to the terminal nodes, along with quantitative information, such as the mean score within a terminal node and the number of answers classified.

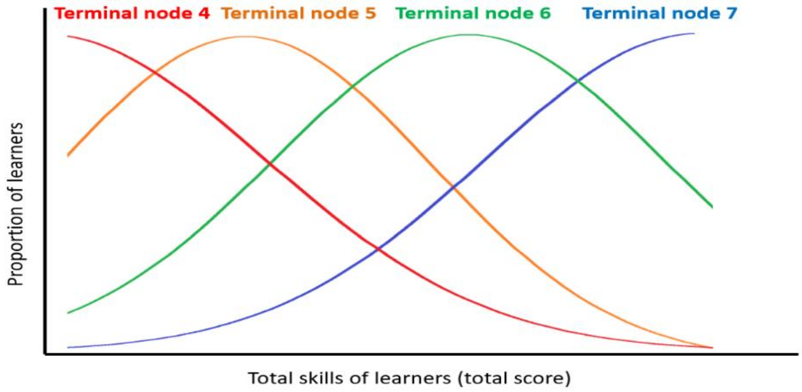


Figure 7. Association Between the Transition of Terminal Nodes and the Total Score of Learners

Further, the structure of the regression tree can be interpreted to express a transition in the learners' comprehension levels. In the example illustrated in **Figure 6**, the mastery of study content expressed in answer characteristic B3 best explains the difference in the levels of comprehension between terminal node 4 and 5; a learner at terminal node 4 can improve his or her comprehension such that it reaches the level represented in terminal node 5 by learning the study content of answer characteristic B3. Likewise, a learner at terminal node 6 can improve his or her comprehension to reach the level expressed in terminal node 7 by learning the study content expressed by answer characteristic B2. Therefore, a learner at terminal node 4 can effectively improve his or her comprehension by understanding answer characteristic B3, which characterizes the difference between terminal nodes 4 and 5; a learner at terminal node 5 can accomplish this as well by understanding answer characteristic B1, which characterizes the difference between terminal nodes 5 and 6, and a learner at terminal node 6 can accomplish this as well by learning answer B2, which characterizes the difference between terminal nodes 6 and 7. Thus, the answer characteristics that characterize the terminal nodes can be understood as study policies that indicate what learners at each stage should do next to improve their understanding.

Further, a positive association between the transition of the answer characteristic at terminal nodes and the total abilities (total score) of the learner is predicted. For example, a learner with a higher total score is predicted to have a greater probability of arriving at the stage of comprehension represented by terminal node 7, rather than terminal node 4 in a problem like that illustrated in **Figure 6**. The association between the transition of the answer characteristic at terminal nodes and total abilities in the case of **Figure 6** as an example is shown in **Figure 7**.

A learner's total comprehension can be grasped by inspecting whether the proportions of learners at each terminal node have the distribution shapes illustrated in **Figure 7**. One technique for visualizing the relationship between learners' understanding for each problem and the total score, as shown in **Figure 7**, is an analysis via an item characteristic chart used in the item analysis for multiple-choice questions. Thus, this study tries to understand the total comprehension of learners for each error type (segment) by applying an item characteristic chart.

Error analysis via item characteristic chart

The significance of considering an item characteristic chart for the analysis of error trends in multiple-choice questions has been demonstrated by Akiyama, Toyota and Iwama (2015). Akiyama, Toyota, and Iwata (2015) state that creating item characteristic curves results in parts that examinees tend to get wrong when their answers are classified into several categories. Since it is difficult to straightforwardly apply an analysis from an item characteristic chart to answers for a test with full descriptive-type questions, such an analysis has not been performed. However, in this study, by understanding the segments (terminal nodes) classified using regression trees as choices in multiple-choice questions, an analysis from the item characteristic chart will be applied to full descriptive questions.

An item characteristic chart can be created by dividing test examinees into approximately 5 groups ordered based on the total test score, with each comprising an approximately equal number of members. The selection rate for each answer category within a group along the vertical axis for the groups along the horizontal axis is plotted; the plotted choices of all groups are then connected with a straight line. For example, consider a multiple-choice question with 4 choices, A, B, C, and D, as explained in the item characteristic chart (**Figure**

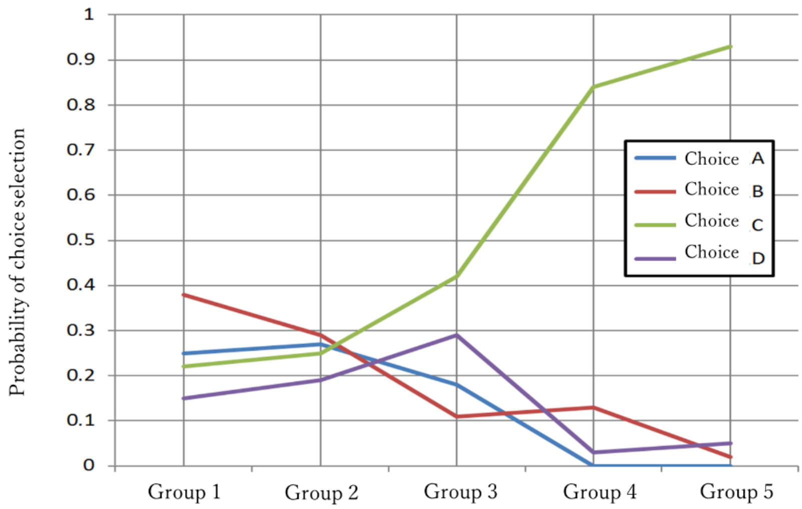


Figure 8. Item Characteristic Chart In 4-multiple-choice Question (Example)

8). Suppose the correct answer is choice C. The examinees were divided into five groups in ascending order according to the total test score.

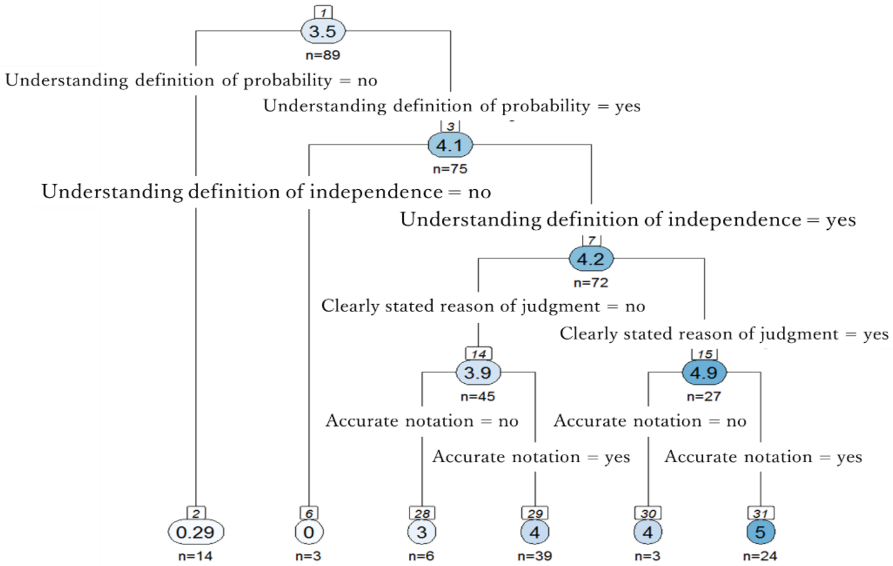
The curve that represents the selection rate of the correct answer is termed the correct response curve, and curves that represent the selection rates of wrong answers are termed wrong response curves. An analysis of the correct response curve allows us to measure the intent of the test creators as well as the adequacy of problems, by considering the position at which the correct response curve starts to become steep curve, as well as the slope. Since the correct response curve in **Figure 8** forms a steep curve between groups 3 and 4, it is thought to be particularly instrumental choice in discriminating the difference in abilities between groups 3 and 4. Regarding the wrong response curves, it is important to interpret why the wrong answers that are selected at high probabilities are indeed selected by the concerned group, based on the content of the wrong answer.

Segmentation by Problem of Learners in the Mid-term Examination and their Comprehension Analysis

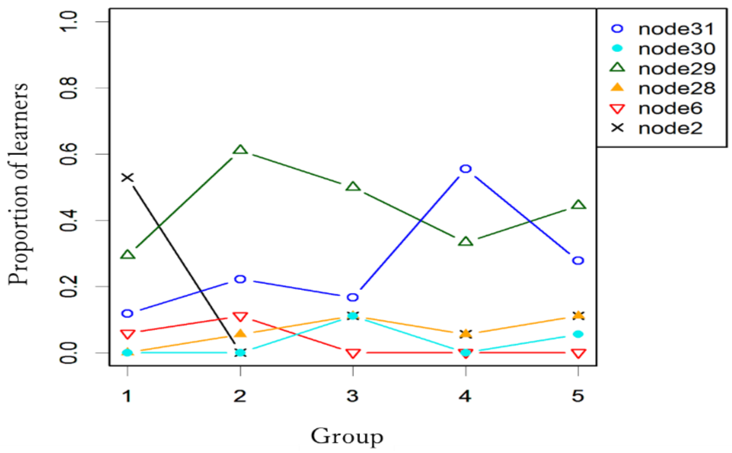
Results of analyses of learners' levels of comprehension based on the answer characteristic variables in Problem 1-(1) (independence, dependence) are shown as an example.

From **Figure 9** (1), learners were classified into 6 terminal nodes. Out of 89 learners (node 1), 75 (node 3) were able to answer correctly regarding the "understanding of probability definition," which represents the comprehension of definitions for concepts such as joint probability. Out of 75 leaners(node3), which 72 learners (node 7) answered correctly for the "understanding of formula for the definition of independence," which tests the comprehension of the concept of independence. Additionally, of the 72 learners who provided correct answers according to two answer characteristics: "understanding of probability definition" and "understanding of formula for the definition of independence," 27 learners (node 15) responded correctly for the "Clearly stated reason of judgment," which asks examinees to properly describe the reason for evaluating the case in the question as independent. At nodes 14 and 15, 39 (node 29) and 24 participants (node 31), respectively, provided a correct response for the "Accurate notation," which is designed to test the knowledge of symbols for concepts such as joint probability.

As shown in **Figure 9** (2), the distribution of learners for each group at each terminal node reveals that group 4 accounts for the highest proportion of learners at terminal node 31, who are considered to have understood the problem better than everyone else; the number declines with group 5. This means that the correct response rate for the "Clearly stated for reason of judgment" is reversed between the high score groups, groups 4 and 5, and is a type of errors that high-scoring learners in group 5 tend to get wrong, compared to those in group 4. Further, the proportion of learners who fail to demonstrate their grasp of "understanding of probability definition," an answer characteristic at terminal node 2, reveals a sharp decline from groups 1 to 2; no group 2 members exist in node 2, while some members from groups 3 to 5 are included. These learners



(1) Segmentation of Learners



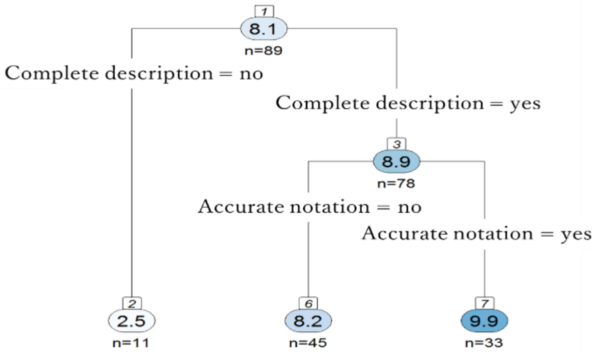
(2) Item Characteristic Chart

Figure 9. Segmentation of Learners in Problem 1-(1) in the Mid-term Examination and Item Characteristic Chart

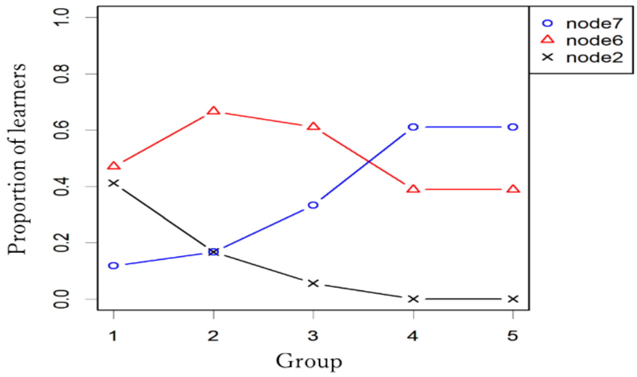
Table 6. Expressions of Terminal Nodes through Answer Characteristic Variables in Problem 1-(1) in the Mid-term Exam

Terminal node	The number of people	Answer characteristic variable			
		Understanding definition of probability	Understanding formula for definition of independence	Clearly stated reason of judgment	Accurate notation
2	14	×			
6	3	○	×		
28	6	○	○	×	×
29	39	○	○	×	○
30	3	○	○	○	×
31	24	○	○	○	○

may not have properly understood the definition of probability and even if they achieved higher scores, an improvement in their understanding is desired.



(1) Learner segmentation



(2) Item characteristic chart

Figure 10. Segmentation of Learners in the Problem 1-(1) in the Final Exam and Item Characteristic Chart

Table 7. Expressions of Terminal Nodes through Answer Characteristic Variables in Problem 1-(1) of the Final Exam

Terminal node	The number of people	Explanatory variable	
		Complete description	Accurate notation
2	11	×	
6	45	○	×
7	33	○	○

Segmentation by Problem of Learners in the Final Examination and their Comprehension Analysis

Results of the analysis of learners' comprehension based on the answer characteristic variables in Problem 1-(1) (maximum likelihood estimate) are shown below for illustration purposes.

As shown in **Figure 10** (1), the learners were divided into three terminal nodes. The answer characteristic that divides the entire group of 89 participants (Node 1) is "Complete description;" of the 78 learners who correctly answered this question (Node 3), 33 were also able to answer "accurate notation" correctly, as represented in terminal node 7.

To examine the proportion of members in each group in the terminal nodes shown in the item characteristic chart (**Figure 10**(2)), groups 4 and 5 are found to have higher proportions of learners who correctly responded to both "Complete description" and "Accurate notation" (Node 7). Further, the proportion of learners in terminal node 2 (who meet neither "Complete description" nor "Correctness of description") declines in the order of groups 1 to 4, and groups 4 and 5 are 0. Of all the learners, the proportion of learners who answered correctly for "Complete description" and incorrectly for "Accurate notation" (Node 6) was relatively high for all groups; 45 learners fall under this terminal node 6, with particularly high numbers from groups 2 and 3.

Table 8. Learning Policy According To Learners' Levels Of Comprehension Of Each Problem
(1) Mid-term examination

Problems	Terminal node	Learning policy 1	Learning policy 2	Learning policy 3
1-(1)	2	Understanding definition of probability		
	6	Understanding if formula for definition of independence		
	28	Accurate notation	Clearly stated reason of judgment	
	29	Clearly stated reason of judgment		
	30	Accurate notation		
1-(2)	2	Understanding definition of conditional probability		
	6	Understanding definition of conditional expectation		
	14	Accurate notation		
1-(3)	2	Understanding definition of expected values for two variables		
	6	Correct numerical calculation		
1-(4)	4	Understanding definition of expected value of a single variable	Understanding definition of variance of a single variable	
	5	Understanding definition of variance of a single variable		
	6	Correct numerical calculation		
2	2	Accurate reading of conditional probability		
3-(1)-1	4	Aware of object of proof	Understanding the property of continuous distribution	
	5	Understanding the property of continuous distribution		
	6	Complete description		
3-(1)-2	8	Understanding definition of variance	Aware of object of proof	Understanding the property of continuous distribution
	9	Aware of object of proof	Understanding the property of continuous distribution	
	5	Understanding the property of continuous distribution		
	12	Aware of object of proof	Complete description	
	13	Complete description		
	14	Understanding calculation rules		
3-(1)-3	2	Understanding definition of distribution function		
3-(2)	4	Complete description	Correct numerical calculation	
	10	Understanding standardization	Correct numerical calculation	
	11	Correct numerical calculation		
4-(1)	8	Understanding Constraint 2	Correct distribution formula manipulation	Understanding limit calculation
	9	Correct distribution formula manipulation	Understanding limit calculation	
	10	Understanding Constraint 1	Understanding limit calculation	
	11	Understanding limit calculation		
	6	Complete description		
4-(2-1)	2	Understanding definition of probability function		
	6	Accurate reading of distributed information		
	14	Accurate notation		
	30	Understanding distribution parameters		
4-(2-2)	2	Correct numerical calculation of second term		
	12	Correct numerical calculation of first term	Understanding definition of discrete distribution probability	
	13	Understanding definition of discrete distribution probability		

Learning Policy According to Learners' Level of Comprehension of Each Problem

A summary of the answer characteristics for each problem in the mid-term and final examinations that characterizes the terminal nodes drawn from the regression tree analysis is shown in **Table 8** as the learning policies for learners at each terminal node.

Table 8 (continued). Learning Policy According To Learners' Levels Of Comprehension Of Each Problem
(2) Final examination

Problem	Terminal node	Learning policy 1	Learning policy 2	Learning policy 3
1-(1)	2	Complete description		
	6	Accurate notation		
1-(2-1)	4	Understanding definition of unbiasedness	Complete description	
	5	Complete description		
	6	Understanding definition of expected value		
	14	Accurate notation		
1-(2-2)	8	Aware of object of proof	Accurate notation	Correct formula manipulation
	9	Accurate notation	Correct formula manipulation	
	10	Complete description	Correct formula manipulation	
	11	Correct formula manipulation		
	6	Aware of object of proof		
	14	Complete description		
1-(2-3)	4	Accurate notation	Correct formula manipulation	
	5	Correct formula manipulation		
	12	Understanding Chebyshev's inequality	Complete description	
	13	Complete description		
	14	Accurate notation		
2-(1)	4	Understanding definition of errors in two-sided test	Understanding definition of power in two-sided test	
	10	Understanding situations where power is increased	Understanding definition of power in two-sided test	
	11	Understanding definition of power in two-sided test		
	12	Correct graphical demonstration of errors in test	Understanding situations where power is increased	
	13	Understanding situations where power is increased		
	14	Correct graphical demonstration of errors in test		
2-(2)	8	Correct graphical demonstration of errors in test	Understanding definition of power in one-sided test	Understanding definition of errors of one-sided test
	9	Understanding definition of power in one-sided test	Understanding definition of power in one-sided test	
	5	Understanding definition of power in one-sided test		
	12	Correct graphical demonstration of errors in test	Understanding difference between power in two-sided test and power in one-sided test	
	13	Understanding difference between power in two-sided test and power in one-sided test		
	14	Correct graphical demonstration of errors in test		
3-(1)	4	Aware of object of derivation	Description of distribution that the statistics follows	
	5	Description of distribution that the statistics follows		
	6	Understanding derivation rules		
3-(2)	4	Understanding definition of unbiased variance	Accurate reading of percentage points in chi-square distribution	
	5	Accurate reading of percentage points in chi-square distribution		
	6	Understanding definition of confidence interval		
	14	Correct numerical calculation		
3-(3)	2	Understanding definition of confidence interval		
	6	Accurate reading of percentage points in t-distribution		
	14	Correct numerical calculation		
4-(1)	8	Clearly stated test hypothesis	Understanding definition of confidence interval	Understanding definition of test statistic
	9	Understanding definition of confidence interval	Understanding definition of test statistic	
	10	Clearly stated test hypothesis	Understanding definition of test statistic	
	11	Understanding definition of test statistic		
	6	Clearly stated test hypothesis		
	14	Understanding how to calculate percentage points in F distribution		
4-(2)	8	Understanding definition of pooled unbiased variance	Accurate reading of percentage points in t-distribution	Understanding definition of test statistic
	9	Accurate reading of percentage points in t-distribution	Understanding definition of test statistic	
	5	Understanding definition of test statistic		
	12	Accurate reading of percentage points in t-distribution	Correct numerical calculation	
	13	Correct numerical calculation		
	14	Accurate reading of percentage points in t-distribution		

ANALYSIS OF THE RELATIONSHIP BETWEEN LEARNING POLICY AND ANSWER CHARACTERISTICS FOP QUALITATIVE IMPROVEMENT IN LEARNERS' LEARNING OF PROBABILITY AND STATISTICS

In this chapter, the relationship between the answer characteristics and the learning policy obtained from the questionnaire survey will be analyzed using Bayesian networks to individually derive the learning policies that are useful in improving errors made by learners. Chen, Feng, Hu and Sun (2019) is to construct a Bayesian network to make causal analysis and then provide personalized interventions for different learners to improve learning. However, they do not model the relationship between the questionnaire survey and the test answer characteristics based on error analysis.

Binarization of Learning Strategies based on Questionnaire Survey Results

As shown in **Table 1** in Chapter 2, the questionnaire survey administered in this study consists of 29 items (16+6+7 items) comprising 5-point scale questions (16 items), numerical input questions (6 items), and the selective question with 7 choices that allow multiple answers is considered as 7 yes-or-no questions. Amongst the latter type of question, all respondents chose No (0: No) for the questions "Video material" and "Others," which represent the materials used. Thus, they were excluded and the remaining 27 items were designated as the variables for the learning strategies.

In this chapter, the learning strategy variables will be binarized to analyze the relationship between the learning strategies and the answer characteristics using Bayesian network models. Regarding 16 questions evaluated on a 5-point scale in **Table 1**, 3 negative answers to the question, 1: Strongly disagree, 2: Disagree, and 3: Undecided, were regarded as 0, and 2 positive answers, 4: Agree, 5: Strongly agree, were regarded as 1. Regarding 6 questions of the numerical input type in **Table 1**, the mean value is understood as a point of reference, since values less than the mean were regarded as 0, and the mean value and those greater than the mean were regarded as 1.

Extraction of Learning Strategies Using Probabilistic Reasoning through Bayesian Network Models

This study proposes a method to quantitatively detect and evaluate learning strategies that are instrumental in eliminating mistakes and qualitatively improving learning by building a Bayesian network model that relates answer characteristics to the learning strategy and answer characteristics for each question and makes a probabilistic reasoning for each individual learner.

Regarding the Bayesian networks, refer to Scutari and Denis (2014), Lqbal, Yin, Hao, IIyas and Ali (2015). In the Bayesian network model M, consider the learning strategy X, which is in direct probabilistic relation to characteristic Y, as the learning strategy that can be deemed useful for learners as a whole by evaluating whether the value of D in the following formula is positive or negative, in addition to its quantity.

$$D = P(Y = 1|X = 1,\ M) - P(Y = 1|X = 0,\ M) \quad (7)$$

The use of the learning strategy X, which causes the value of D in formula (7) to be positive, is considered to promote understanding of the learned content, as represented in the answer characteristic Y, as well as the required improvement in associated skills. Moreover, if there is more than one learning strategy X for a single answer characteristic Y, which causes the solution of formula (7) to be positive, the learning strategy X, which maximizes the value of D in formula (7), is deemed most effective.

Extraction of Learning Strategies for Qualitative Improvement in Learning for Learners in the Mid-term Exam

In the structural search for Bayesian network models, an AIC-based Greedy Search algorithm included in the "bnlearn" R package was adopted. Here, to obtain a better model while preventing local optimization, the structural search for Bayesian network models was repeated 1000 times while randomly changing the initial values, and a model minimizing the AIC was adopted.

Let us explain the constraints imposed on the model structure of the Bayesian network model in the mid-term examination. First, it is reasonable to assume that the comprehension of the learned content, as well as the mastery of associated skills as represented by the answer characteristic variables, is a consequence of learning using the strategies represented by the learning strategy variables. It is therefore inappropriate to

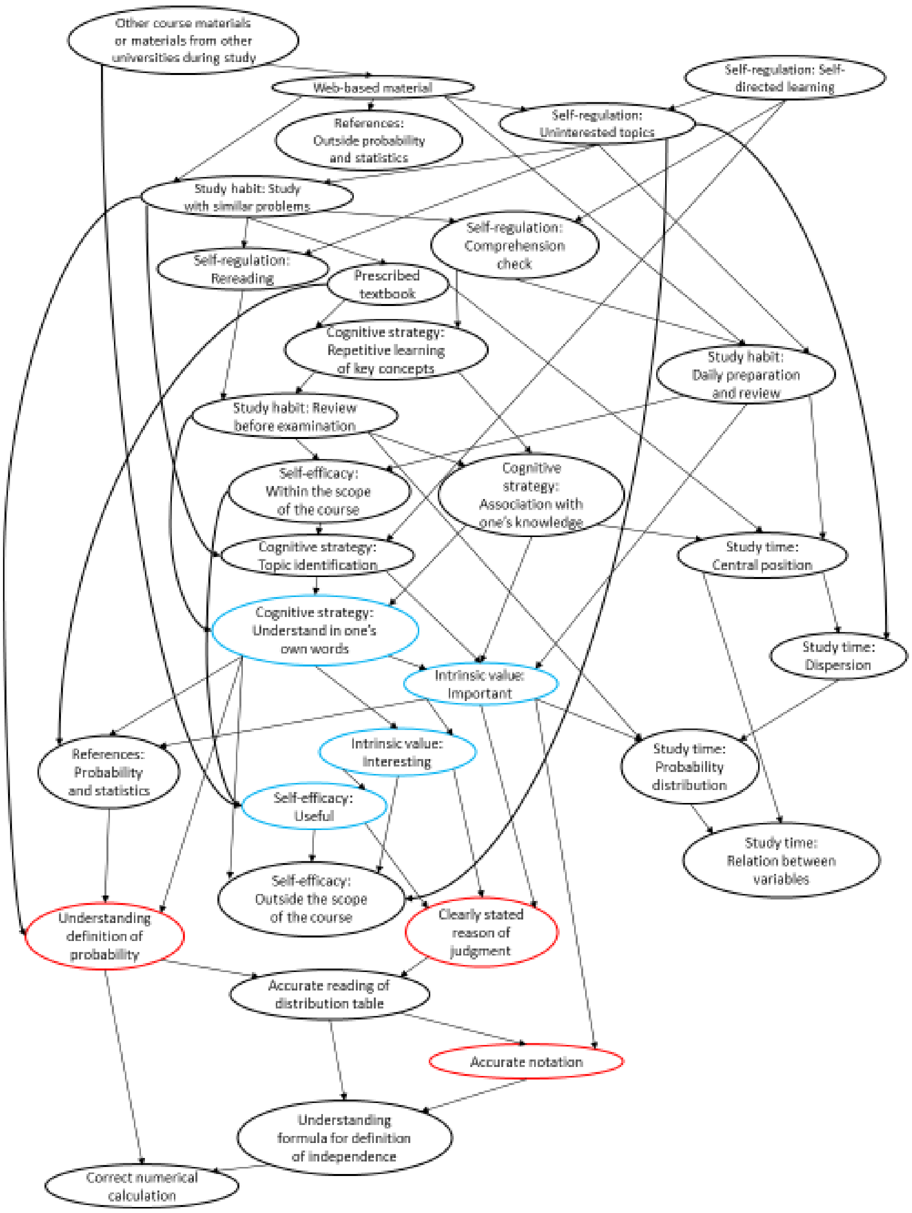


Figure 11. Structuration Of A Bayesian Network For The Answer Characteristics And Learning Strategies Of Problem 1-(1)

use the type of models with an edge structure in which the answer characteristic variable occupies the parent node and the learning strategy variable occupies the child node; hence, a constraint is imposed on all answer characteristic variables by disallowing them from possessing learning strategy variables as their child nodes. Regarding the two learning strategy variables, "Study time: Statistical estimation" and "Study time: Statistical testing," both of which represent the hours of study spent on topics tested in the final examination, it is appropriate to conclude that they do not affect learners' mid-term examination results. Thus, of all 27 parameters that express learning strategies, two variables, "Study time: Statistical estimation," and "Study time: Statistical testing", were excluded, which reduced the number of learning strategy variables to 25.

In the figure of the Bayesian network model below, the nodes in the blue and red boxes are learning strategy variables and answer characteristic variables, respectively, which were observed to have a direct positive probabilistic dependent relationship.

Here, the results of extracting the learning strategies to improve the answer characteristics in Problem 1-(1) (independence, dependence) are shown as an example.

Table 9. Conditional Probabilities At The Edges That Were Observed To Have Direct, Positive Probabilistic Dependent Relationships With The Answer Characteristic Variables

Learning strategy variable X	Answer characteristic variable Y	D = P(Y=1|X=1, M) − P(Y=1|X=0, M)
Cognitive strategy: Understand in one's own words	Understanding of probability definition	0.125
Intrinsic value: Important	Accurate notation	0.169
Intrinsic value: Important	Clearly stated reason of judgment	0.078
Intrinsic value: Interesting	Clearly stated reason of judgment	0.178
Self-efficacy: Useful	Clearly stated reason of judgment	0.477

Of all pairs of nodes observed to have a direct, probabilistic dependent relationship from the learning strategy variable to the answer characteristic variable, those with a positive relationship are shown in **Table 9**. For "Understanding of probability definition," which expresses the most basic understanding of this Problem, "Cognitive strategy: Understand in one's own words," i.e., a strategy of studying basic concepts in one's own words, is suggested as effective (D=0.125). An awareness of the significance of learning probability and statistics as expressed by "Intrinsic value: Important" is considered to affect the understanding of "Accurate notation" and "Clearly stated reason of judgment" (D=0.169, D=0.078). Furthermore, "Clearly stated for reason of judgment" is most probabilistically affected by "Self-efficacy: Useful" (D=0.477), delineating the importance of an actual awareness of the usefulness of probability and statistics.

As for problems in the mid-term examination, of all pairs of nodes observed to have a direct, probabilistic dependent relationship from the learning strategy variable to the answer characteristic variable, those with a positive relationship are summarized in **Table 10** to examine the relationships between the learning strategies and answer characteristics.

Regarding the learning strategy "Self-efficacy," "Self-efficacy: Useful" was revealed as effective for "Clearly stated reason of judgment" in Problem 1-(1) and "Correct numerical calculation" in Problems 1-(2) and 2. These answer characteristics can be answered correctly through an understanding of basic learning content for problems which mainly deal with relationships between probabilistic parameters. Hence, these relationships suggest the importance of having self confidence that, for further improvement on the understanding of fundamental concepts learned regarding the relationship between the probabilistic variables, probability and statistics can be properly applied to the actual states of affairs. Meanwhile, "Self-efficacy: Within the scope of the course" and "Self-efficacy: Outside the scope of the course" are the learning strategies considered effective for answer characteristics regarding the properties of the probability distribution, including "Aware object of proof" in Problem 3-(1)-1, which mainly deals with probability distribution, "Understanding properties of continuous distribution" in Problem 3-(1)-2, and "Understanding properties of normal distribution" in Problem 3-(1)-3.

As for the learning strategy "Intrinsic value," "Intrinsic value: Important" was suggested as an effective learning strategy for "Accurate notation" and "Clearly stated reason of judgment" in Problem 1-(1), which mainly deals with relationships between probability variables, as well as for "Properties of continuous distribution" in Problem 3-(1)-1, which deals with probability distribution, and "Aware object of proof" in Problem 3-(1)-2; meanwhile, "Intrinsic value: Interesting" was revealed as effective for Problems 1-(1) and 1-(2), which mainly deal with relationships between probability variables.

In terms of the learning strategy "Cognitive strategy," "Cognitive strategy: Understand in one's own words" was revealed as an effective learning strategy, much like "Self-efficacy: Useful," for the improvement of answer characteristics in Problems 1-(1), 1-(2), and 2. However, the answer characteristics for which the use of "Cognitive strategy: Understand in one's own words" is revealed as effective are "Understanding of probability definition," "Understanding definition of conditional probability," and "Understanding definition of normalized constant," which implies that this strategy is effective for understanding definitions at the initial stage of learning. This contrasts with "Self-efficacy: Useful," which is effective in promoting further improvement after a given learner understands the basics required to solve problems. "Cognitive strategy: Identification of content" was shown as effective for "Notation" in Problem 2, "Aware object of proof" in Problem 4-(1), and "Correctness of second item calculation" in Problem 4-(2-2). "Aware object of proof" in Problem 4-(1) is a variable that represents the accurate awareness of the relationship between the binominal distribution and Poisson distribution, which is the object of the proof, and the association of learned contents is interpreted as a product of the application of the learning strategy to identify the content.

Table 10. The Relationship Between Effective Learning Strategies And Answer Characteristics In The Mid-term Examination

	Problem											
	1-(1)	**1-(2)**	**1-(3)**	**1-(4)**	**2**	**3-(1)-1**	**3-(1)-2**	**3-(1)-3**	**3-(2)**	**4-(1)**	**4-(2-1)**	**4-(2-2)**
Self-efficacy: Useful	Clearly stated reason of judgment	Correct Numerical calculation			Correct Numerical calculation							
Self-efficacy: Within the scope of the course						Aware of object of proof	Understand Properties of continuous distribution					
Self-efficacy: Outside the scope of the course								Understand Properties of Normal distribution				
Intrinsic value: Important	Accurate notation clearly stated reason of judgment					Understand properties of continuous distribution	Aware of object of proof					
Intrinsic value: Interesting	Clearly stated reason of judgment	Understand definition of conditional probability Accurate reading of the distribution table										
Cognitive strategy: Understand in one's own words	Understand definition of probability	Understand definition of conditional probability			Understand definition of normalized constants							
Cognitive strategy: Identification of content					Accurate notation					Aware of object of proof		Correct second item calculation
Cognitive strategy: Association with one's knowledge					Complete description Accurate reading of prior probability			Understand properties of normal distribution				
Cognitive strategy: Repetitive learning of key concepts			Understand definition of expected values of two variables							Aware of object of proof		
Self-regulation: Comprehension check				Understand definition of variance of two variables			Aware of object of proof			Understand Constraint 1		
Self-regulation: Self-directed learning		Accurate notation										
Self-regulation: Rereading						Understand properties of continuous distribution	Complete description					
Self-regulation: Uninterested topics						Complete description		Accurate graphic drawing				
Study habit: Study with similar problems						Understand calculation rules						

Table 10 (continued). The Relationship Between Effective Learning Strategies And Answer Characteristics In The Mid-term Examination

	Problem											
	1-(1)	1-(2)	1-(3)	1-(4)	2	3-(1)-1	3-(1)-2	3-(1)-3	3-(2)	4-(1)	4-(2-1)	4-(2-2)
Study time: Central position											Understand definition of probability function	Correct first item calculation
Study time: Dispersion			Correct numerical calculation		Accurate reading of conditional probability		Understand definition of variance					
Study time: Probability distribution								Understand properties of normal distribution		Correct distribution formula manipulation	Accurate notation	
References: Probability and statistics											Accurate reading of distributed information	
References: Outside probability and statistics							Complete description					
Web-based material		Understand definition of conditional expectation								Complete description		
Other course materials or materials from other universities												Understand definition of discrete distribution probability

As for the learning strategy "Self-regulation," "Self-regulation. Comprehension check" is considered an effective learning strategy for a range of answer characteristics, including "Understanding definition of variance of two variables" in Problem 1-(4), which mainly deals with the expected value and variance of two variables, "Aware object of proof" in Problem 3-(1)-2; which relates to the probability distribution and "Understanding constraint 1" in Problem 4-(1).

Of all 4 learning strategy variables that express the study time, "Study time: Dispersion" and "Study time: Probability distribution" were extracted for a range of answer characteristics, the study time indicating that for dispersion and probability distribution was essential. Problems 3 and 4, which deal with probability distribution, are the problems for which "Study time: Probability distribution" was extracted as an effective learning strategy, demonstrating that study time directly affects the understanding of the subject matter. Conversely, "Study time: Dispersion" was extracted for Problem 1-(3), which is a calculation of the expected values of two variables, for Problem 2 on the Bayesian theorem, and for Problem 3-(1)-2, which mainly deals with variance. This highlights the particular importance of study time for dispersion in the study of probability and statistics.

Learning Strategy Extraction for Qualitative Improvement of Learning in the Final Examination

Regarding the Bayesian model used for the final examination, the constraints imposed on the model structure are explained below. Similar to the case of the mid-term examination, since the type of models with an edge structure in which the answer characteristic variable occupies the parent node and the learning strategy variable occupies the child node were deemed inappropriate, constraints were imposed on all answer characteristic variables by disallowing them from having learning strategy variables as their child nodes. Moreover, regarding the learning strategy variables, constraints were imposed on the "Study time: Statistical estimation" and "Study time: Statistical testing" for the study content learned in the second half such that they could not possess "Study time: Central position," "Study time: Dispersion," "Study time: Probability distribution," and "Study time: Relationship between variables" of mid-term examination topics as their child nodes.

Here, the results of extracting the learning strategies to improve the answer characteristics in Problem 1-(1) (maximum likelihood estimate) are shown as an example.

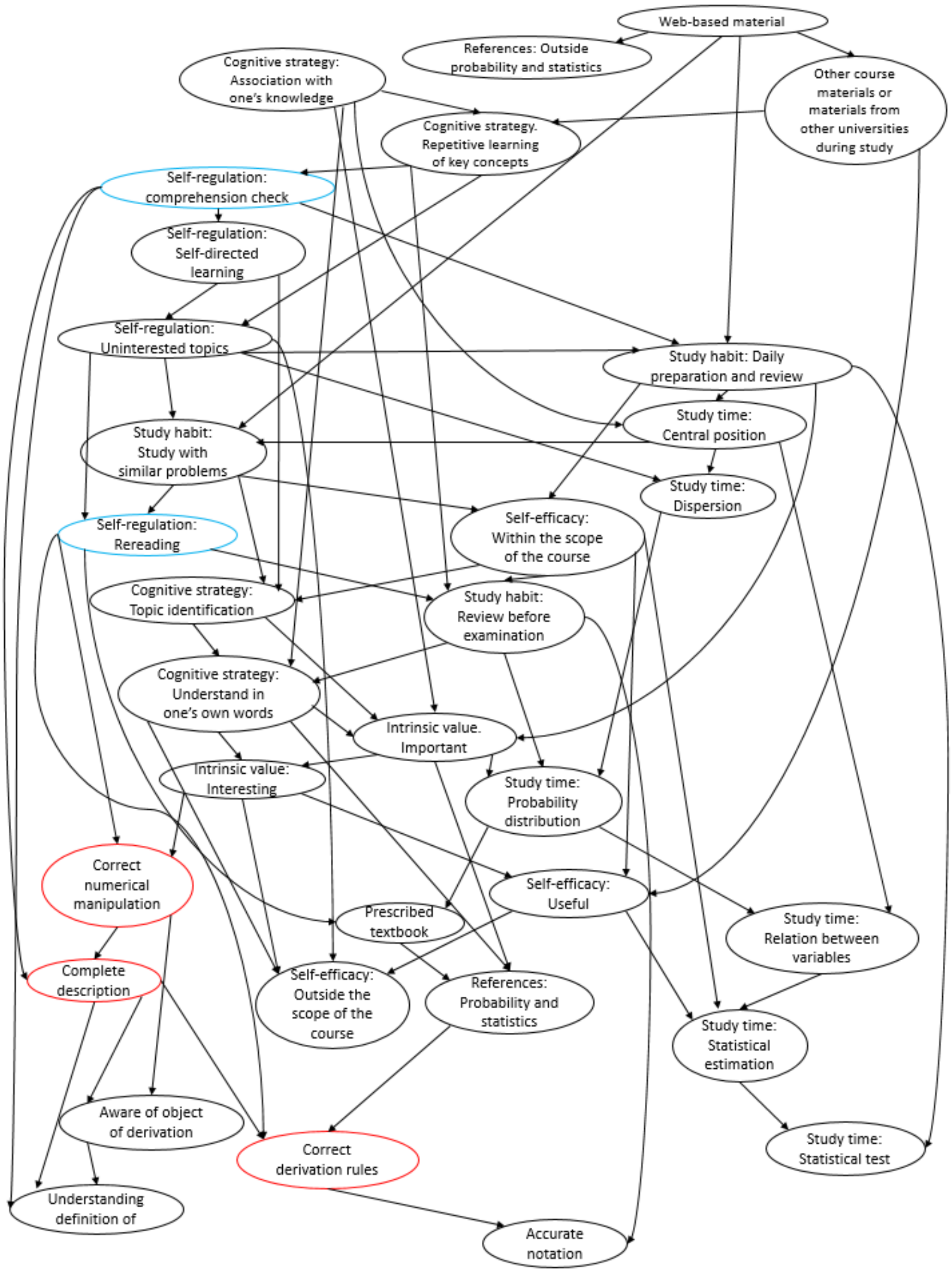


Figure 12. Structuration of a Bayesian Network for Answer Characteristics and Learning Strategies of Problem 1-(1)

Of all the pairs of nodes that were observed to have a direct, probabilistic dependence relationship from the learning strategy variable to the answer characteristic variable, those with a positive relationship are shown in **Table 11**. Regarding "Complete description," which expresses the completeness of the description for derivation, the importance of "Self-regulation. Comprehension check," that is, the importance of pursuing learning while checking one's comprehension, was suggested (D=0.180). Regarding "Correct formula manipulation," which expresses correct formula manipulation during the process of derivation, and "Understanding derivation rules," which expresses the learner's proper understanding of the derivation rules, "Self-regulation. Rereading," i.e., pursuit of learning by reviewing what has been learned (D=0.128, 0.222) was found to be important.

Table 11. Difference between Conditional Probabilities at Edges that were Observed to have Direct, Probabilistic Dependence Relationships with the Answer Characteristic Variables

Learning strategy variable X	Answer characteristic variable Y	D = P(Y=1|X=1, M) - P(Y=1|X=0, M)
Self-regulation. Comprehension check	Complete description	0.180
Self-regulation. Rereading	Correct formula manipulation	0.128
Self-regulation. Rereading	Understanding derivation rules	0.222

Table 12 shows that the learning strategy "Self-efficacy: Outside the scope of the course" was effective for "Complete description" in Problem 1-(2-2), and "Description of Distribution that the statistic follow to," "Understanding definition of confidence interval," and "Accurate reading of percentage points in t-distribution" in Problem 3. Seen from each Problem, this is considered important primarily for the understanding of statistical estimation. Problems 1-(2-2) and 3-(1) are shown as effective for answer characteristics concerning description, which are therefore thought to comprise an important strategy for devising complete expressions.

The learning strategy "Cognitive strategy: Identification of content" is shown as effective for the answer characteristics in Problem 1-(2-1), which is concerned with unbiased estimates, Problem 2-(1), which is concerned with the concept of a two-sided test, and Problem 4-(1), for testing the equality of variance. By contrast, "Cognitive strategy: Association with one's knowledge" was shown as effective in improving answer characteristics in Problems 2-(1) and 4-(2), both of which are concerned with the testing of the hypothesis. Unlike "Cognitive strategy: Identification of content," it is considered effective for answer characteristics on relatively basic matters such as "Understanding definition of error in two-sided test," and "Understanding definition of test statistics." "Cognitive strategy: Repetitive learning of key concepts" is considered an effective strategy across a wide range of questions, including "Understanding definition of Chebyshev's inequality" in Problem 1-(2-3), "Understanding definition of two-sided test," "Understanding definition of one-sided test" in Problems 2-(1) and (2), and "Aware of object of derivation" in Problem 3-(1).

Of the six learning strategy variables for study time, the study time for contents in the Central position, Dispersion, and Probability distribution, which were learned before the mid-term exam, and the study time

Table 12. The Correspondence between the Effective Learning Strategies and Answer Characteristics in the Final Examination

	Problem										
	1-(1)	1-(2-1)	1-(2-2)	1-(2-3)	2-(1)	2-(2)	3-(1)	3-(2)	3-(3)	4-(1)	4-(2)
Self-efficacy: Within the scope of the course				Accurate notation							
Self-efficacy: Outside the scope of the course			Complete description				Description of Distribution that the statistic follows	Understanding definition of confidence interval	Accurate reading of percentage points in t-distribution		
Cognitive strategy: Understand in one's own words										Understanding how to calculate percentage points in F distribution	
Cognitive strategy: Identification of content		Accurate notation			Correct graphical demonstration of errors of test					Understanding definition of confidence interval	
Cognitive strategy: Association with one's knowledge					Understanding definition of errors in two-sided test						Understanding definition of testing statistics
Cognitive strategy: Repetitive learning of key concepts				Understanding Chebyshev's inequality	Understanding definition of errors in two-sided test	Understanding definition of errors in one-sided test	Aware of object of derivation				
Self-regulation: Comprehension check	Complete description					Correct graphical demonstration of power					
Self-regulation: Self-directed learning									Correct numerical calculation		

Table 12 (continued). The Correspondence between the Effective Learning Strategies and Answer Characteristics in the Final Examination

	Problem										
	1-(1)	**1-(2-1)**	**1-(2-2)**	**1-(2-3)**	**2-(1)**	**2-(2)**	**3-(1)**	**3-(2)**	**3-(3)**	**4-(1)**	**4-(2)**
Self-regulation: Rereading	Correct formula manipulation Correct derivation rules	Understanding definition of expected value									
Study habit: Daily preparation and review						Correct graphical demonstration of errors of test					
Study habit: Study with similar problems			Explaining grounds for minimum variance Aware of object of proof							Understanding definition of errors in two-sided test	Accurate reading of percentage points in t-distribution
Study time: Central position						Understanding definition of power in one-sided test				Understanding definition of confidence interval	
Study time: Dispersion								Aware of object of derivation		Clearly stated test hypothesis	
Study time: Probability distribution			Complete description								
Study time: Statistical estimation			Correct formula manipulation		Understanding situations where power is increased	Understanding definition of errors in one-sided test					
Web-based material				Accurate notation			Aware of object of derivation			Understanding definition of errors in two-sided test	Understanding definition of pooled unbiased variance
Other course materials or materials from other universities					Correct graphical demonstration of errors of test						

for contents concerning statistical test, which were learned later, were confirmed as effective for improving the answer characteristics. "Study time: Statistical test" was observed as effective for Problem 1-(2-2), which deals with the minimum variance of estimator, and for "Understanding situations where power is elevated" and "Understanding definition of error in one-sided test," which are the answer characteristics of Problems 2-(1) and (2), respectively, for the concept of testing a hypothesis. The study time spent on hypothesis testing is therefore considered particularly effective for understanding the testing errors, which comprise one of the most fundamental issues in hypothesis test.

Regarding the learning strategy variables of the study materials used, the "Web-based material" was revealed as effective across a range of questions, including Problems 1-(2-3), 3-(1), 4-(1), and 4-(2).

Learning Strategy Extraction using Individual Probabilistic Reasonings based on the Bayesian Network Model

As a next step, a probabilistic reasoning conditioned on the comprehension of each learner and his use of a given learning strategy was performed individually, to quantitatively assess any learning improvements that may prove effective for learner. Here, suppose that a learner i fails to correctly answer the question q in the fth answer characteristic $Y_{i,q,f}$. Further, suppose that learning strategy $\boldsymbol{X}_i$ of the learner i is represented as

$$\boldsymbol{X}_i = [x_{i,1}, \dots, x_{i,m-1}, 0, x_{i,m+1}, \dots, x_{i,r}]$$

and element $X_{i,m}$ of the learning strategy is 0. In addition, let answer characteristics other than f-th answer characteristic $Y_{i,q,f}$ of the learner i be expressed as

$$\boldsymbol{Y}_{i,q}(-f) = [Y_{i,q,1}, \dots, Y_{i,q,f-1}, Y_{i,q,f+1}, \dots, Y_{i,q,p}]$$

Then, the probability that the learner i will correctly answer the f-th answer characteristic $Y_{i,q,f}$ under the learning strategy he or she has currently adopted can be expressed as

$$P(Y_{i,q}, f = 1 | X_i,\ \ Y_{i,q} = (-f)) \tag{8}$$

Table 13. Results of Probabilistic Reasonings for Student 07

	Understanding the definition formula for independence	Understanding definition of probability	Accurate reading of distribution table	Correct numerical calculation	Clearly stated reason for judgement	Accurate notation
Self-efficacy: Useful					0.875	0.000
Self-efficacy: Within the scope of the course					0.000	0.000
Self-efficacy: Outside the scope of the course					0.000	0.000
Intrinsic value: Important					0.208	0.200
Intrinsic value: Interesting					0.330	0.000
Cognitive strategy: Understand in one's own words						
Cognitive strategy: Identification of content					0.000	0.000
Cognitive strategy: Association with one's knowledge						
Cognitive strategy: Repetitive learning of key concepts						
Self-regulation: Comprehension check						
Self-regulation: Self-directed learning					0.000	0.000
Self-regulation: Rereading					0.000	0.000
Self-regulation: Uninterested topics					0.000	0.000
Study habit: Daily preparation and review					0.000	0.000
Study habit: Review before examination						
Study habit: Study with similar problems					0.000	-0.165
Study time: Central position						
Study time: Dispersion					0.000	0.000
Study time: Probability distribution					0.000	0.000
Study time: Relation between variables					0.000	0.000
Prescribed textbook						
References: Probability and statistics					0.000	0.000
References: Outside probability and statistics					0.000	0.000
Web-based material						
Other course materials or materials from other universities during study					0.000	0.000

Here, if learner i improves the element $X_{i,m}$ of the learning strategy and the learning strategy at this point is expressed as $\boldsymbol{X}_i'(x_{i,m} = 1)$, then we obtain

$$X_i' = [x_{i,1}, \dots , x_{i,m-1}, 1, x_{i,m+1}, \dots , x_{i,r}] \tag{9}$$

and the probability that learner i will correctly answer the f-th answer characteristic $Y_{i,q,f}$ under the condition that he or she improved on the m-th learning strategy is as follows.

$$P(Y_{i,q,f} = 1 | Xi'(x_{i,m} = 1),\ \ Y_{i,q} =(-f)) \tag{10}$$

Then, if we take the difference between formulas (10) and (8)

$$P(Y_{i,q,f} = 1 | Xi'(x_{i,m} = 1),\ \ Y_{i,q} =(-f)) - P(Y_{i,q,f} =1 | X_i,\ \ Y_{i,q} =(-f)) \tag{11}$$

then the learning strategy m, which is effective for improving the element f of the learning characteristic in which learner i is interested, would be detected and assessed.

From the mid-term examination, an example of results related to probabilistic reasoning given changes tailored for a learner in the conditions of the learning strategy and answer characteristic parameters is shown in **Table 13**. Since the learning strategies that had already been used and answer variables that the learner had gotten right did not require improvement, diagonal lines were drawn in the relevant cells.

Student 07 failed to answer "Clearly stated reason of judgment" and "Correctness of notation" correctly. The probability that this student will give a correct answer for "Clearly stated reason of judgment" correctly with the current learning strategy and level of comprehension is expected to increase by approximately 0.875

Table 14. Results of Probabilistic Reasonings for Student 12

	Understanding the definition of likelihood	Correct numerical manipulation	Accurate natation	Complete description	Awareness of object of derivation	Correct derivation rules
Self-efficacy: Useful			0.000			0.000
Self-efficacy: Within the scope of the course			0.000			0.000
Self-efficacy: Outside the scope of the course			0.000			0.000
Intrinsic value: Important			0.000			0.000
Intrinsic value: Interesting			-0.065			-0.175
Cognitive strategy: Understand in one's own words						
Cognitive strategy: Identification of content			0.000			0.000
Cognitive strategy: Association with one's knowledge			0.000			0.000
Cognitive strategy: Repetitive learning of key concepts						
Self-regulation: Comprehension check			0.056			0.152
Self-regulation: Self-directed learning			0.000			0.000
Self-regulation: Rereading			0.092			0.249
Self-regulation: Uninterested topics			0.000			0.000
Study habit: Daily preparation and review			0.000			0.000
Study habit: Review before examination						
Study habit: Study with similar problems			0.000			0.000
Study time: Central position						
Study time: Dispersion						
Study time: Probability distribution						
Study time: Relation between variables			0.000			0.000
Study time: Statistical estimation						
Study time: Statistical test			0.000			0.000
Prescribed textbook						
References: Probability and statistics			0.032			0.086
References: Outside probability and statistics			0.000			0.000
Web-based material						
Other course materials or materials from other universities during study			0.000			0.000

with improvement in "Self-efficacy: Useful," by approximately 0.208 with improvement in "Intrinsic value: Important," by approximately 0.300 with improvement in "Intrinsic value: Interesting." Moreover, the probability that he will give a correct answer for "Accurate notation" is expected to increase by about 0.2, with an improvement in "Intrinsic value: Important."

The example of the final examination is shown in **Table 14**.

Student 12 failed to give a correct answer for "Accurate notation" and the "Correctness of derivation rules." The probability that this student will give a correct answer for "Accurate notation" and the "Correctness of derivation rules" under the current learning strategy and level of comprehension is expected to increase by approximately 0.056 and 0.152, respectively, with improvements in "Self-regulation. Comprehension check," and by approximately 0.092 and 0.249 with improvements in "Self-regulation. Rereading," by approximately 0.032 and 0.086, with the supportive use of "References: Probability and Statistics."

SUMMARY

This study propose the method to make variables of the level of comprehension of learners as answer characteristics in a class of probability and statistics based on actual student mistakes in answers to full descriptive questions; and an analytical method was proposed and examined that aimed to qualitatively improve learners' skills by giving feedback to both the educators and the learners. First, the method of making variables of answer characteristics from errors of answers in the mid-term and final examinations in probability and statistics was proposed and examined. Second, a regression tree analysis was conducted to extract answer characteristics that depict learners' level of comprehension for each question, in addition to providing a possible study policy to improve comprehension of the question for learners. Furthermore, item characteristic charts were used to visualize the relationship between comprehensive ability and the answer characteristics and a comprehensive analysis of learner comprehension levels requiring improvement was carried out to provide feedback for educators. Additionally, the relationship between the students' learning strategy and the answer characteristic was structured for each question using a BN model, and learning strategies effective for qualitative improvement in comprehension for both learners as a whole and as individuals were extracted and evaluated. Our findings suggested the presences of effective learning strategies separately for each concept in each question, as well as for each level of expertise, whether it was basic (understanding definition, etc.) or applied (Aware object of derivation, Complete description, etc.), in answer characteristics. This indicates that a learning strategy's importance differs according to the learning stage. Furthermore, improvement in dispersion based on study time was revealed to be effective in a wide range of questions in both the mid-term and final examinations.

Disclosure statement

No potential conflict of interest was reported by the authors.

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APPENDIX

1) Mid-term Examination

Problem 2:
Suppose that there are four AI (artificial intelligence) robots (A, B, C and D) that deliver some services. The AI robots A, B, C and D deliver 20%, 25%, 25% and 30% of the entire services respectively and out of services they deliver, they are empirically known to carry out inadequate services at 3%, 3%, 2% and 1% each. When sampling just one service out of the entire services, calculate the probability that it is an inadequate service. Furthermore, when it is found to be inadequate, calculate the probability that the service was carried out by the AI robot D (Round up to 4 decimal places).

Question 3:
(1) Calculate the mean and variance of the random variable X that obeys the normal distribution N (μ, σ2). In addition, find the distribution function of this normal distribution. Note that the probability density function for the normal distribution N (μ, σ2) is as follows.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

(2) Calculate P(formula omitted) for the random variable X that follows the normal distribution N (μ , σ 2). (Round up to 4 decimal places.) The standard normal distribution table is shown in the right page.

Question 4:
(1) Prove that the Poisson distribution is an extreme distribution of binomial distribution. That is, in the binomial distribution {b(k; n, p)}, when n → ∞ while keeping np=λ (λ: constant) constant, prove that the following obtains.

$$b(k; n, p) \to p(k; \lambda)$$

Here, the probability distribution of the binomial distribution {b(k; n, p)} can be expressed as

$$P(X = k) = b(k; n, p) = \binom{n}{k} p^k q^{n-k} \ (k = 0,1,2, \dots, n)$$

Likewise, the probability distribution of the Poisson distribution {p(k; λ)} can be expressed as

$$p(k; \lambda) = e^{-\lambda} \frac{\lambda^k}{k!} \ (k = 0,1,2, \dots ; \lambda > 0)$$

(2) In a service delivery field, services are carried out 100 times a day, of which about 2% are inadequate services.
(2-1) In this service field, suppose that the number of inadequate services for tomorrow is Y, Y follows b(k; 100, 0.02). Find the probability function of this binomial distribution.

(2-2) Suppose that Y follows a Poisson distribution where λ=np=2, calculate P(0≤Y≤1). Round up to 3 decimal places. Note that e-2=0.135.

2) Final examination

Question 2:
(1) Explain the Type I error and Type II error, as well as power in case of a two-sided test in testing a statistical hypothesis using drawing too. Further, explain briefly the kinds of situations where power is elevated.
(2) Explain the Type I error and Type II error, as well as power in case of a one-sided test in testing a statistical hypothesis using drawing too.

Problem 3: When services made by an AI robot was randomly chosen and examined, the service duration was as follows.
10.5 10.9 11.1 10.7 10.8
Suppose that these service durations follow with the normal distribution $N(\mu, \sigma^2)$, answer the following questions.
Note that in this case, both the mean μ and variance σ^2 are unknown.
(1) Show how to lead the confidence interval $(\hat{\mu}_L, \hat{\mu}_U)$ of the mean μ, where the confidence coefficient is β.
(2) Calculate the 95% confidence interval of the variance σ^2.
(3) Calculate the 95% confidence interval of the mean μ.
Refer to the distribution table shown in the right page.

Problem 4
In a soccer world cup, the average goal attempts per game was 15.1915 and the SD was 2.7312 for 16 top teams that advanced into the final tournament, while it was 12.2708 and 3.0773 respectively for 16 teams that failed to advance into the final tournament. Suppose that the total number of goal attempts per game for 16 teams that advanced into the final and for 16 teams that failed to do so follow $N(\mu_A, \sigma_A^2)$, $N(\mu_B, \sigma_B^2)$.
(1) Perform a two-sided test for equality of (σ_A^2=σ_B^2) of two population variances (significance level=5%).
(2) According to the test result of (1), perform a one-sided test on the difference in the population means (significance level =5%)

Note that $F_{15}^{15}(0.025) = 2.862$ and refer to the distribution table shown in the right page.

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